Dynamics and Stability AE3-914

Answers to recommended exercises—Week 2

2.1 (a) 4; (b) 6; (c) rod: 3; spring: 4; (d) 2; (e) 3; (f) 2; (g) 5; (h) 3; (i) 2; (j) 1.

2.16 $(\cos^3\theta - 2\cos\theta)\ddot{\theta} + \sin\theta(\cos^2\theta - 4)\dot{\theta}^2 + \dfrac{g}{b}\cos^3\theta(1-\sin\theta) = 0$
Of course there are other ways of simplifying the trigonometric functions.

2.17 $4ma\ddot{\theta} + (2mg - ka)\sin\theta + kl\sin\frac{\theta}{2} = 0$
Hint: Identify the isosceles triangle for which the spring is the basis. Then you can state the extension of the spring in terms of the position θ of the pendulum (that is the way you get $\theta/2$ in the formulas). Do not forget the gravity g and the natural length l of the spring in V.

2.20 Taking the angle θ between the vertical diameter and the apothem (the line joining the centre of the circumference and the midpoint of the bar) as a generalised coordinate, the Lagrangian becomes

$$L = \frac{1}{2}m\left\{\left[\frac{1}{2}a^2 - \frac{1}{12}l^2 - \left(\frac{1}{2}a^2 - \frac{1}{6}l^2\right)\cos 2\theta\right]\omega^2 + \left(a^2 - \frac{1}{6}l^2\right)\dot{\theta}^2\right\} + mg\sqrt{a^2 - \frac{1}{4}l^2}\cos\theta$$

which renders the equation of motion

$$\left(a^2 - \frac{1}{6}l^2\right)\ddot{\theta} + g\sqrt{a^2 - \frac{1}{4}l^2}\sin\theta - \left(\frac{1}{2}a^2 - \frac{1}{6}l^2\right)\omega^2\sin 2\theta = 0$$

Hint: Express the angular velocity vector of the bar in terms of ω and $\dot{\theta}$ in a moving coordinate system for which the inertia tensor is simple. Project the angular velocity on this system to find the rotational part of the kinetic energy and do not forget that the mass centre of the bar has velocity components in terms of θ, $\dot{\theta}$ and ω as well.

2.22 Take the coordinates x and y of the mass center, together with s and θ as generalised coordinates. Among the different ways of writing the kinetic energy we believe that the most adequate is

$$T = \frac{2M+m}{2}(\dot{x}^2 + \dot{y}^2) + \frac{Mm}{2M+m}\dot{s}^2 + \frac{M}{2(2M+m)}\left\{Ml^2 + m\left[(l-s)^2 + s^2\right]\right\}\dot{\theta}^2$$

because it is explicit in the generalised velocities and because it is obvious that all terms are positive. The potential energy is much easier,

$$V = \tfrac{1}{2}k(l/2 - s)^2$$

continued overleaf

Hint: The kinetic energy of a system of particles is written in terms of the velocity of the center of mass and the relative velocities of each mass by means of equation (4/4), page 275 of Meriam & Kraige's Dynamics, 4th edition. Keep in mind that the relative position of the three particles with respect to the mass center depends on the s coordinate.

2.24

$$
\begin{aligned}
&4\ddot{x} && +2b\ddot{\theta}_1\cos\theta_1 && +b\ddot{\theta}_2\cos\theta_2 && -2b\dot{\theta}_1^2\sin\theta_1 && -b\dot{\theta}_2^2\sin\theta_2 && =0\\
&2\ddot{x}\cos\theta_1 && +2b\ddot{\theta}_1 && +b\ddot{\theta}_2\cos(\theta_1-\theta_2) && +2g\sin\theta_1 && +b\dot{\theta}_2^2\sin(\theta_1-\theta_2) && =0\\
&\ddot{x}\cos\theta_2 && +b\ddot{\theta}_1\cos(\theta_1-\theta_2) && +b\ddot{\theta}_2 && +g\sin\theta_2 && -b\dot{\theta}_1^2\sin(\theta_1-\theta_2) && =0
\end{aligned}
$$

Hint: It is tough, but it is good for you to do it at least once in a life time.

2.31 Taking θ_1 as the angle between the bars and a horizontal line (notice that they are always parallel) and θ_2 as the angle between the connecting rods and a vertical line we get

$$
\begin{aligned}
\ddot{\theta}_1 &= 0\\
\ddot{\theta}_2 + \frac{g}{a}\sin\theta_2 &= 0
\end{aligned}
$$

Hint: What is the kinetic energy of a rigid body experiencing translation and rotation?