Dynamics and Stability AE3-914

Answers to recommended exercises—Week 3

2.53 The position of the collar is given by generalised coordinate s. Considering that the spring is unstretched when $s = 0$ and that this position also corresponds to the zero level of gravitational potential energy, we get

$$\begin{array}{lllllll} 3\ddot{s}\sin\theta & +2L\ddot{\theta} & & -L\dot{\phi}^2\sin 2\theta & +3g\sin\theta & & =0 \\ M\ddot{s} & +\frac{1}{2}ML\ddot{\theta}\sin\theta & & +\frac{1}{2}ML\dot{\theta}^2\cos\theta & +K_e s & +Mg & =0 \\ & & \frac{1}{6}ML^2\ddot{\phi}(1-\cos 2\theta) & +\frac{1}{3}ML^2\dot{\theta}\dot{\phi}\sin 2\theta & +K_t\phi & & =0 \end{array}$$

Hint: The bar is experiencing translation and rotation. The velocity of the mass centre depends on $\dot{s}$ too.

2.59

$$h = \sum_{k=1}^{coord} \dot{q}_k \frac{\partial L}{\partial \dot{q}_k} - L - \sum_{j=1}^{constr} \lambda_j f_j$$

with

$$\frac{\partial f_j}{\partial q_k} = a_{kj}$$

Hint: It has something to do with the way you get generalised potentials. The terms containing reaction forces are smartly added to the Lagrangian.

2.60 (a) $m\ddot{y} - m\omega^2 y + ky = 0$
Hint: Read the statement carefully. The axes are *painted* on the disc. What is the absolute velocity of the particle for an arbitrary position y, keeping in mind that the relative velocity is $\dot{y}$?

(b) $h = \frac{1}{2}m\dot{y}^2 + \frac{1}{2}(k - m\omega^2)y^2 - \frac{1}{2}md^2\omega^2$
(c) The three cases asked for are, obviously, $k > m\omega^2$, $k = m\omega^2$ and $k < m\omega^2$. The corresponding phase portraits will be a family of ellipses, horizontal lines and hyperbolas respectively, which I am not sketching here because if you see them in advance this part of the exercise becomes useless. The physical interpretation of the results would be:

$k > m\omega^2$ The mass experiences periodical motion.

$k = m\omega^2$ The mass moves with constant velocity forever (until it reaches the end of the slot)

$k < m\omega^2$ The mass drifts away from the y-axis while its absolute velocity increases (until it reaches the end of the slot, of course)

2.71 (a)

$$\begin{array}{llll} 4\ddot{x} & +l\ddot{\theta}\cos\theta & -l\dot{\theta}^2\sin\theta & = 0 \\ 3\ddot{x}\cos\theta & +2l\ddot{\theta} & -3g\sin\theta & = 0 \end{array}$$

(b)

$$\dot{\theta} = \pm\sqrt{\frac{24g(1-\cos\theta)}{l(8-3\cos^2\theta)}}$$

Hint: State the Routhian and derive the Jacobi integral from it. The integrals of motion appearing in the process can be substituted by their actual value given by the initial conditions as soon as they are explicitly present in the formulas.

2.86 Just do it. Notice that $\ddot{\theta}^2$ is wrong and it should be $\dot{\theta}^2$. The conditions for steady motion are

$$m_1 r\dot{\theta}^2 = m_2 g = k(s+r-l)$$

2.87

$$\frac{k_1 r_1 - k_2(r_2-r_1)}{m_1 r_1} = \frac{k_2(r_2-r_1)}{m_2 r_2} = \dot{\phi}^2$$

2.90

$$h = q_2\dot{q}_1^2 + m\dot{q}_1^2 + \frac{\dot{q}_2^2}{4q_2}$$

Look up in the book why this could have been predicted to be exactly equal to the Lagrangian!

Extra

Accelerations:

$$\ddot{x} = g\sin\alpha\cos\alpha \qquad \ddot{y} = -g\sin^2\alpha$$

Reaction force:

$$mg\cos\alpha \qquad \text{(not surprisingly)}$$

Hint: The equations of motion for x and y receive right hand terms with a Lagrange multiplier times the partial derivative of the constraint with respect to x and y respectively.