Dynamics and Stability AE3-914

Answers to recommended exercises—Week 5

4.23 At the represented configuration the reactions at the bearings amount to $\frac{Mr^2}{8l}\omega^2 \sin 2\alpha$. On the left bearing it is upward and on the right bearing it is downward. These reactions evolve in time as

$$R_{\text{left}} = \frac{Mr^2}{8l}\omega^2 \sin 2\alpha \cos \omega t \qquad R_{\text{right}} = -\frac{Mr^2}{8l}\omega^2 \sin 2\alpha \cos \omega t$$

Hint: Direct application of Euler equations for a body with a misalignment. Notice that the acceleration terms are zero for any time. Do not confuse action and reaction.

4.25 $\omega_3 = 39\,\mathrm{s}^{-1}$. The terms involving the square of the precession can be neglected because the angular momentum associated with $\dot\phi$ is much smaller than that associated with $\dot\psi$

4.26 The same kind of exercise, but the other way around now:

$$\dot\phi = 0.35\,\mathrm{s}^{-1}$$

4.27 For the cone we have

$$I = \frac{3}{20}ma^2 + \frac{1}{10}mh^2 \qquad I_s = \frac{3}{10}ma^2 \qquad z_G = \frac{3}{4}h \qquad \dot\phi = 2\pi/\tau$$

and the spin follows from

$$\dot\psi = \frac{(I - I_s)\dot\phi^2 \cos\theta + mgz_G}{\dot\phi I_s}$$

Hint: Derive equation (4.41) in page 223 by yourself from the Routhian in page 221 (thus, imposing the conditions for steady motion). Notice that the equation on top of page 223 is wrong and that it should be

$$\dot\theta = 0 \qquad \text{or} \qquad \theta = \text{const.}$$

4.34 See page 221 of the book and identify the error in formula (4.38).

4.73 $F = 145\,\mathrm{N}$ at each bearing. Notice that, since the sense of the turn and the spin are not given, the sense of the reactions cannot be determined.

Hint: Use Euler's equations of motion and notice that the vertical component of the angular velocity, which corresponds to the turn, is constant if viewed from an inertial reference system. This should help you to get rid of the moments of inertia not given in the problem statement. See the handout on kinematics as well.

4.74 Choose the x-axis pointing to the left, the y-axis pointing upward and the z-axis pointing forward. Call the spin rate of the turbine Ω (with the proper sign). The mass of the turbine is m and its radius of gyration k. Then the moment exerted by the turbine *on* the hull is

$$M = -mk^2\omega_b\Omega$$

Hint: Euler's equations again, but the constant component of the angular velocity is horizontal now and the sense is given ("raising the bow at steady rate ω_b"). See the handout on kinematics. The action by the turbine on the hull is asked for, not the reaction by the hull on the turbine.

4.75 Choose the same axes as in 4.74. The turbine spins at rate Ω viewed from the cockpit and the plane makes a turn to the right at a speed v with a radius r. The mass of the turbine is m and the radius of gyration is k. The gyroscopic moment exerted *on* the rotor bearings then is

$$M = -mk^2\frac{v}{r}\Omega$$

about the x-axis, which is equivalent to a pitching moment tending to rise the nose. Substitute the proper units by yourselves, because I do not know the conversion by heart.

Hint: The same kind of hints as for 4.73 and 4.74

4.76 Consider the propeller to be a slender bar in order to find the missing moments of inertia. The bending moment in the shaft is $M = -2I\omega\Omega\sin\omega t$. The moments *transmitted* to the supports are

$$M_x = -\frac{1}{2}I\Omega^2\sin 2\omega t \qquad M_y = -I\omega\Omega(1-\cos 2\omega t) \qquad M_z = I\omega\Omega\sin 2\omega t$$

Hint: Find the bending moment for a coordinate system attached to the propeller by means of Euler's equations. Keep in mind that Ω will project on the y- and z-axes. See the handout on kinematics.