Dynamics and Stability AE3-914

Answers to recommended exercises—Week 7

3.23 The equation for deflection of the beam is

$$EI\frac{d^4w}{dx^4} + \kappa w = -f(x),$$

where the upward deflection w is assumed to be positive. If you took the downward deflection as positive you should get

$$EI\frac{d^4w}{dx^4} + \kappa w = f(x).$$

Hint: The potential energy stored in the beam itself is $\frac{EI}{2}\left(\frac{d^2w}{dx^2}\right)^2$. The applied force provides a virtual work $\delta W = \int_0^l -f(x)\delta w(x)\, dx$ (for upward w positive) for which a generalised potential V_g must be found such that $\delta V_g = -\delta W$. The generalised potential V_g is then added to the potential energy V and $\delta V = 0$ is imposed.

3.24 The equation for deflection of the beam is

$$EI\frac{d^4w}{dx^4} = -f(x),$$

where the upward deflection w is assumed to be positive. The natural boundary conditions at $x = l$ are

$$\left.\frac{d^2w}{dx^2}\right|_{x=l} = 0; \qquad EI\left.\frac{d^3w}{dx^3}\right|_{x=l} = kw(l)$$

Hint: The elastic potential energy of the spring is $\frac{1}{2}k[w(l)]^2$ and it is added to the potential energy functional *outside* the integral. When taking $\delta V = 0$ keep in mind that $u(l)$ is not fixed.

3.37 Notice that they mean "the displacement $v(L, t)$". The natural boundary condition is found to be

$$\left.\frac{\partial v}{\partial x}\right|_{x=L} = 0,$$

which means that the string will always remain horizontal at the right end if no restriction is imposed to the motion there.

3.40 The constants are

$$A = \frac{F}{2m} \qquad B = \frac{2ma - F\tau^2}{2m\tau} \qquad C = 0$$

Hint: See the handout on Ritz method. The kinetic energy of a particle is, of course, $\frac{1}{2}m\dot{x}^2$. The boundary conditions $x(0) = 0$ and $x(\tau) = a$ provide $C = 0$ and $A\tau^2 + B\tau = a$. Substitution of $x(t) = At^2 + Bt + C$ in the action functional and elaboration of the integral renders a function of A, B and C. Substitute $C = 0$ and B as a function of A in the action to obtain a function of A only. Differentiate with respect to A, make equal to zero in order to find an extremum and solve for A.

3.34(a) The natural boundary condition found in Week 6 is $y'(1) = 0$. The Ritz solution with the specified shape functions is

(i) $\bar{y}(x) = 1 - \frac{5}{7}x + \frac{5}{14}x^2$

(ii) $\bar{y}(x) = 1 - \frac{252}{347}x + \frac{130}{347}x^2$

In case (i) the approximate solution is required to fulfil the natural boundary condition in advance. In case (ii) the natural boundary condition is *approximated* together with the solution to the variational problem. Both solutions are acceptable as approximation. Case (ii) is actually illustrating what is understood as natural boundary condition: the condition that provides an extremal when no other restriction is specified at the boundary. Notice that the actual value of $\bar{y}\prime(1)$ amounts to $\frac{8}{347}$ in case (ii). An approximation with more degrees-of-freedom would provide a closer approximation of the boundary condition (and of the extremal, of course).

If you have time left, you can make a plot of these two approximated solutions and compare with the exact solution given by

$$y(x) = \frac{\cosh(1 - x)}{\cosh(1)},$$

but only if you have time left.