

# Chapter 5: use of theory

ct5308 Breakwaters and Closure Dams

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March 29, 2012

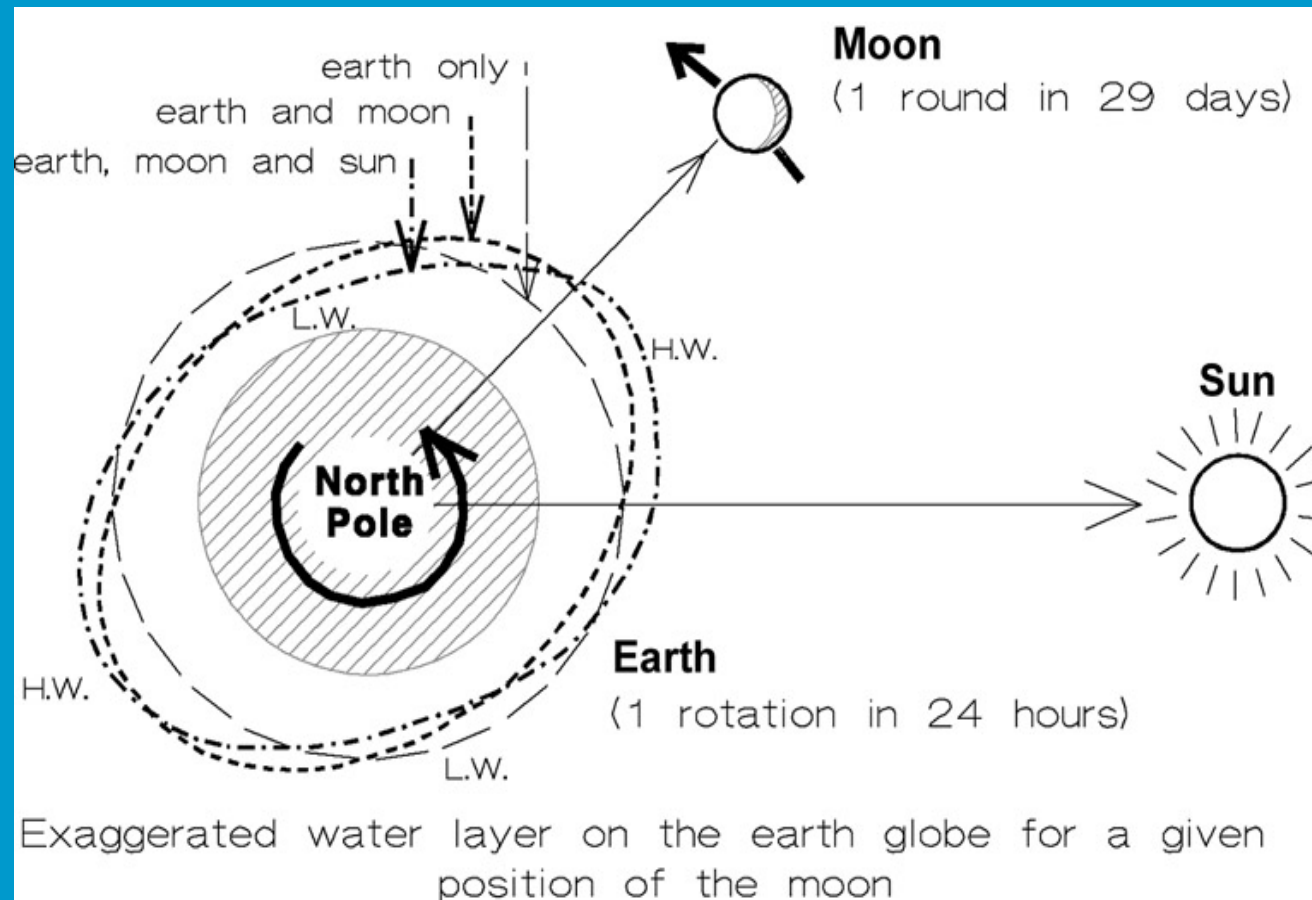


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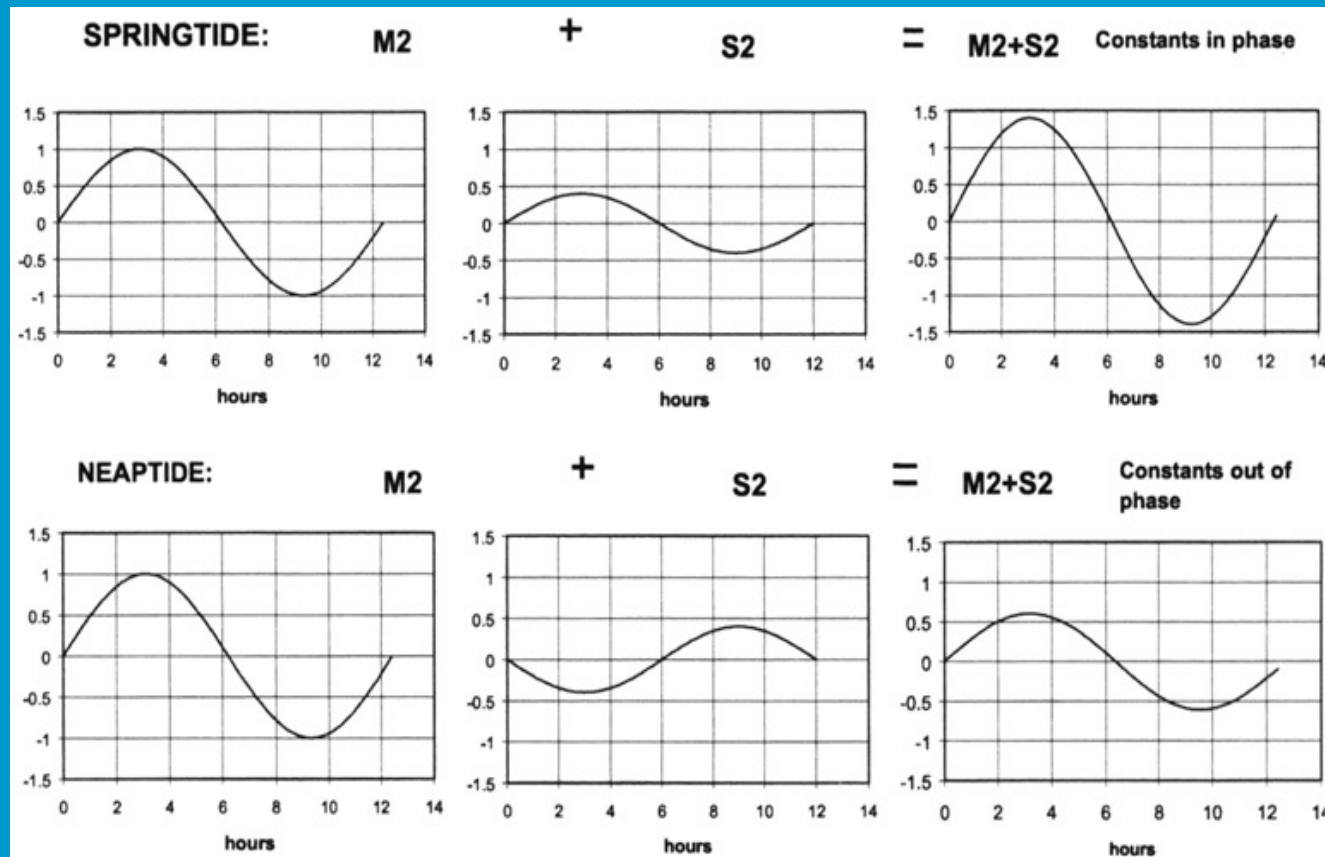
# Theoretical background needed

- waterlevels (tides)
- flow trough gaps
- stability of floating objects
- waves
  - basics
  - refraction, shoaling, breaking, diffraction, reflection
  - wave statistics
    - short term statistics (Rayleigh)
    - long term statistics
- Geotechnics
  - sliding
  - squeeze
  - liquefaction

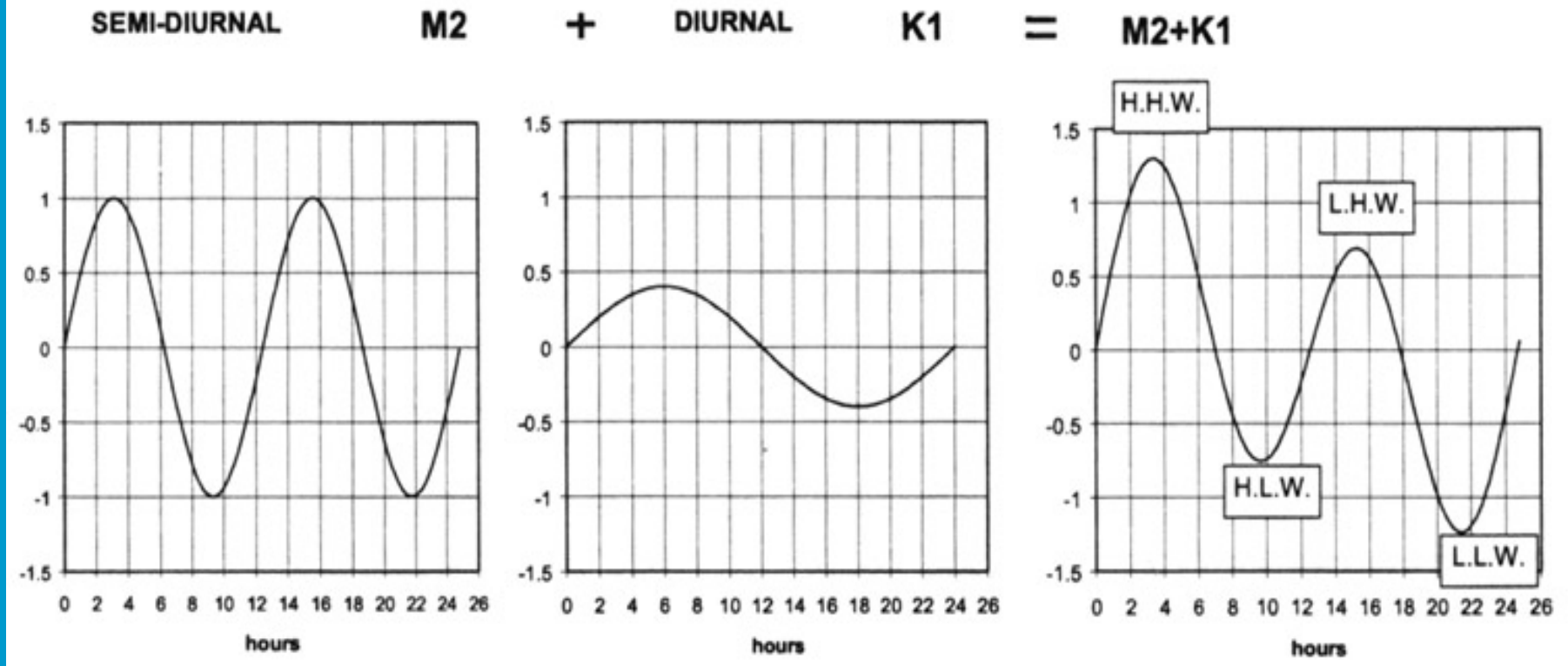
# Initial tidal wave by the moon and the sun



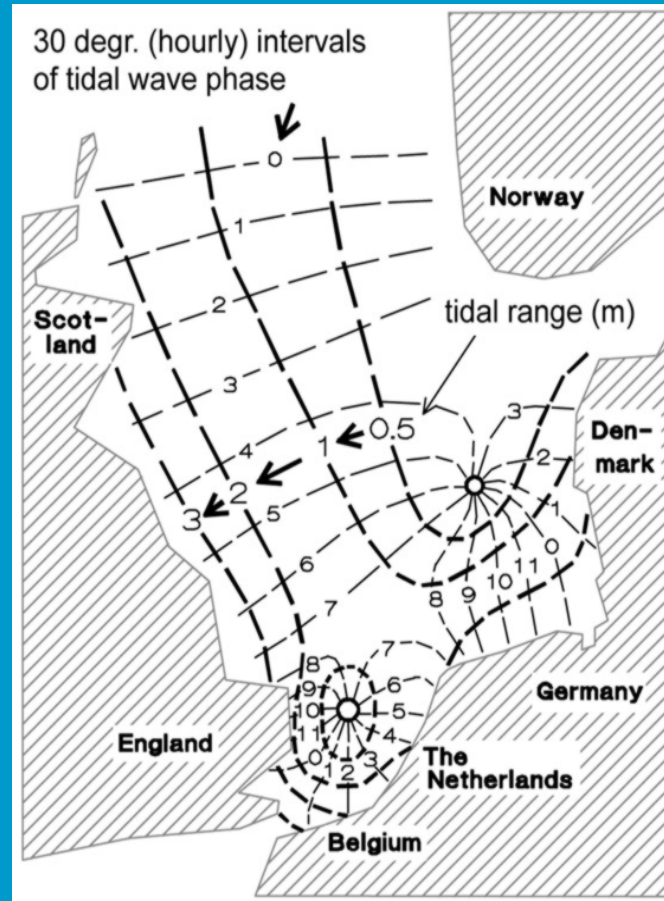
# Adding semi-diurnal constants resulting in spring and neap tide



# Adding diurnal to semi-diurnal constant

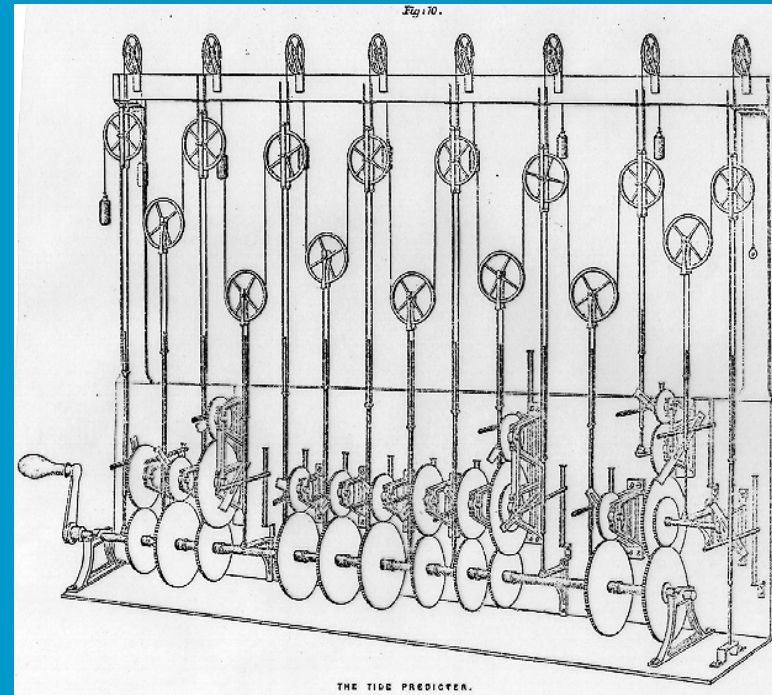
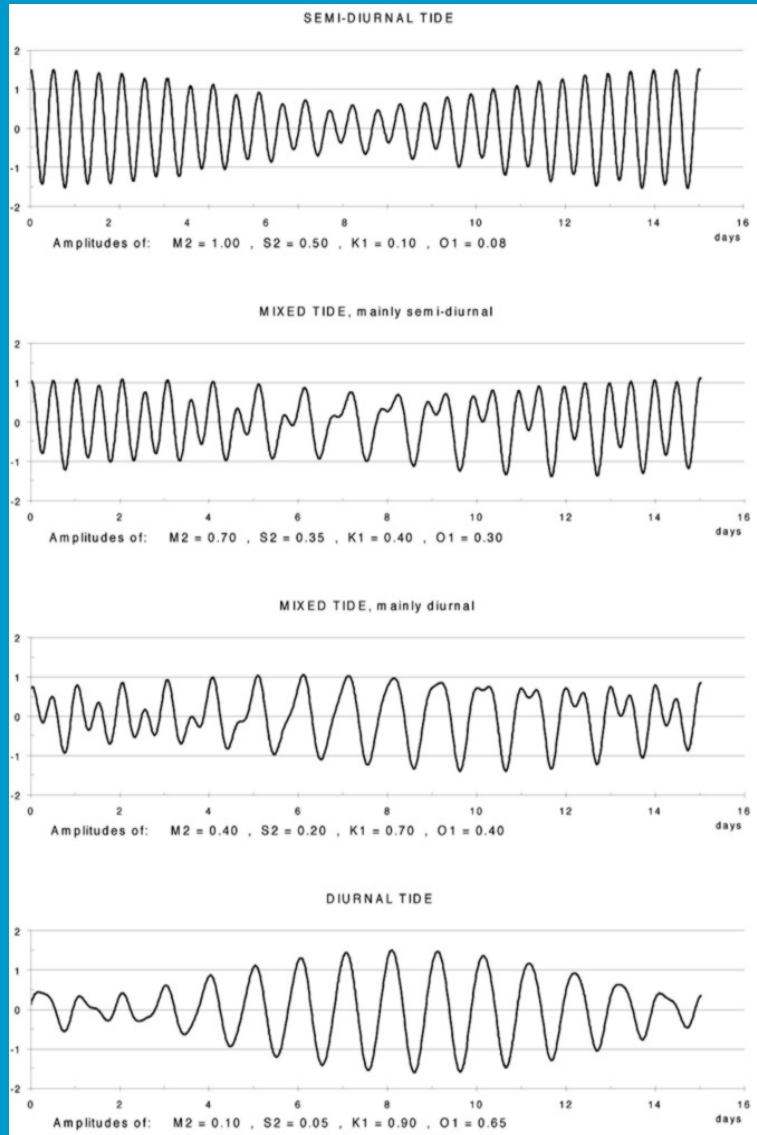


# Amphidromy in the North Sea

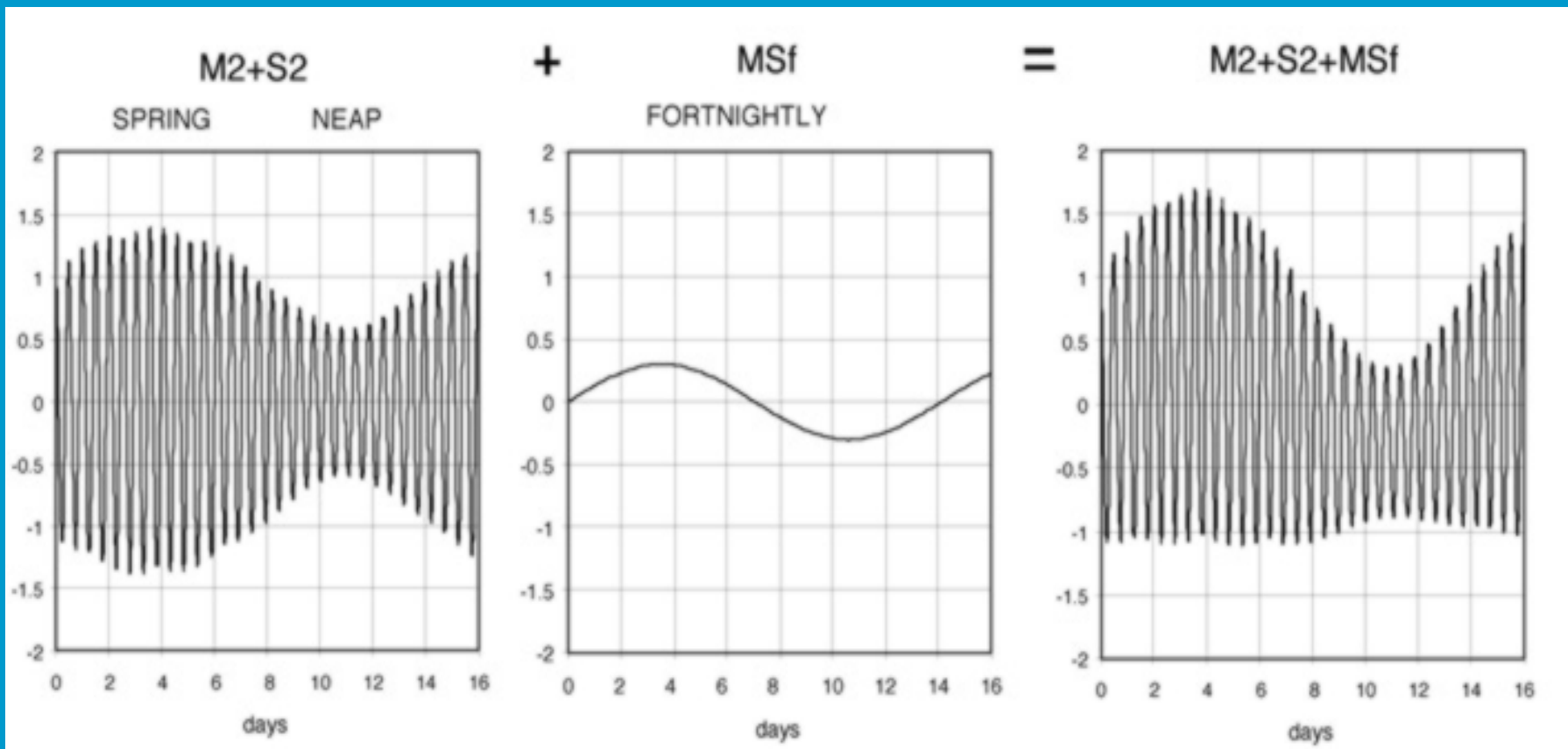




# typical tides

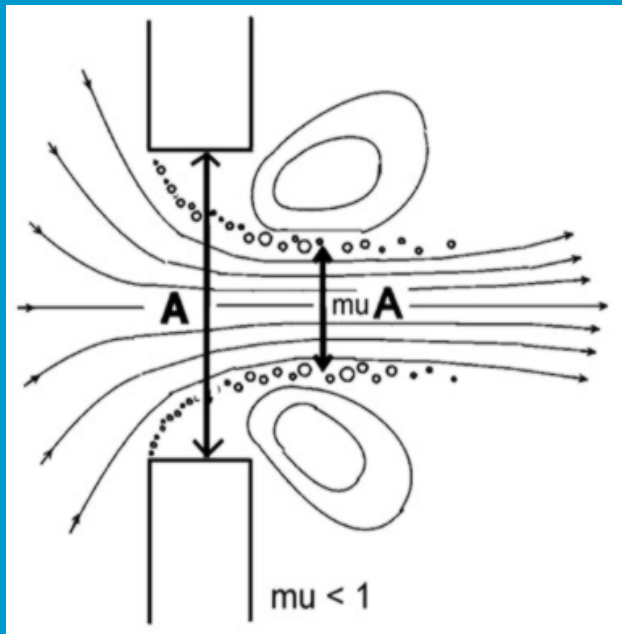


# adding the fortnightly constant

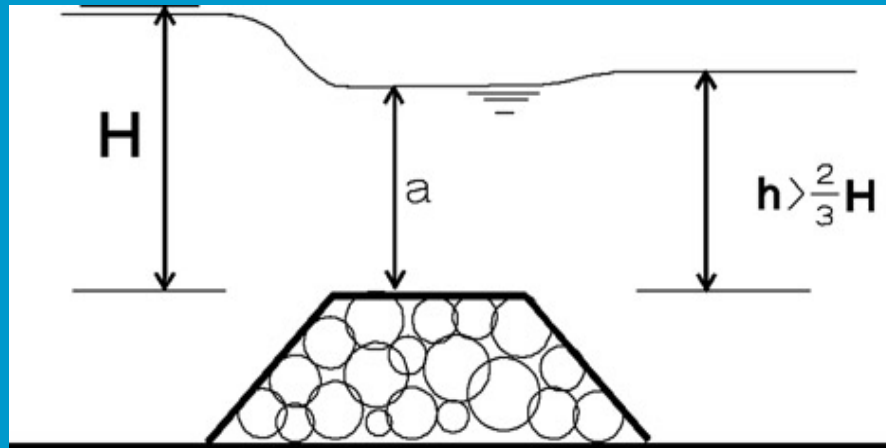




# flow pattern in a gap



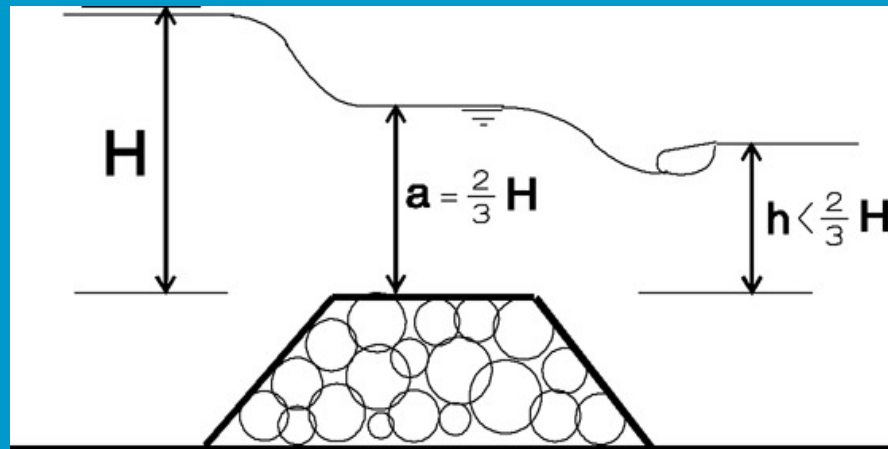
# Flow over a sill



## subcritical flow

$$Q = m B h \sqrt{2g(H-h)}$$

$$u = \frac{Q}{B a} = m \frac{h}{a} \sqrt{2g(H-h)}$$



## critical flow

$$Q = m B a \sqrt{2g \frac{1}{3} H}$$

$$Q = m B \left(\frac{2}{3} H\right) \sqrt{2g \left(\frac{1}{3} H\right)}$$

$$u = m \sqrt{2g \left(\frac{1}{3} H\right)}$$

# modelling

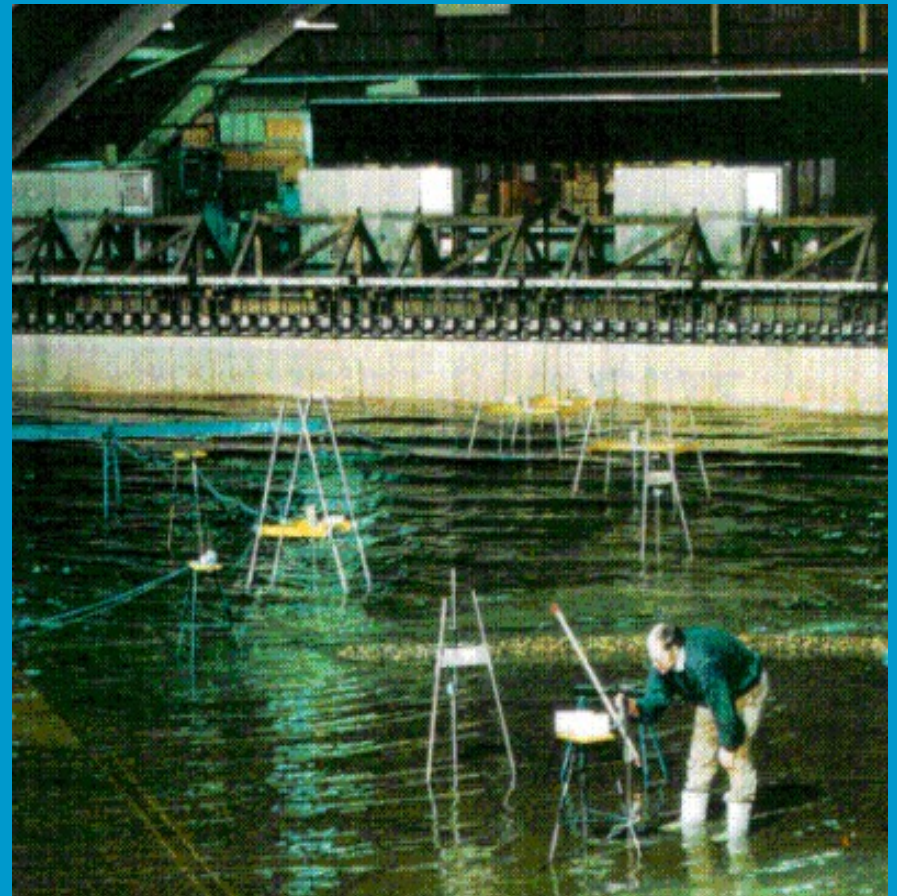
$$\frac{\delta Q}{\delta x} + B_x \frac{\delta H}{\delta t} = 0$$

$$\frac{\delta Q}{\delta t} + \frac{\delta(\alpha Q u)}{\delta x} + g A \frac{\delta H}{\delta x} + \frac{g |Q| Q}{C^2 A R} - W_x = 0$$

Solving these equation by:

- physical model
- mathematical model
  - 2 d model
  - 1 d model
  - storage area approach

# Physical model

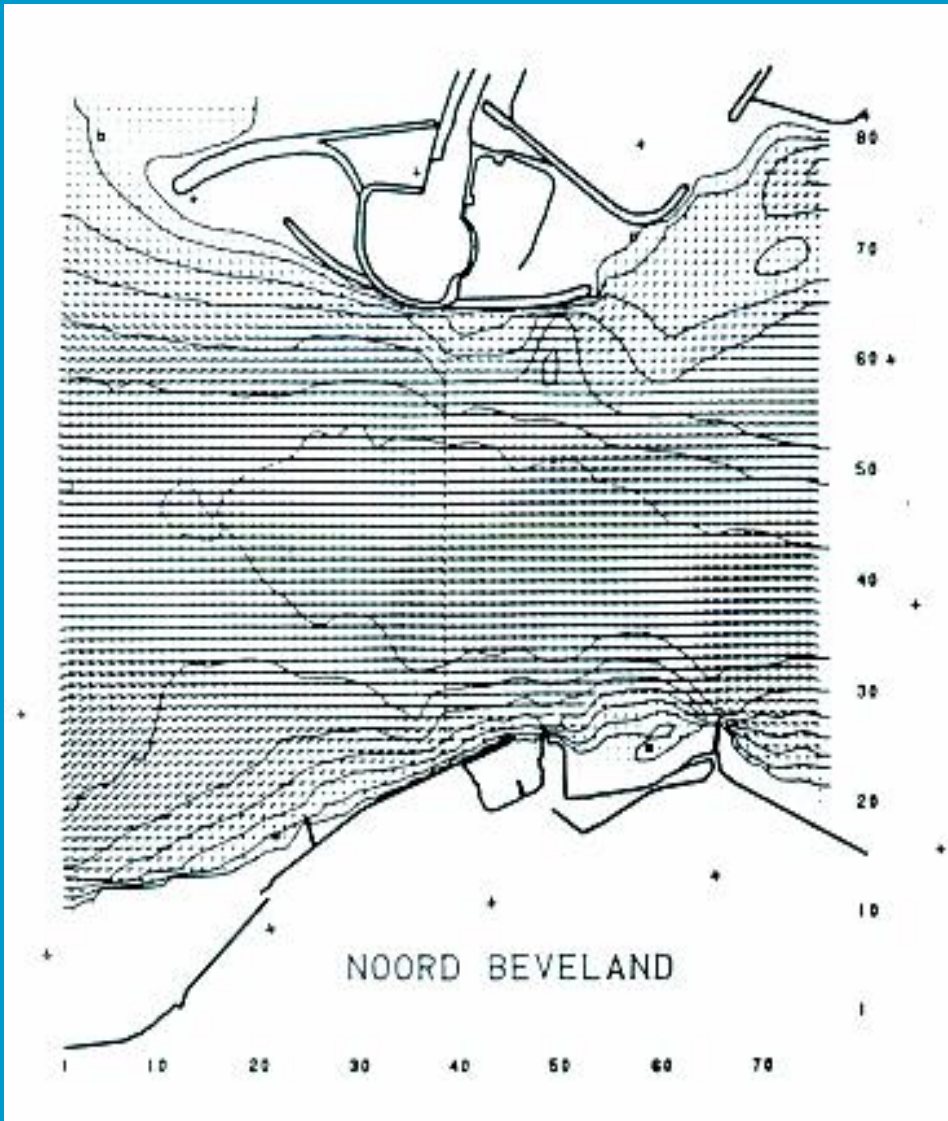


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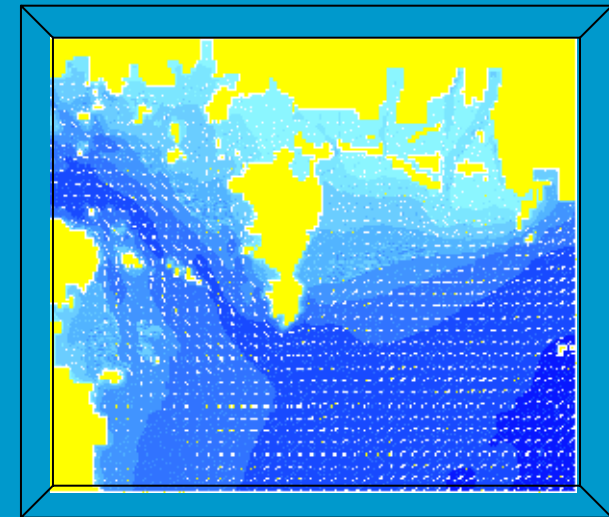
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# two dimensional model

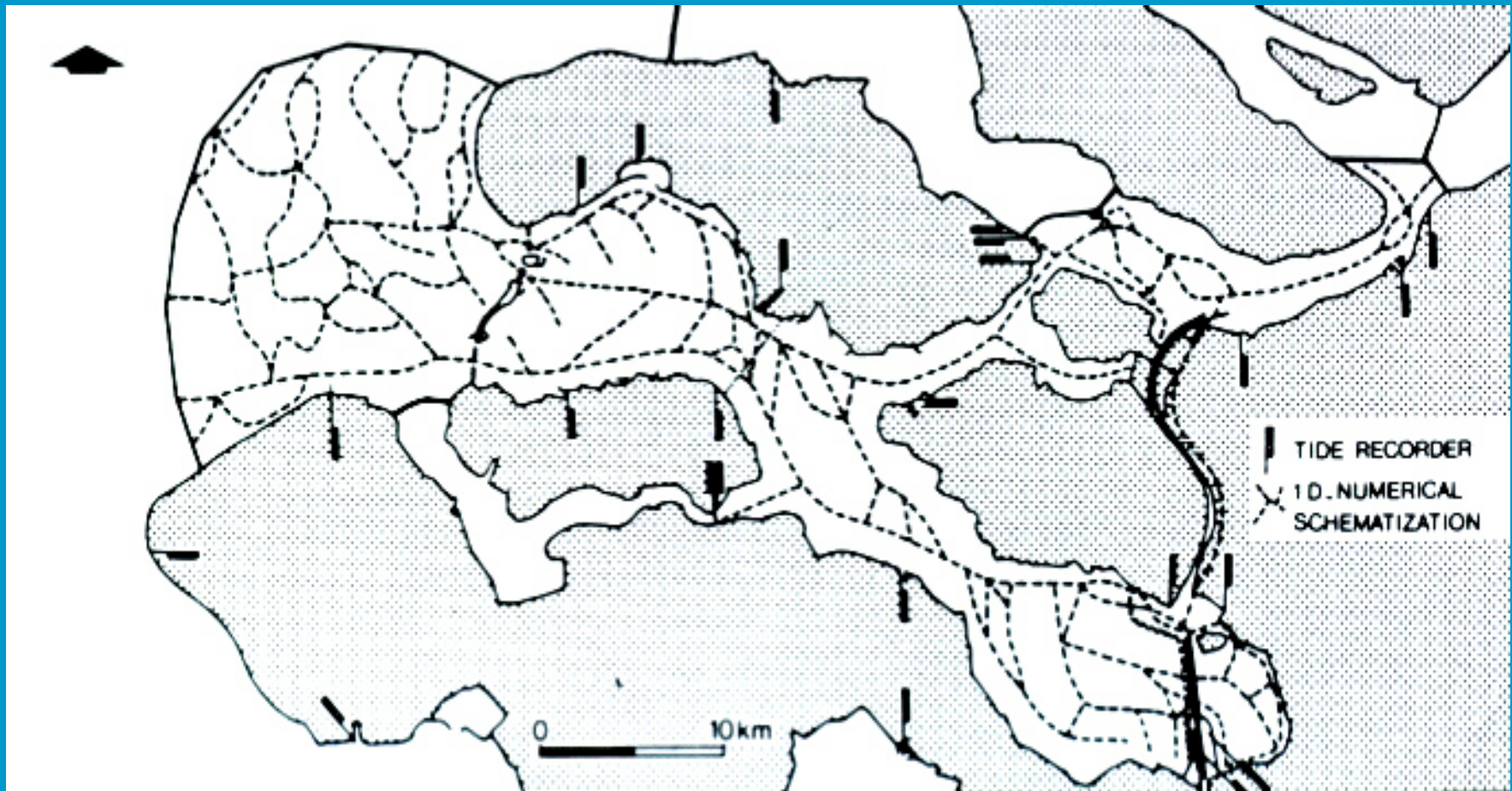


Oosterscheldewerken, Waqua, Rijkswaterstaat/WL



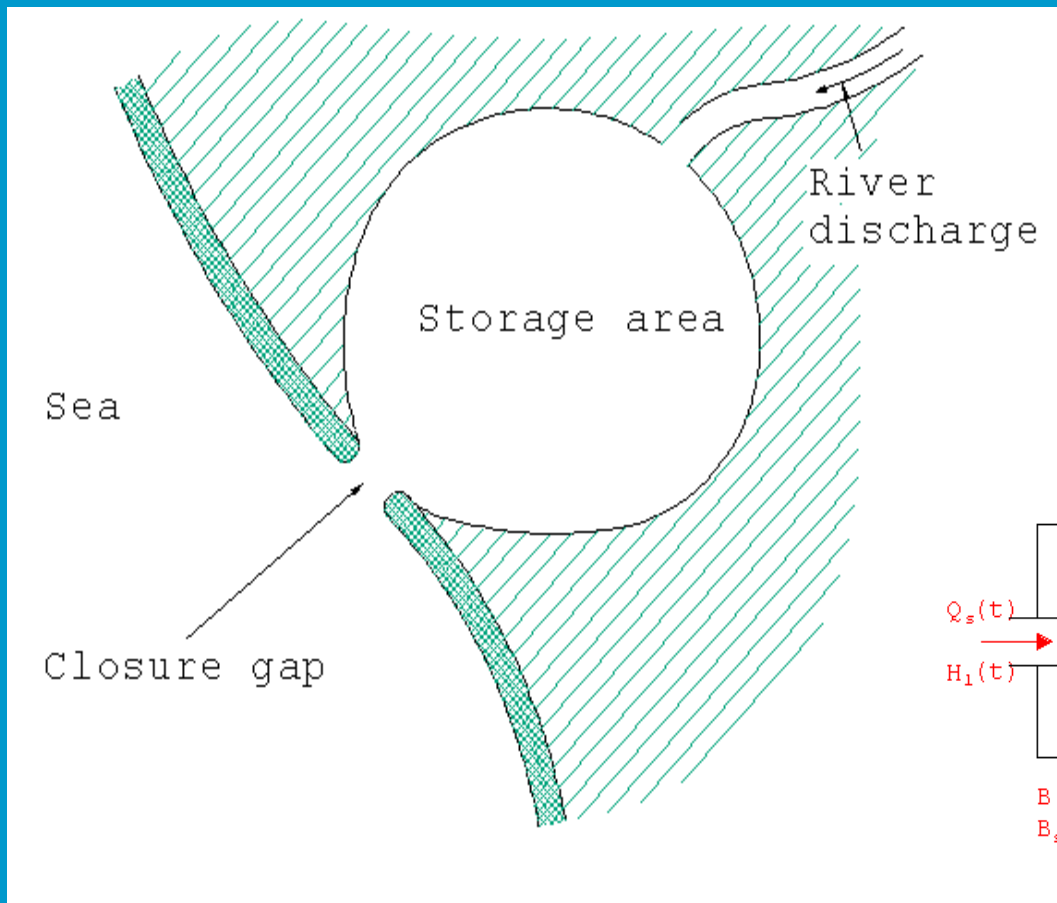
Korea, Gaduk port, Mike21, DHI

# one-dimensional model

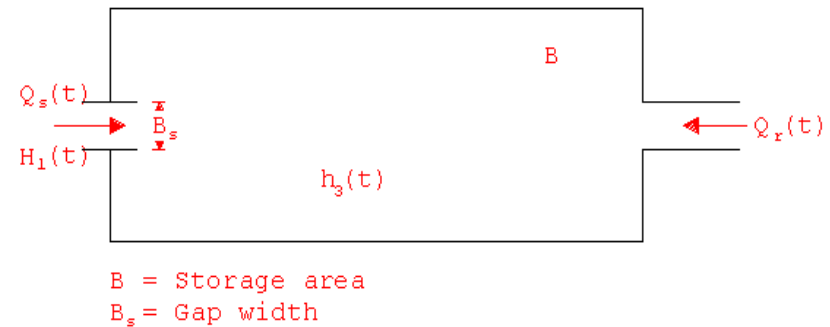




# storage/area approach



$$\frac{\delta Q}{\delta x} + B_x \frac{\delta H}{\delta t} = 0$$



# validity of storage/area approach

$$\begin{aligned}\text{length of tidal wave: } L &= c * T = \sqrt{gh} * T \\ &= \sqrt{10 * 10} * 12 * 3600 \\ &= 432 \text{ km}\end{aligned}$$

$$\text{basin} < 0.05 L = 20 \text{ km}$$

# equations for storage/area approach

$$\mu A_g \sqrt{2g(H_1 - h_2)} = B \frac{dh_3}{dt} - Q_R(t)$$

$$h_2 = h_3 \quad \text{for } h_3 > \frac{2}{3}H_1$$

$$h_2 = \frac{2}{3}H_1 \quad \text{for } h_3 < \frac{2}{3}H_1$$

$A_g$  and  $B$  can be combined to one input parameter

# parameters needed

- water level in the sea
- river discharge
- ratio between storage area and width of closure gap
- sill height
- discharge coefficient of the gap

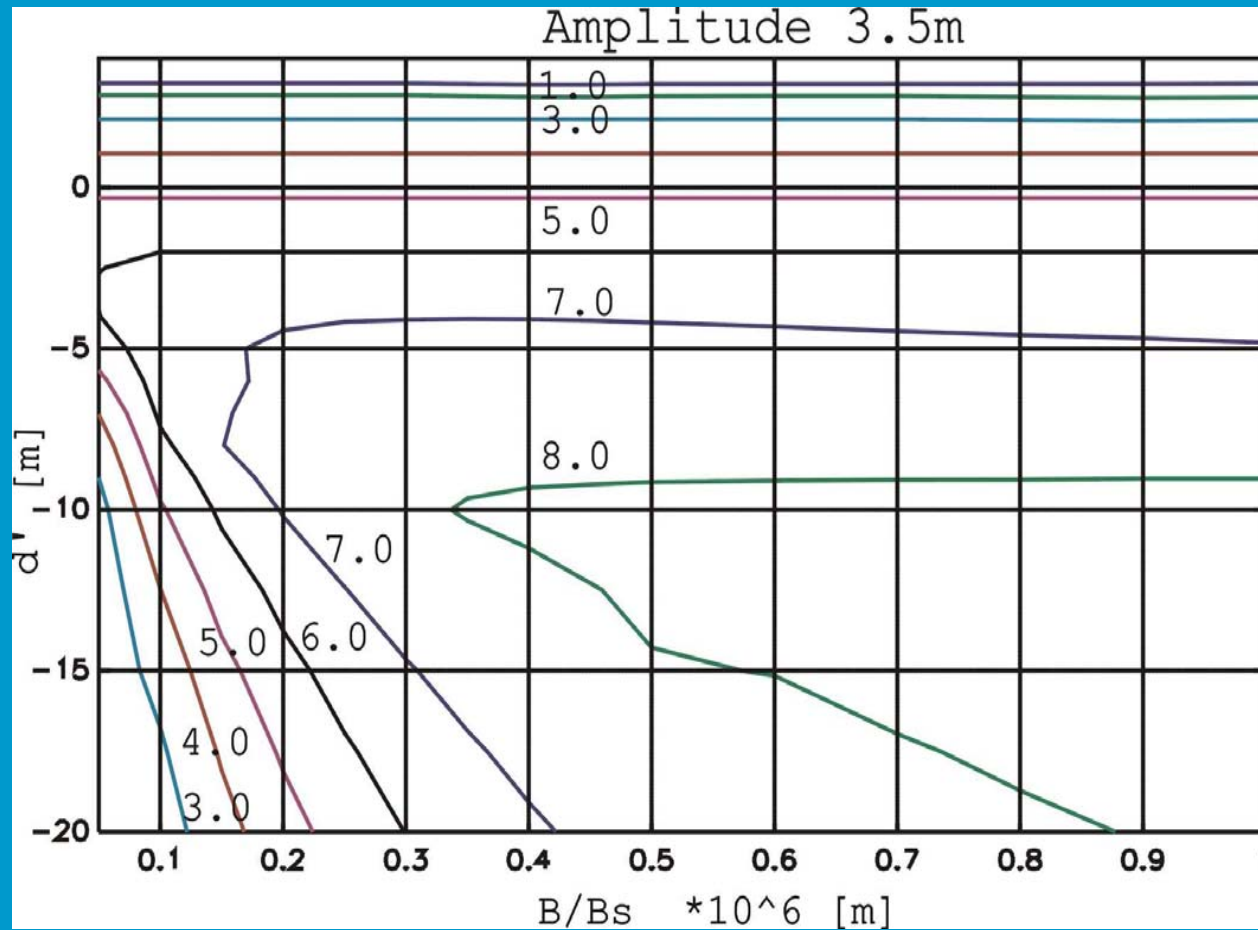
Assume for the time being that the river discharge is zero and that the tide is always semi-diurnal

Set the discharge coefficient of the gap to 1

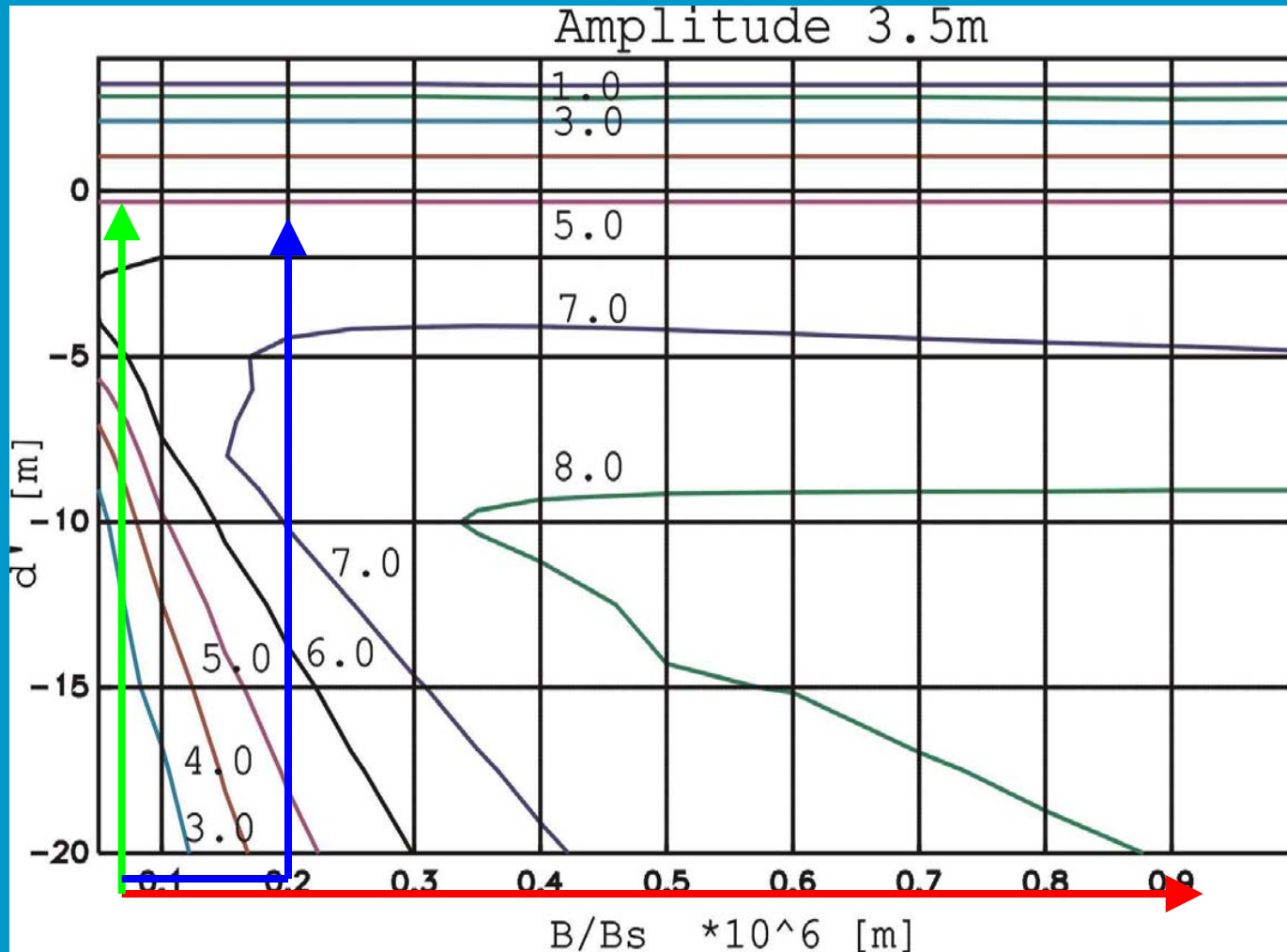
Remaining parameters:

- tidal difference
- ratio storage area/gap width
- sill height

# design graph for the velocity

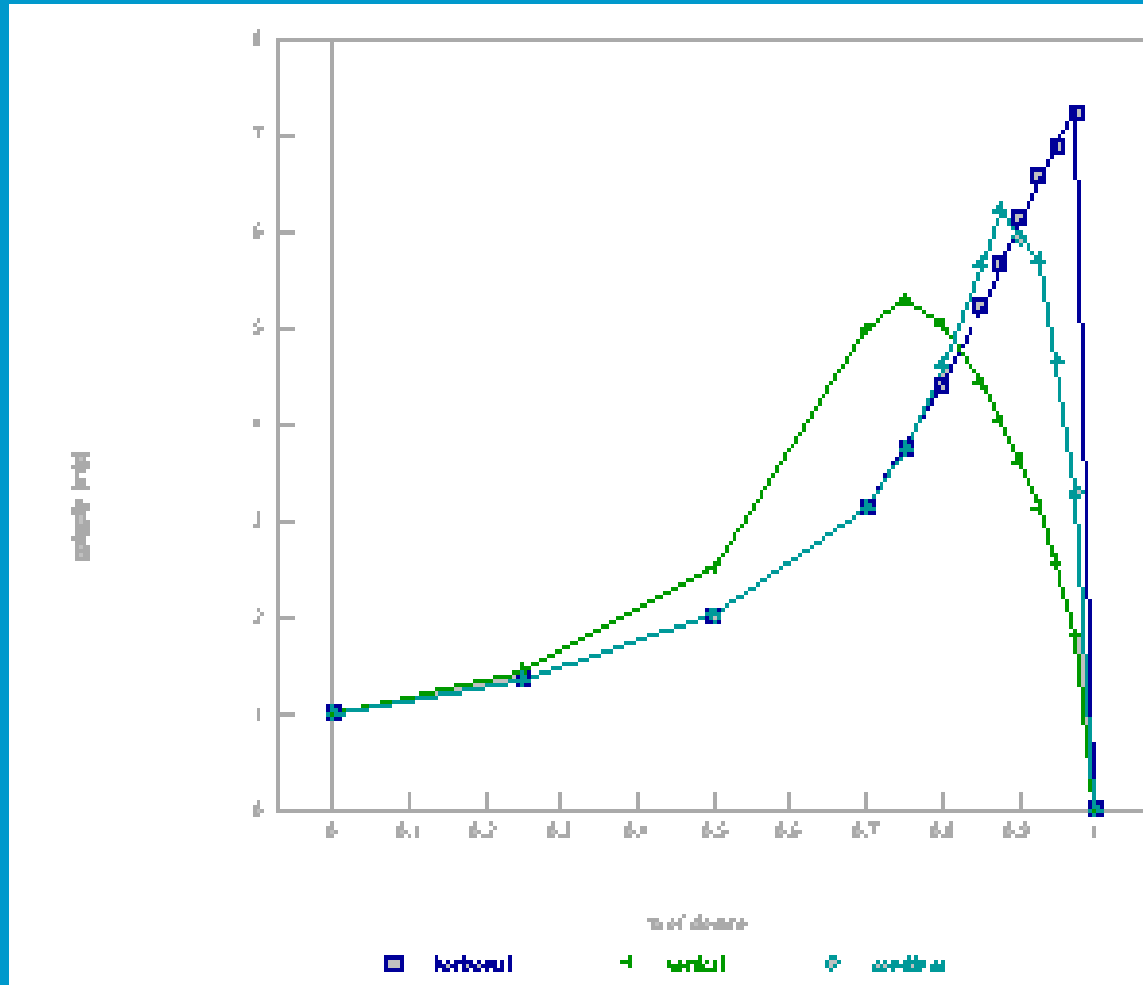


# example of the use of a design graph

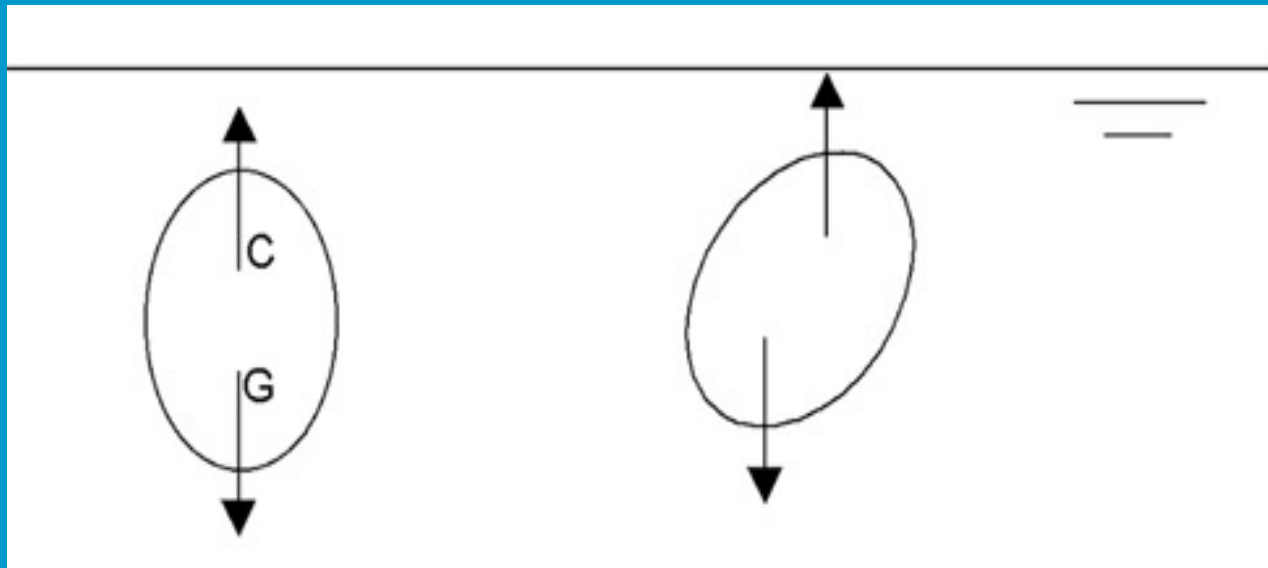




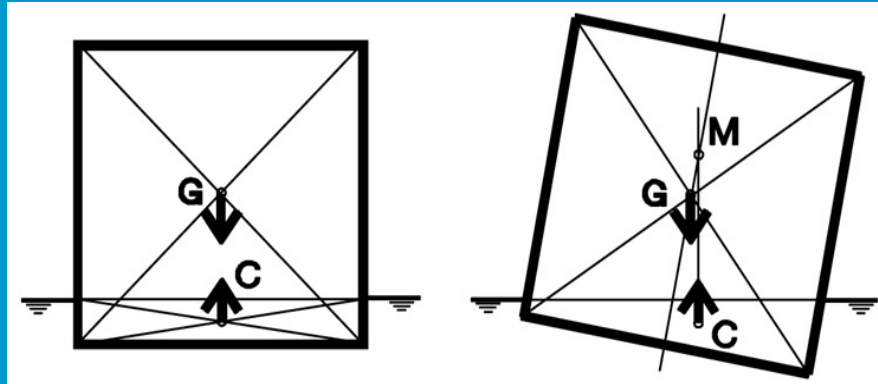
# velocity as a function of the closure



# Stability of a submerged object



# Stability of a floating object

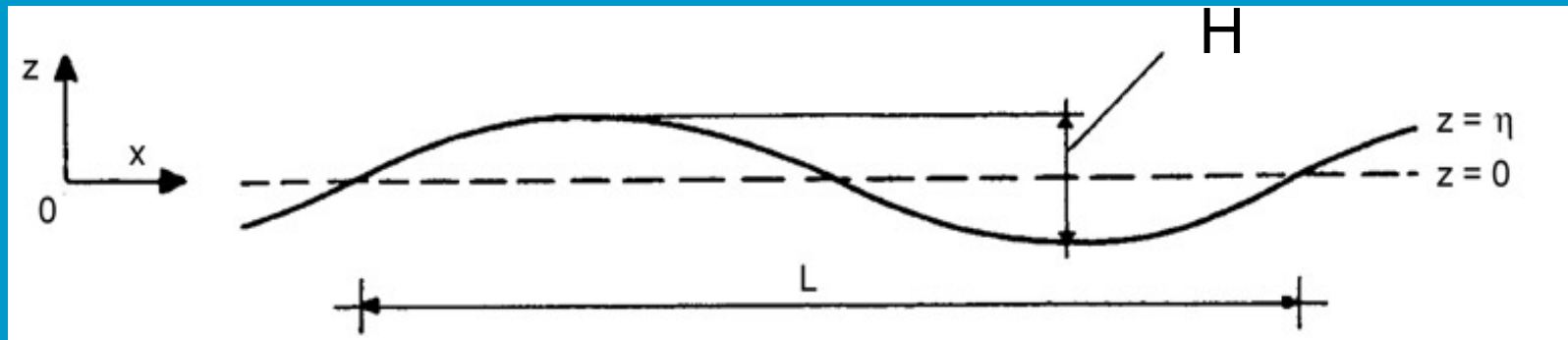


$$MC = \frac{I}{V}$$

$$I = \int_{-0.5b}^{+0.5b} yx^2 dx = \frac{1}{12} LB^3$$

$$V = \frac{G}{\rho g}$$

# Definition of a regular wave



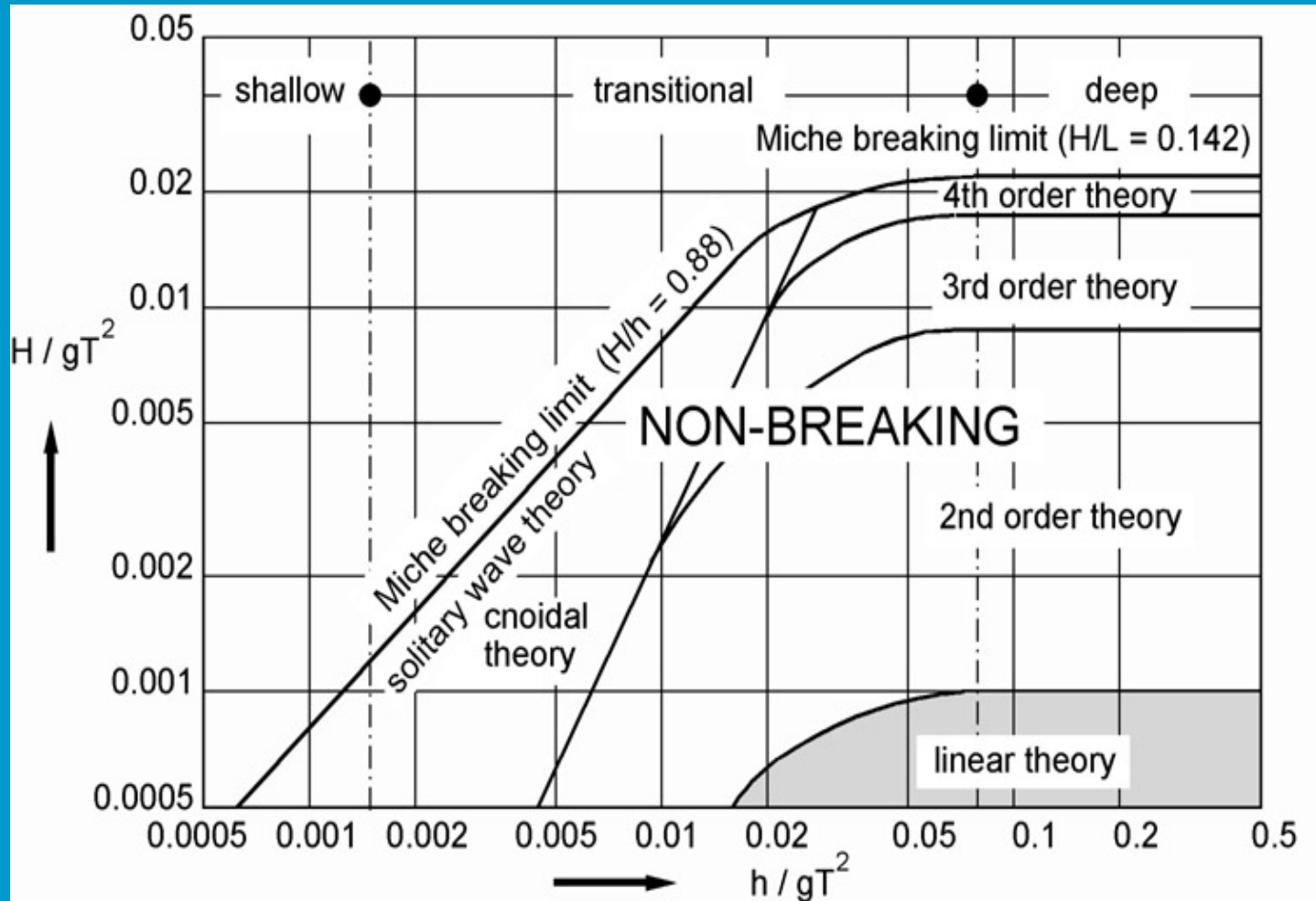
H wave height  
T wave period  
L wave length

$$\eta = \frac{H}{2} \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$

$$c = \sqrt{\frac{gL}{2\pi} \tanh\left(\frac{2\pi h}{L}\right)}$$

$$L_0 = \frac{gT^2}{2\pi} = 1.56T^2 \quad c = \sqrt{gh}$$

# validity for wave theories

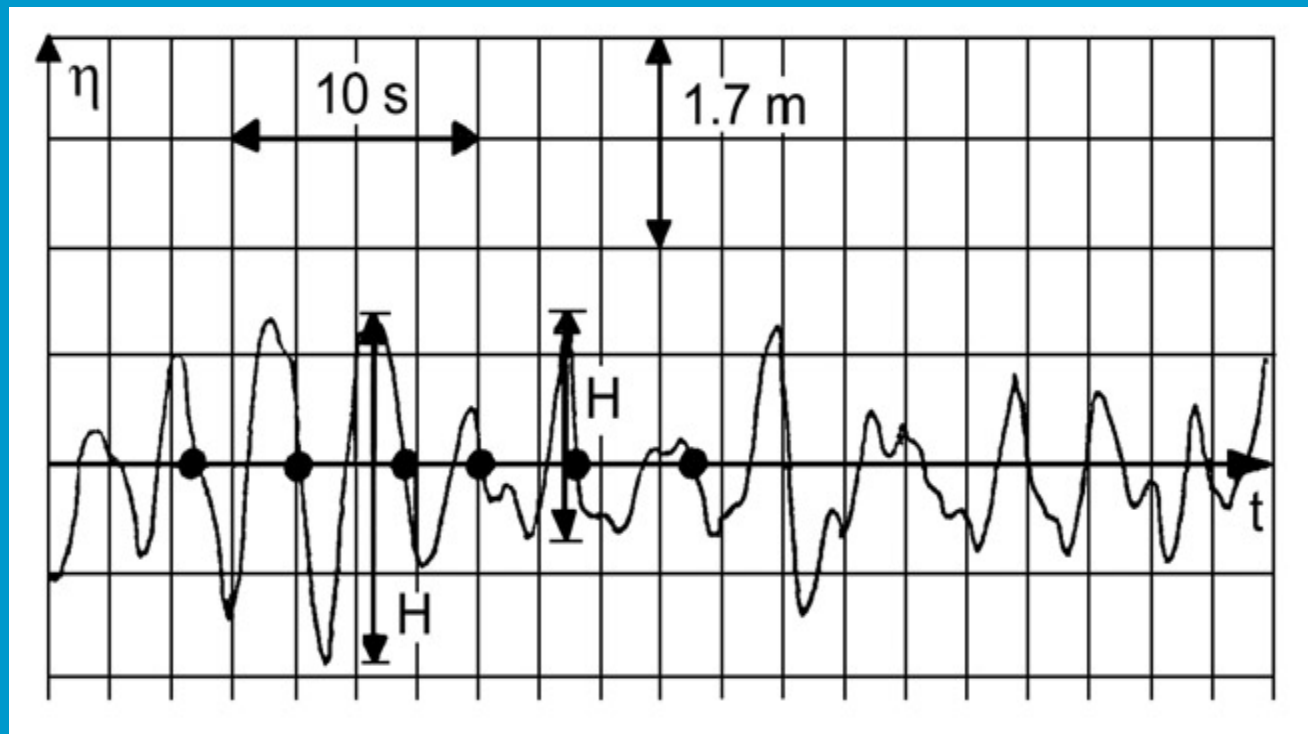


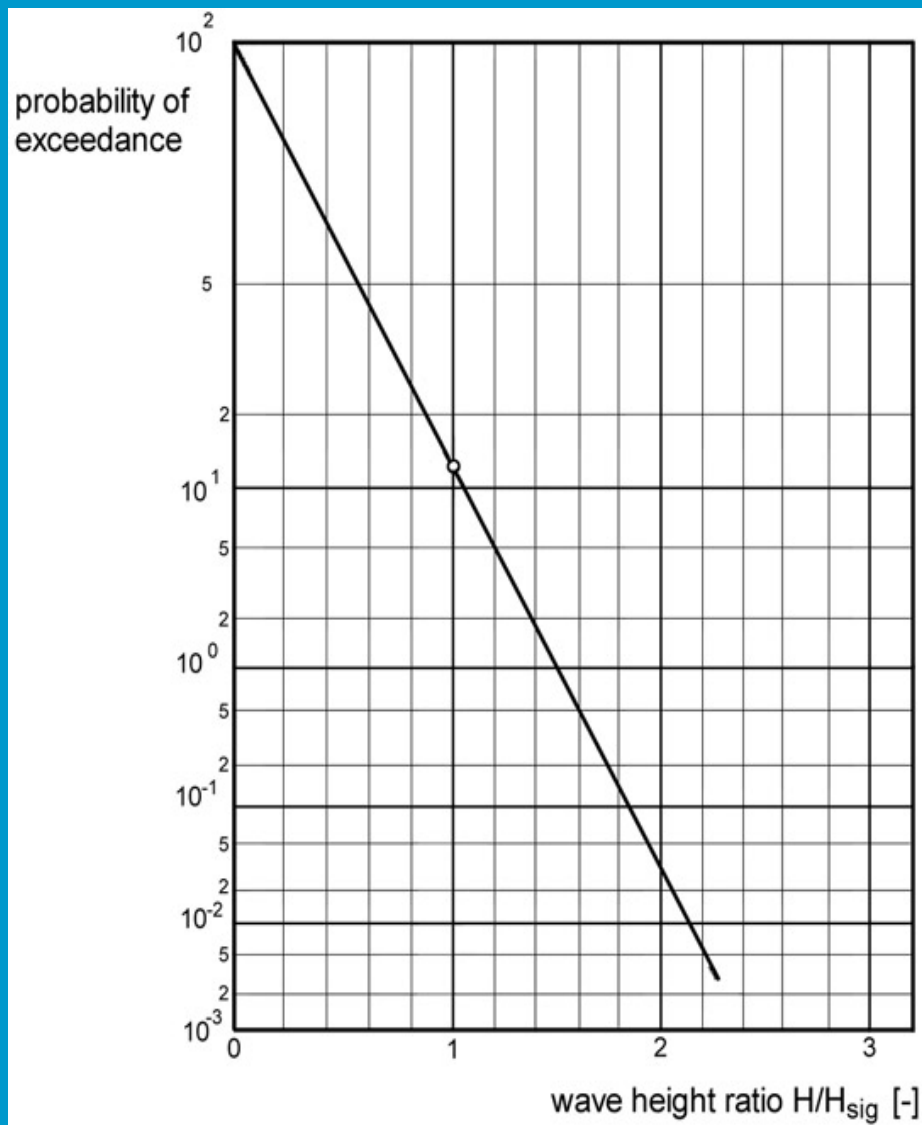
# breaking

by steepness  $H/L < 0.14$   
by depth  $H/h < 0.78$  but.....



# Irregular wave





# Rayleigh graph paper

$$P(\underline{H} > H) = e^{\left[-2\left(\frac{H}{H_s}\right)^2\right]}$$

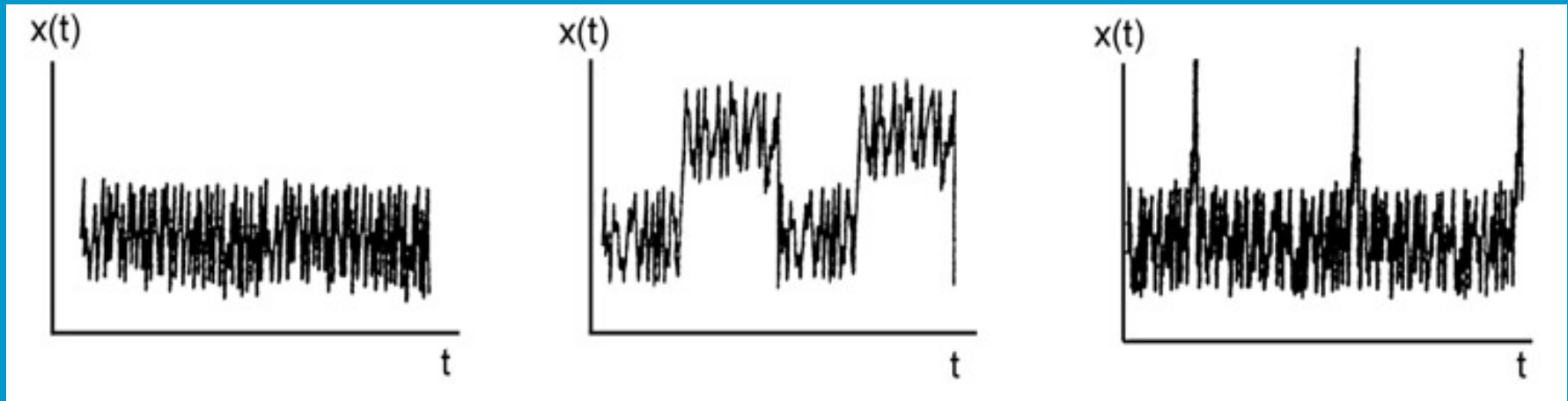
# characteristic wave heights

| Name                            | Notation                   | $H/\sqrt{m_0}$  | $H/H_s$ |
|---------------------------------|----------------------------|-----------------|---------|
| Standard deviation free surface | $\sigma_\eta = \sqrt{m_0}$ | 1               | 0.250   |
| RMS height                      | $H_{rms}$                  | $2\sqrt{2}$     | 0.706   |
| Mean Height                     | $H = H_1$                  | $2\sqrt{\ln 2}$ | 0.588   |
| Significant Height              | $H_s = H_{1/3}$            | 4.005           | 1       |
| Average of 1/10 highest waves   | $H_{1/10}$                 | 5.091           | 1.271   |
| Average of 1/100 highest waves  | $H_{1/100}$                | 6.672           | 1.666   |
| Wave height exceeded by 2%      | $H_{2\%}$                  |                 | 1.4     |

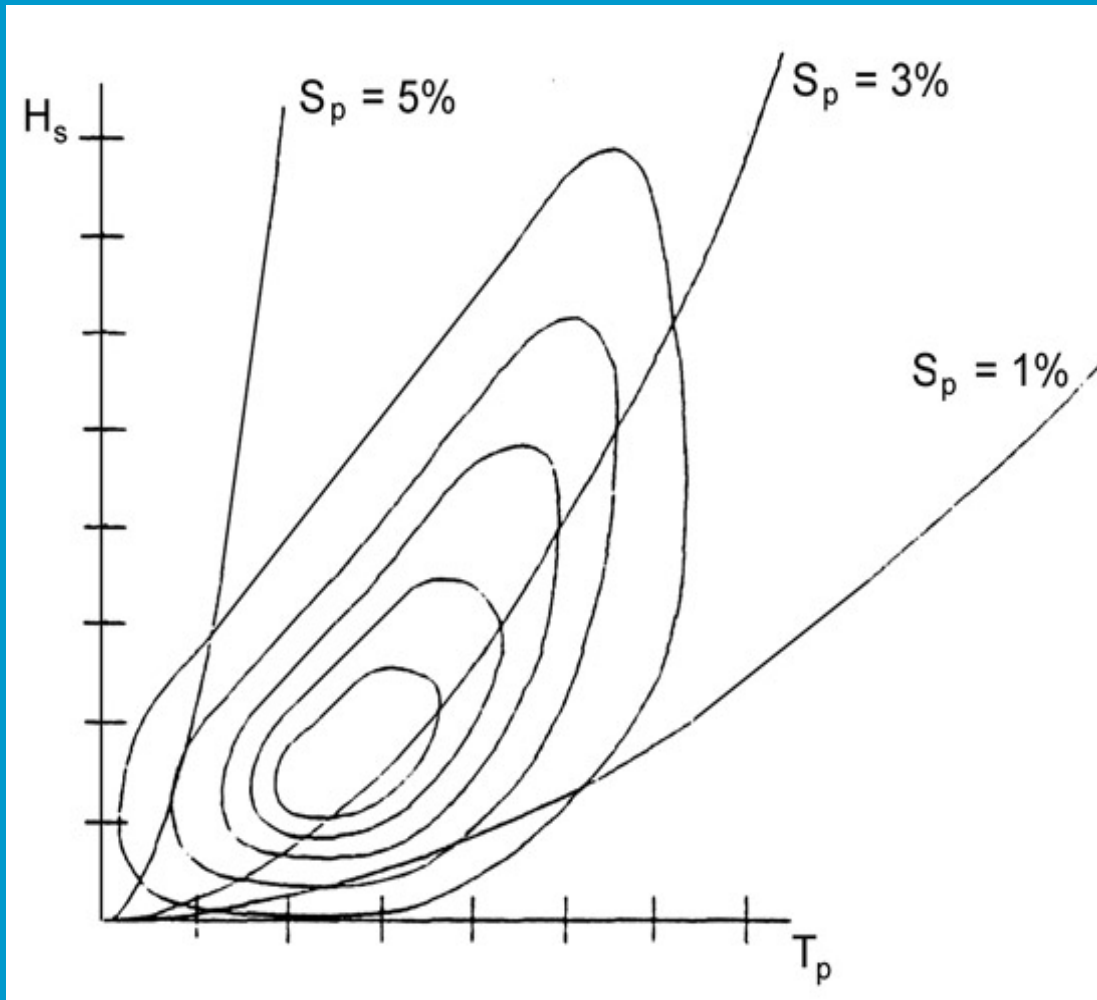
# characteristic wave periods

| Name               | Notation | Relation to spectral moment | $T/T_p$      |
|--------------------|----------|-----------------------------|--------------|
| Peak period        | $T_p$    | $1/f_p$                     | 1            |
| Mean period        | $T_m$    | $\sqrt{m_0/m_2}$            | 0.75 to 0.85 |
| Significant period | $T_s$    |                             | 0.9 to 0.95  |

# typical types of wave statistics patterns



# H/T-diagram





# waves in shallow water

shoaling  
refraction  
breaking  
diffraction  
reflection

$$\frac{H}{H_0} = \sqrt{\frac{1}{\tanh(2\pi h/L)} + \frac{1}{1 + \frac{(4\pi h/L)}{\sinh(4\pi h/L)}}} = k_{sh}$$

# the iribarren number (surf similarity parameter)

$$\xi = \frac{\tan \alpha}{\sqrt{H/L_0}}$$

$\tan \alpha$  slope of the shoreline/structure  
 $H$  wave height  
 $L_0$  wave length at deep water

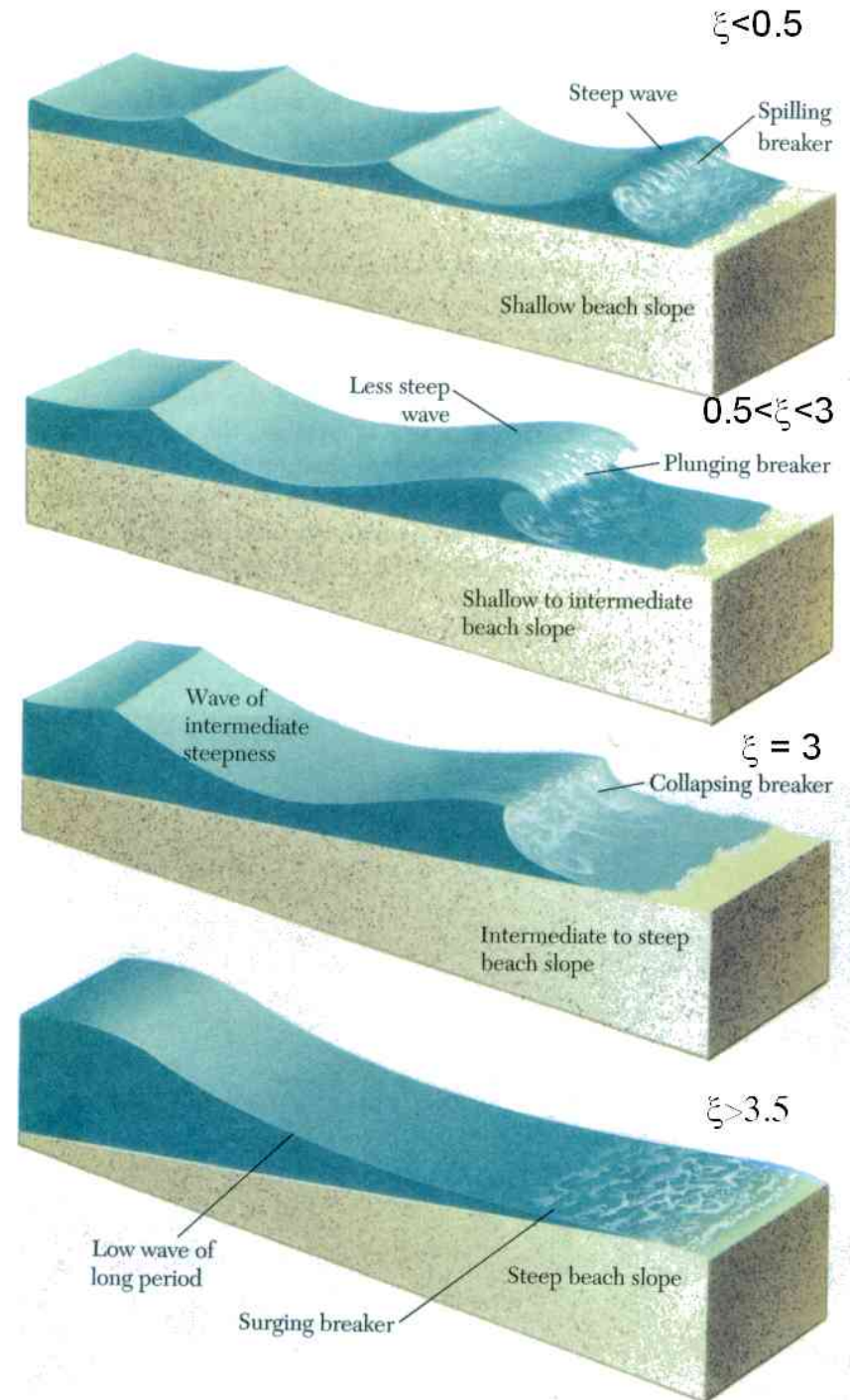
# breaker types (2)

spilling  $\xi < 0.5$

plunging  $0.5 < \xi < 3$

collapsing  $\xi = 3$

surging  $\xi > 3$



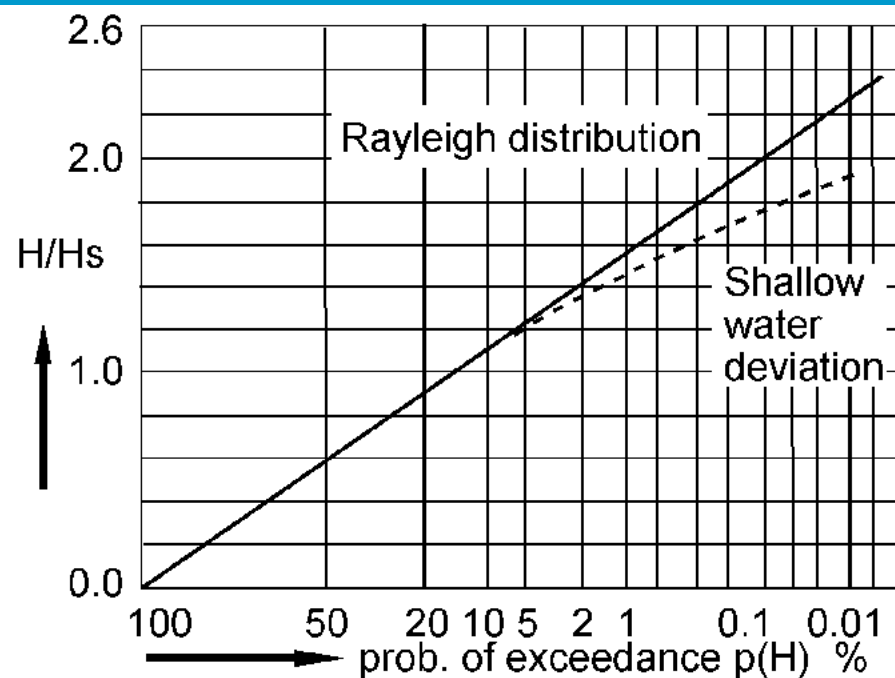
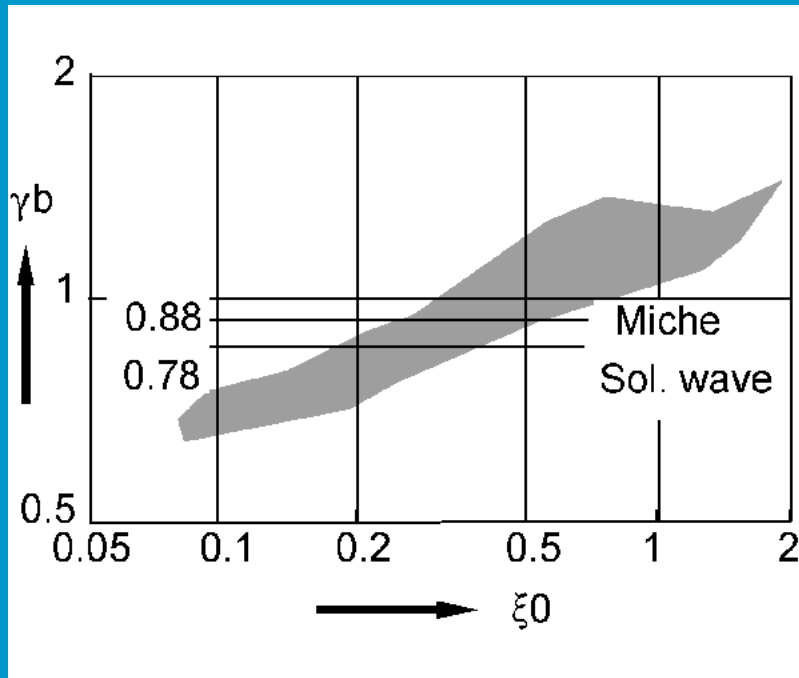
# breaking waves

$$H_b = 0.142 L \tanh\left(\frac{2\pi}{L} h\right)$$

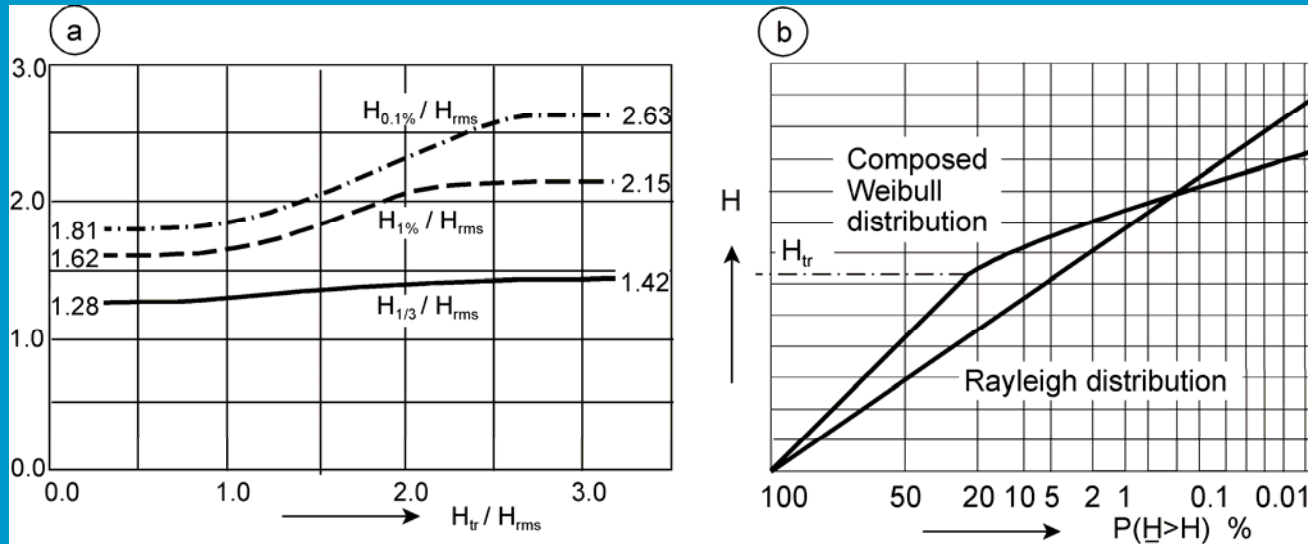
$$\frac{H_b}{h} \approx 0.78 \text{ (solitary wave)}$$

$$\frac{H_s}{h} \approx 0.4 - 0.5$$

# change of distribution in shallow water

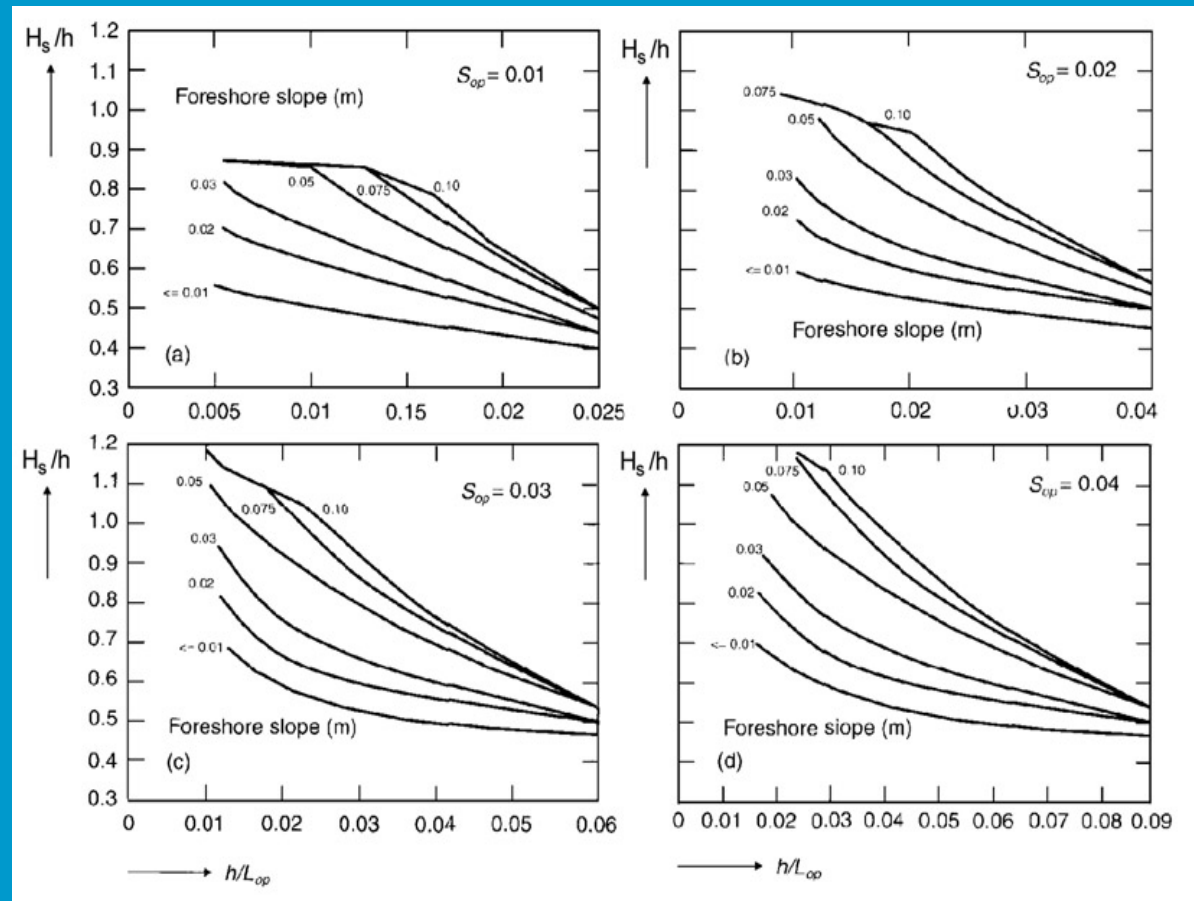


# Battjes Jansen method

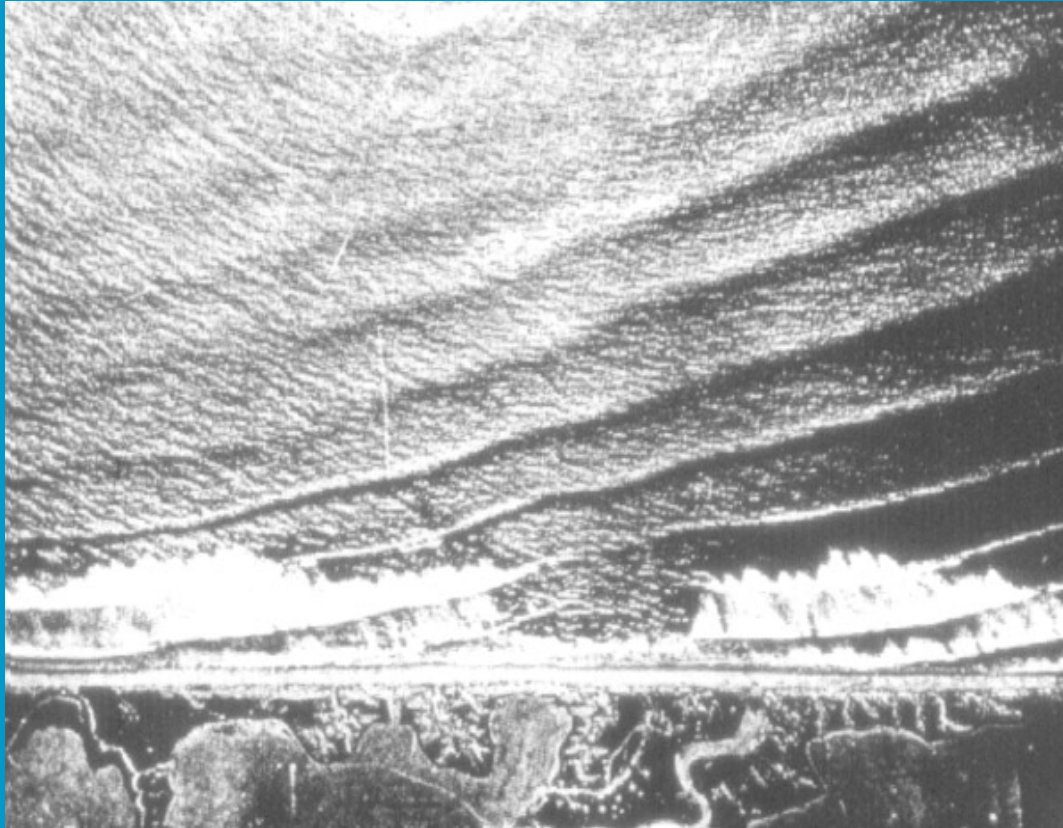


$$\Pr \{ \underline{H} \leq H \} = \begin{cases} F_1(H) = 1 - \exp \left[ - \left( \frac{H}{H_1} \right)^2 \right] & H \leq H_{tr} \\ F_2(H) = 1 - \exp \left[ - \left( \frac{H}{H_2} \right)^{3.6} \right] & H > H_{tr} \end{cases}$$

# Influence of shallow water on the wave height



# Wave refraction



$$\sin \alpha_2 = \left( \frac{c_2}{c_1} \right) \sin \alpha_1$$

$$\frac{H_2}{H_1} = \sqrt{\frac{b_1}{b_2}}$$



# Diffraction behind a detached breakwater

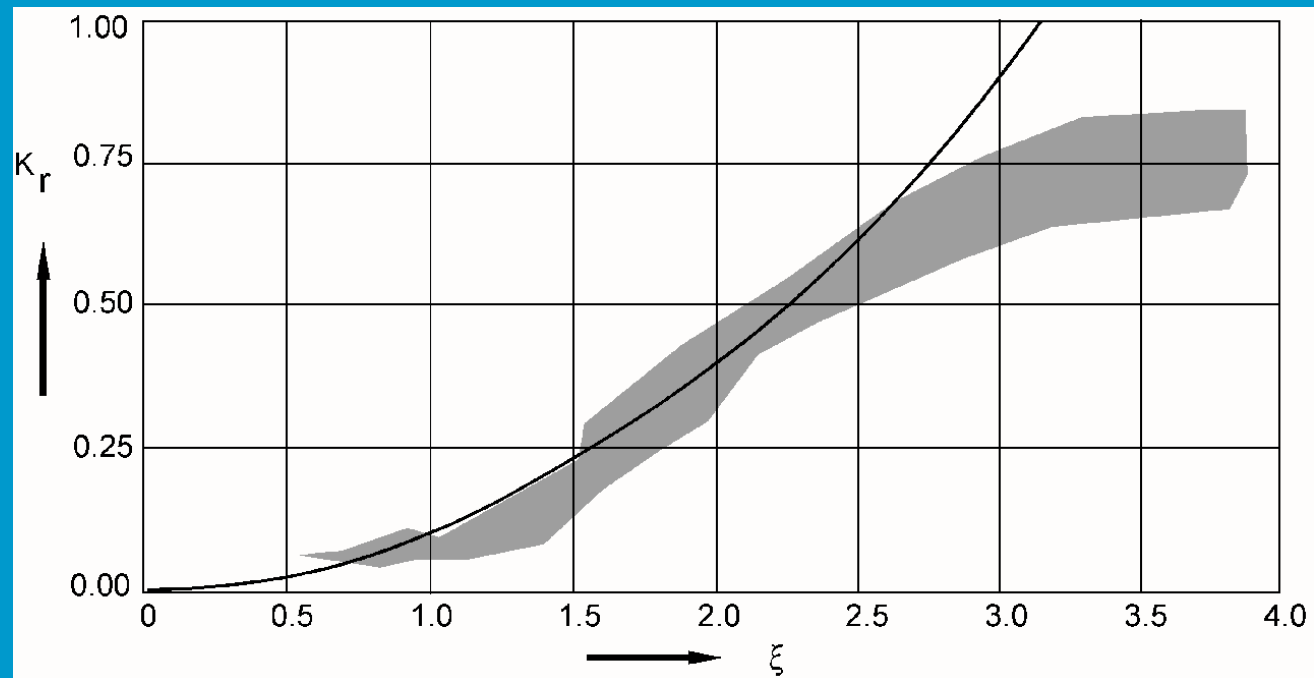


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# reflection

$$K_r = \frac{H_R}{H_I} \approx 0.1\xi^2$$

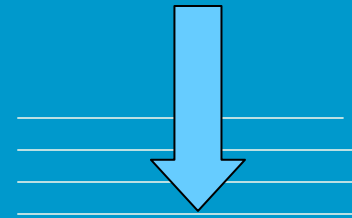


$$\eta_{tot} = \eta_i + \eta_r = (1+r) \frac{H_i}{2} \cos\left(\frac{2\pi x}{L}\right) * \cos\left(\frac{2\pi t}{T}\right) + (1-r) \frac{H_i}{2} \sin\left(\frac{2\pi x}{L}\right) * \sin\left(\frac{2\pi t}{T}\right)$$

# Example with Cress



run demo Cress



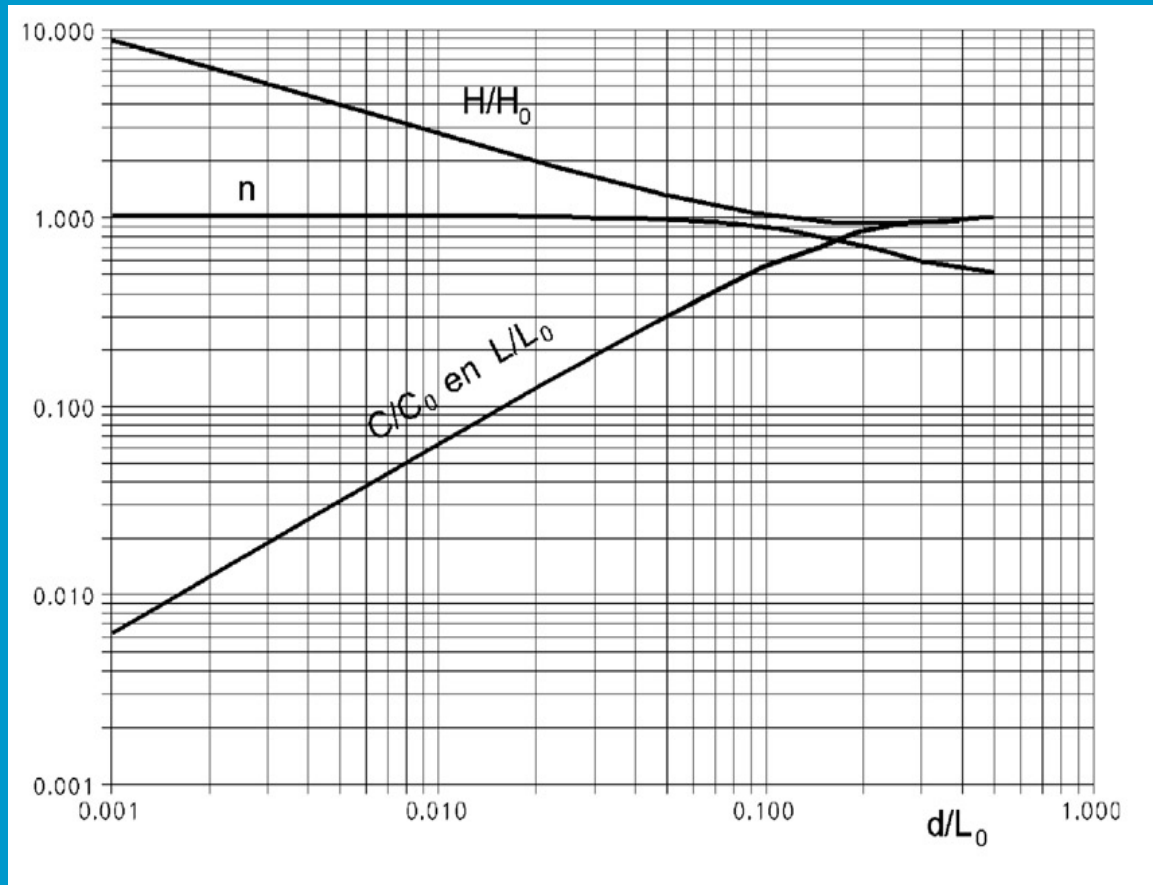
refraction  
shoaling, etc  
diffraction



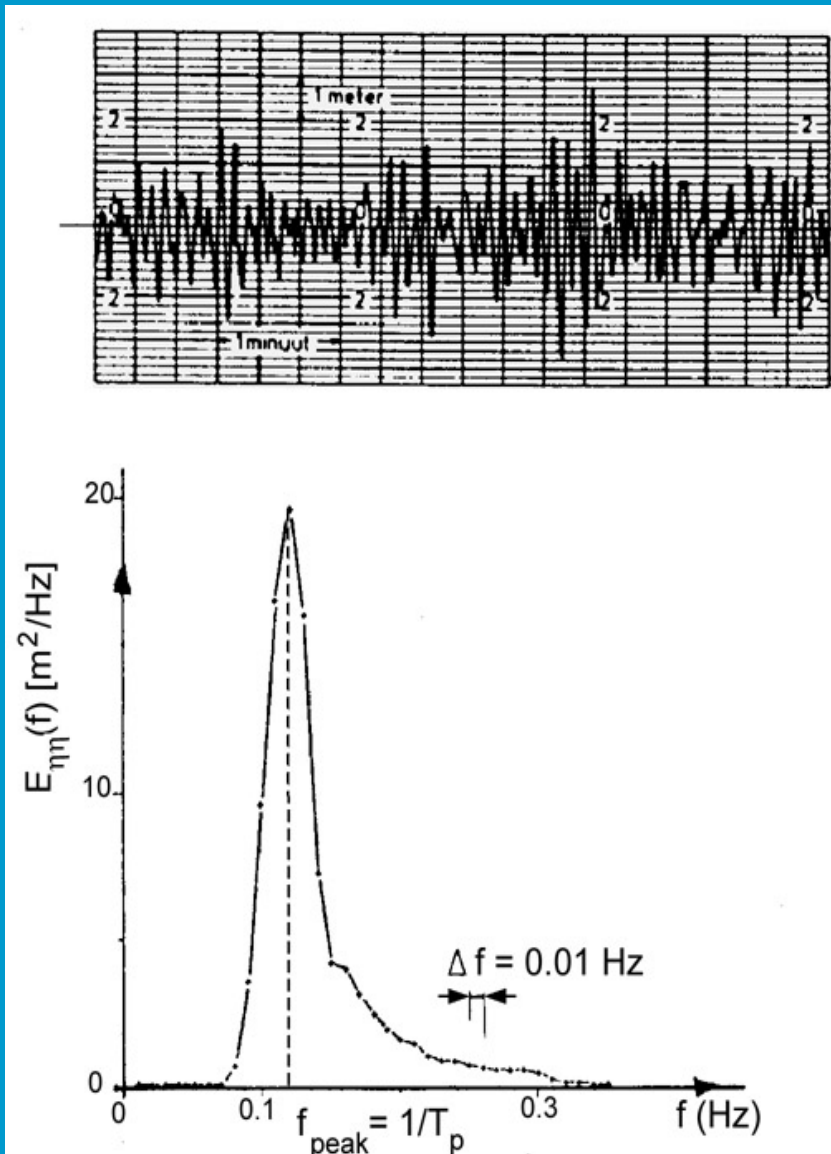
←  
y (-200,200)

↓  
x(50-200;4)

# The effect of shoaling on wave parameters



# Typical wave record of the North Sea



$$\eta(t) = \sum a_i \cos(2\pi f_i t + \varphi_i)$$

$$S(\omega) = \frac{1}{2} \sum a_i^2 / \Delta\omega$$

$$H_s = 4\sqrt{m_0} = H_{13.5\%}$$

# Spectral wave periods

The use of different wave parameters to obtain better results for wave structure interaction

$$m_n = \int_0^{\infty} f^n S(f) df$$

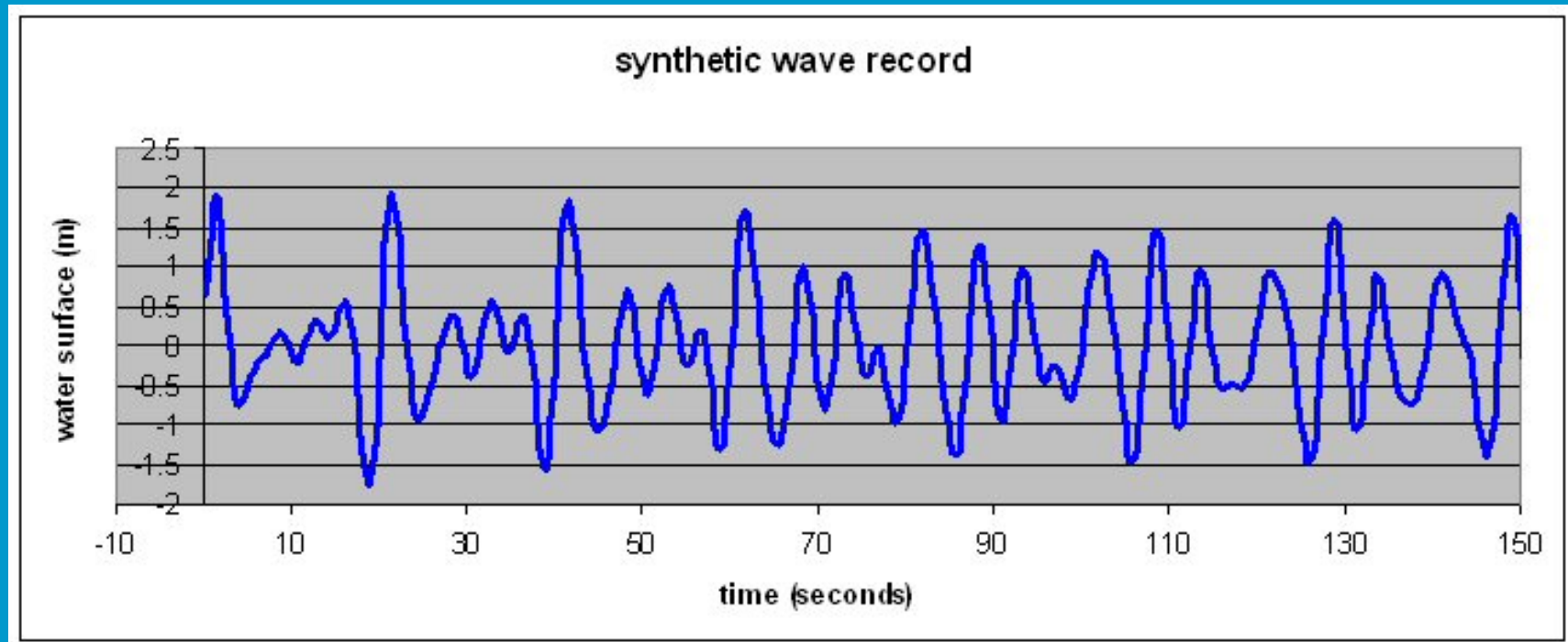
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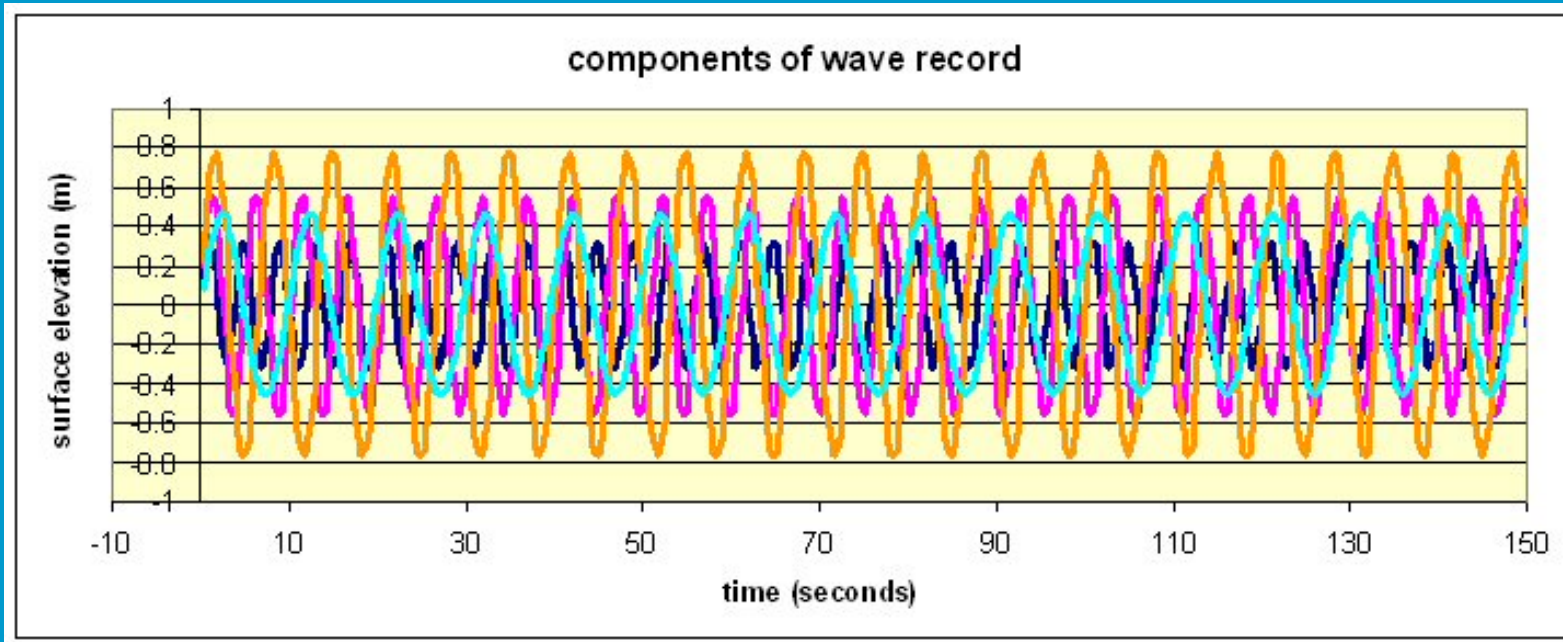
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# Example wave record



28 waves,  $H_s =$  "13% wave",  $H_s =$  wave nr 4,  $H_s \approx 3.8$   
28 waves in 150 seconds, so  $T_m = 5.3$  s

# composition of the record



$$H_1 = 0.63 \text{ m}$$

$$T_1 = 4 \text{ sec}$$

$$H_2 = 1.80 \text{ m}$$

$$T_2 = 5 \text{ sec}$$

$$H_3 = 1.55 \text{ m}$$

$$T_3 = 6.67 \text{ sec}$$

$$H_4 = 0.90 \text{ m}$$

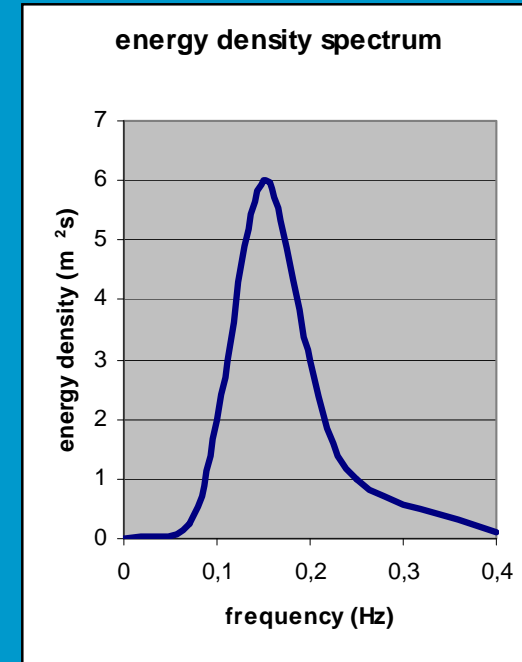
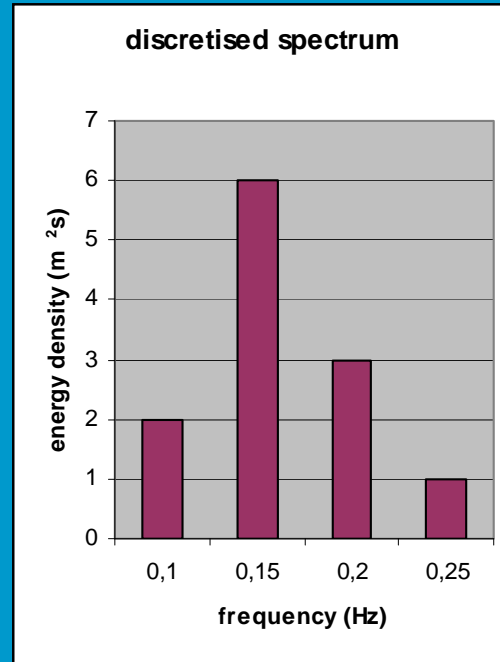
$$T_4 = 10 \text{ sec}$$

$$T_m = 5.3 \text{ sec}$$



# Spectrum

$$\frac{1}{2} a^2 = S \cdot \Delta f$$



$$H = \sqrt{8S \cdot \Delta f} \quad S = \frac{H^2}{8\Delta f} = \frac{1.55^2}{8 \cdot 0.05} = 6 [m^2 s]$$

# Calculation of $m_0$

$$0.05 * 2 \quad 0.10$$

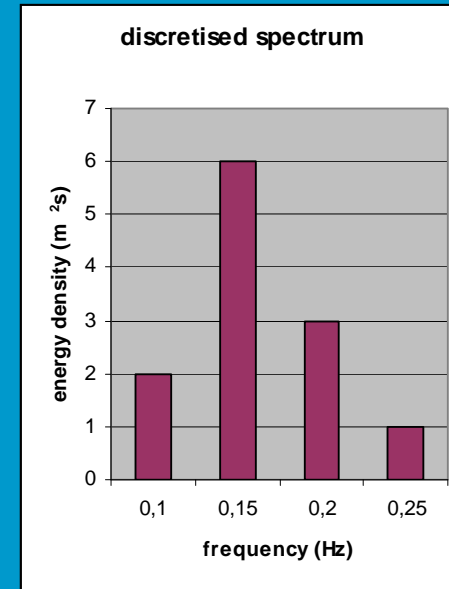
$$0.05 * 6 \quad 0.30$$

$$0.05 * 3 \quad 0.15$$

$$0.05 * 1 \quad 0.05$$

---

$$0.60$$



$$4\sqrt{m_0} = 3.1 \text{ m}$$

$$m_n = \int_0^{\infty} f^n S(f) df$$

# Calculation of $m_1$

dist \*  $S\Delta f$

0.10\*0.10      0.010

0.15\*0.30      0.045

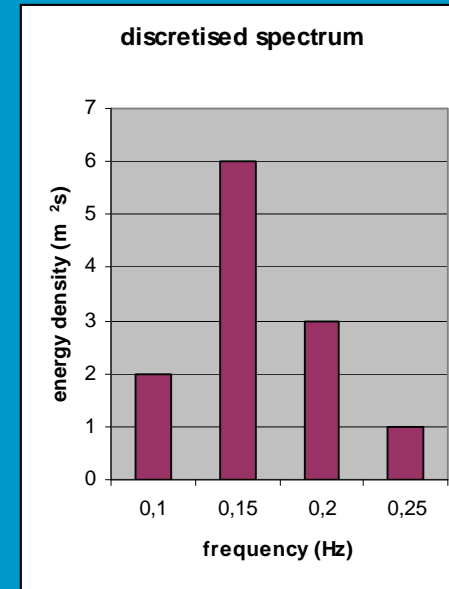
0.20\*0.15      0.030

0.25\*0.05      0.013

---

0.098

$$m_n = \int_0^{\infty} f^n S(f) df$$



# Calculation of $m_2$

dist<sup>2</sup> \* SΔf

$$0.10^2 * 0.10 \quad 1.00 \quad 10^{-3}$$

$$0.15^2 * 0.30 \quad 6.75 \quad 10^{-3}$$

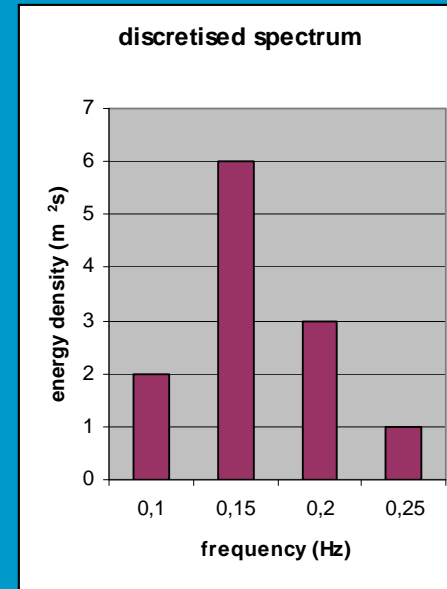
$$0.20^2 * 0.15 \quad 6.00 \quad 10^{-3}$$

$$0.25^2 * 0.05 \quad 3.12 \quad 10^{-3}$$

---


$$1.69 \quad 10^{-3}$$

$$m_n = \int_0^{\infty} f^n S(f) df$$



$$T = \sqrt{\frac{m_0}{m_2}} = 10 \sqrt{\frac{0.60}{1.69}} = 5.69 \text{ sec}$$

# Calculation of $m_{-1}$

$$1/\text{dist} * S\Delta f$$

$$1/0.10 * 0.10 \quad 1.0$$

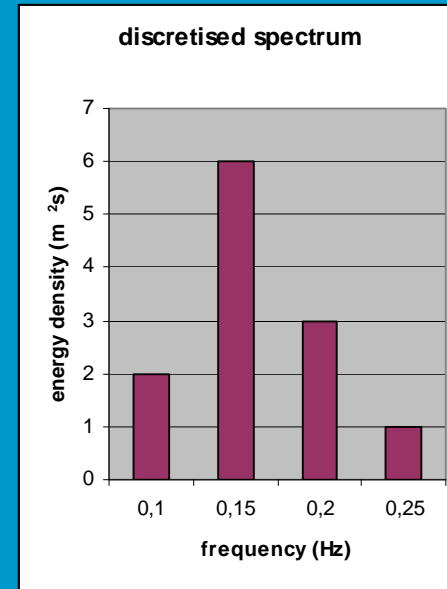
$$1/0.15 * 0.30 \quad 2.0$$

$$1/0.20 * 0.15 \quad 0.75$$

$$1/0.25 * 0.05 \quad 0.20$$

---


$$3.95$$



$$T_{m^{-1},0} = \frac{m_{-1}}{m_0} = \frac{3.95}{0.60} = 6.58 \text{ sec}$$

$$m_n = \int_0^{\infty} f^n S(f) df$$

# Overview

- $H_{m0} = 3.1 \text{ m}$   
( $1.55+1.10+0.90+0.63=4.18$ )

- $T_{m0} = 5.69 \text{ sec}$

- $T_{m-1,0} = 6.58 \text{ sec}$

- $T_{\text{peak}} = 6.67 \text{ sec}$

- $\frac{T_{m-1,0}}{T_{m0}} = \frac{6.58}{5.69} = 1.16$

- $T_m = 5.35 \text{ sec}$

- $\frac{T_{m0}}{T_m} = \frac{5.69}{5.35} = 1.06$

Usual assumptions:

$$T_{m0} = T_p$$
$$T_{1/3} = T_m$$

For standard spectra:

Goda:  $T_p = 1.1 T_{1/3}$

PM:  $T_p = 1.15 T_{1/3}$

Jonswap:  $T_p = 1.07 T_{1/3}$

TAW (vdMeer):  $T_p = 1.1 T_{m-1,0}$

Old Test (vdMeer):  $T_p = 1.04 T_{m-1,0}$

Also:  $T_{m-1,0} = 1.064 T_{1/3}$

# Overview to determine shallow water wave condition

- Determine deep water wave condition, this gives wave height, peak period and spectrum shape type (e.g. Jonswap)
- Calculate shallow water condition using spectral model (e.g. with SWAN), this gives  $H_{m0}$ ,  $T_{m0}$  and  $T_{m-1,0}$
- Use Battjes-Jansen method to determine  $H_{2\%}$

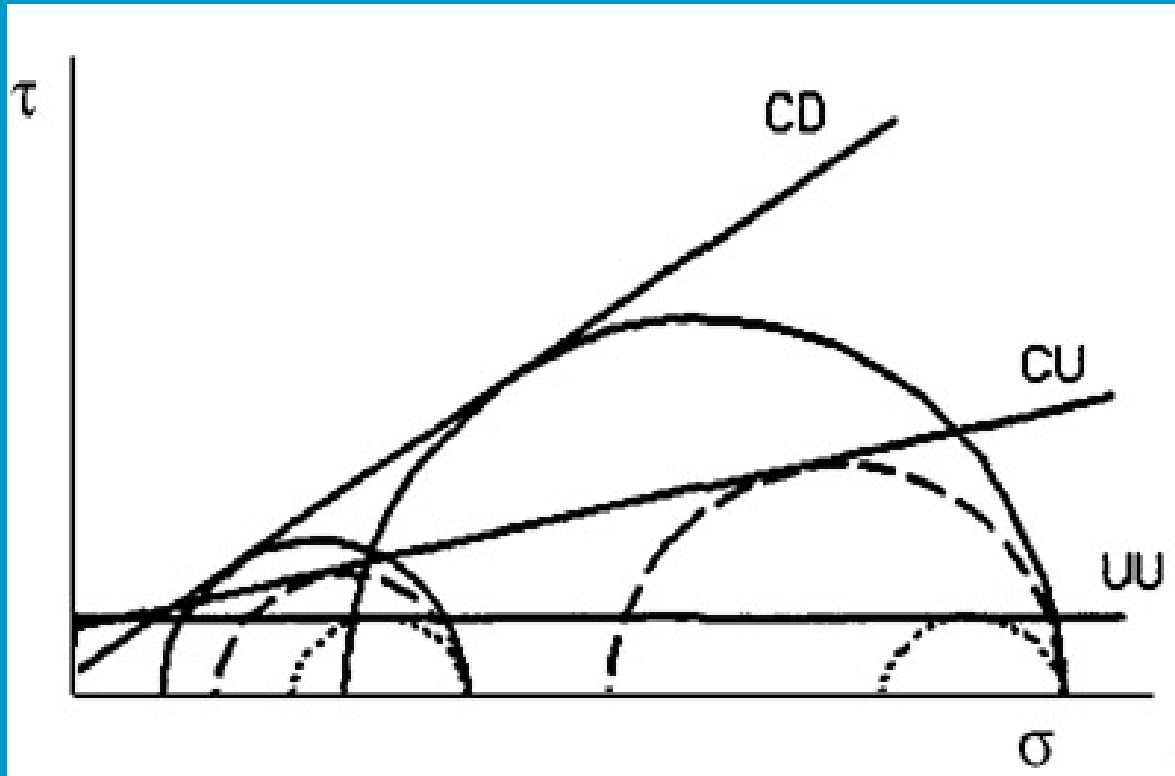
# Why these parameters ?

$$\frac{H_{2\%}}{\Delta d_{n50}} = c_{pl} P^{0.18} \left( \frac{S}{\sqrt{N}} \right)^{0.2} (s_{m-1,0})^{0.25} \sqrt{\cot \alpha} \quad \text{for plunging waves}$$

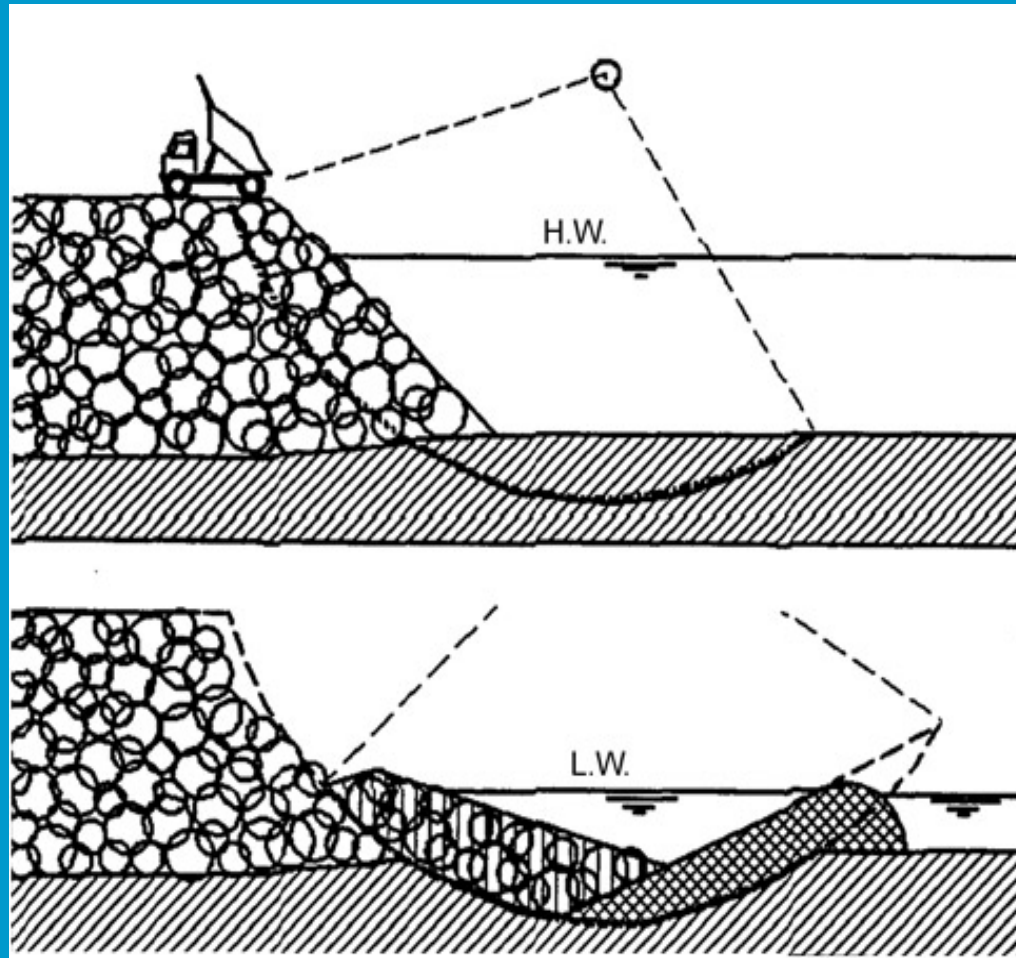
$$\frac{H_{2\%}}{\Delta d_{n50}} = c_s P^{-0.13} \left( \frac{S}{\sqrt{N}} \right)^{0.2} (s_{m-1,0})^{-0.25} (\xi_{s-1,0})^{P-0.5} \quad \text{for surging waves}$$



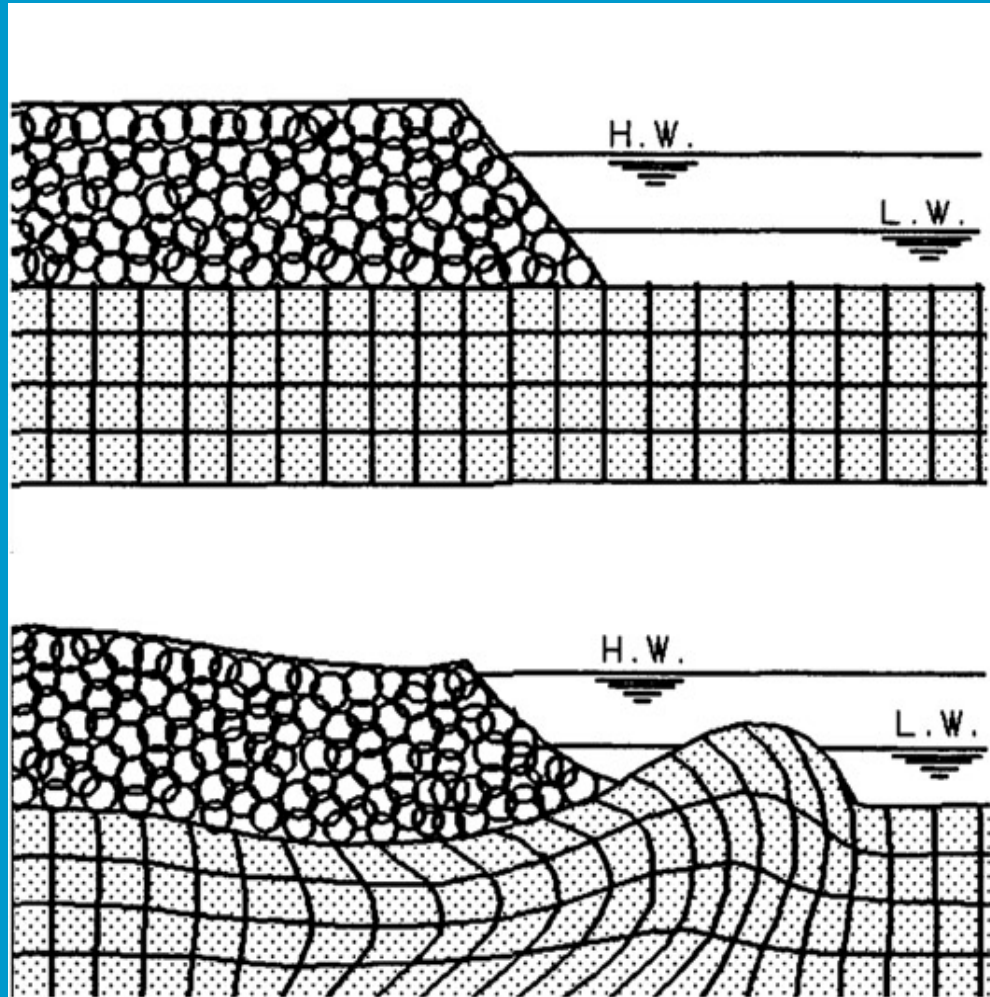
# stress relations determined by soil testing



# Dam profile after the slide



# Squeeze



# Liquefied sand

