Non-equilibrium thermodynamics
Exercise 2
Entropy Production

1

Equation 1 and 2 show two different ways to express the entropy production:

$$\sigma = J_q \left(\frac{\partial}{\partial x} \frac{1}{T} \right) + \sum_{j=1}^{n} J_j \left(-\frac{\partial}{\partial x} \frac{\mu_j}{T} \right) + j \left(-\frac{1}{T} \frac{\partial \phi}{\partial x} \right) + r \left(-\frac{\Delta_r G}{T} \right) \tag{1}$$

$$\sigma = J_q' \left(\frac{\partial}{\partial x} \frac{1}{T} \right) + \sum_{j=1}^{n} J_j \left(-\frac{1}{T} \frac{\partial \mu_{j,T}}{\partial x} \right) + j \left(-\frac{1}{T} \frac{\partial \phi}{\partial x} \right) + r \left(-\frac{\Delta_r G}{T} \right) \tag{2}$$

Each equation gives a set of conjugate fluxes and forces. Using equation 1, what is the conjugate flux of:

a) $\left(-\frac{\partial}{\partial x} \frac{\mu_j}{T}\right)$?

b) $\left(-\frac{\Delta_r G}{T}\right)$?

Using equation 2, what is the conjugate force of:

c) the measurable heat flux, J_q'?

d) the electric current density, j?

2

a) What is the expression for the entropy production for the diffusion of one component in a system at constant temperature?

b) What is the expression for the entropy production for a metallic conductor where there is transport of heat and charge?

c) Describe the difference between the thermodynamic driving forces, which are the forces defined by the entropy production, and the driving forces from the simple transport laws. Can the two descriptions be compatible?

3

a) Derive the rate of change in the local entropy density for a one-dimensional transport process.

b) What is the balance equation of a volume element with one-dimensional transport processes for:
- components?
- charge?
- internal energy?

c) Start with Gibbs' equation and derive Gibbs' equation on local form.

d) Use the answers from question a to c to derive equation 1.

e) What do we assume in order to use the local form of Gibbs' equation?

f) What must be the sign of the sum of fluxes times forces?

4

a) What is the difference between the total heat flux, J_q, and the measurable heat flux, J_q'?

b) Show how equation 2 can be derived from equation 1. Start by using the chain rule on $\frac{\partial}{\partial x}\frac{\mu_j}{T}$. The Maxwell relation $\left(\frac{\partial \mu}{\partial T}\right)_{P,N} = -\left(\frac{\partial S}{\partial N}\right)_{P,T}$ can be useful.

5

An electric conductor has conductivity 500 kS/m. A current of $1{\cdot}10^4$ A m^{-2} goes through the conductor. The temperature is constant at 300 K. What is the local entropy production in the conductor?

6

Find the entropy production due to the heat flux through the bottom of an aluminium pan containing boiling water. The surface of the electric plate is 150 °C. The pan bottom is 1.0 cm thick and the area is 3.0 dm^2. The Fourier type thermal conductivity for aluminium is 237 Js^{-1}m^{-1}K^{-1}. What is the total lost work when the temperature of the surroundings is 20 °C?