1D integrated acceleration : double $\to$ single integral

$$\dot{x}(t) = \dot{x}(t_0) + \int_{t_0}^{t} \ddot{x}(\rho)\,d\rho$$

$$x(t) = x(t_0) + \int_{t_0}^{t} \dot{x}(\tau)\,d\tau = x(t_0) + \dot{x}(t_0)(t-t_0) + \int_{t_0}^{t}\int_{t_0}^{\tau} \ddot{x}(\rho)\,d\rho\,d\tau$$

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integration by parts : $\int f'g = [fg] - \int f g'$

(functions of τ) $\quad f : \tau \quad g : \int_{t_0}^{\tau} \ddot{x}(\rho)\,d\rho \quad$ hence $g' : \ddot{x}(\tau)$

then $$\int_{t_0}^{t}\int_{t_0}^{\tau} \ddot{x}(\rho)\,d\rho\,d\tau = \left[\tau \int_{t_0}^{\tau} \ddot{x}(\rho)\,d\rho\right]_{t_0}^{t} - \int_{t_0}^{t} \tau\,\ddot{x}(\tau)\,d\tau$$

$$= t\int_{t_0}^{t} \ddot{x}(\rho)\,d\rho - \cancel{t_0 \int_{t_0}^{t_0} \ddot{x}(\rho)\,d\rho} - \int_{t_0}^{t} \tau\,\ddot{x}(\tau)\,d\tau = \int_{t_0}^{t} (t-\tau)\,\ddot{x}(\tau)\,d\tau$$

$$x(t) = x(t_0) + \dot{x}(t_0)(t-t_0) + \int_{t_0}^{t} (t-\tau)\,\ddot{x}(\tau)\,d\tau$$

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<u>note</u> with $t_0 = 0$ and $\ddot{x}(\tau) = \ddot{x}$ (just constant)
the integral gives the 'famous' $\frac{1}{2}\ddot{x}t^2$