2. Water transport through pipes

2.1 Introduction

One of the basic rules of nature is that water flows from a high energy level to a low energy level. The high energy level can for instance be a storage tank at a high level as a water tower or a tank on a hill. It can also be the high pressure induced by pumps. For the drinking, sewerage and irrigation practise we focus on the flow in open channel or partially filled pipes and flow through surcharged filled closed pipes. These are the most common phenomena in the water transport in these fields.

Fig. 2.1 - Water tower

Characteristic for water transport through partially filled pipes or open channels is that pressure at the fluid surface is atmospheric. Consequently open channel flow is always induced by surface fluid level difference between the upstream boundary and the downstream boundary. The upstream level may, for instance, be an open water surface or an inflow from a house manifold at a sewer system. A downstream flow can be an outflow over a weir. Flow is induced by gravity.

Open channel flow allows for storage of water within the profile, through changes of water level. Upstream inflow of water doesn't necessarily instantaneously

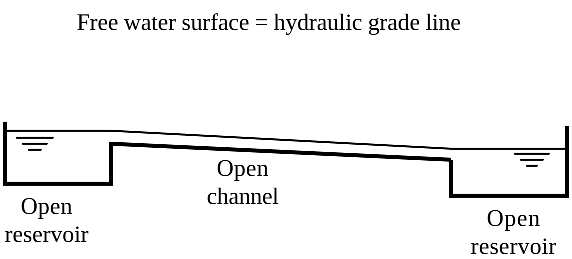


Fig. 2.2 - Two open reservoirs with open channel

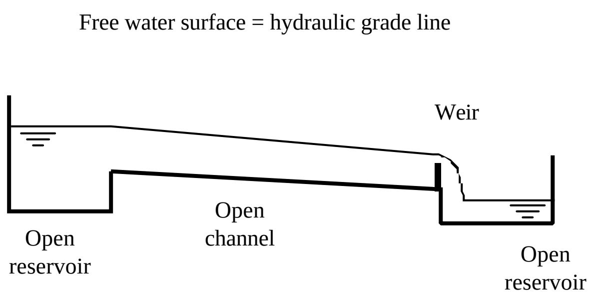


Fig. 2.3 - Flow with a weir

cause an out flow at the down stream end, but water can be stored in the channels causing a rise in water level. In urban drainage systems a deliberate storage is wanted or even needed in the system, for instance by putting weirs at the down stream boundary.

In open channel flow the storage capacity is an important describing factor.

The energy difference in pressurized flow in surcharged and closed pipes is mostly induced by an energy input at the upstream boundary by pumps or by an elevated reservoir.

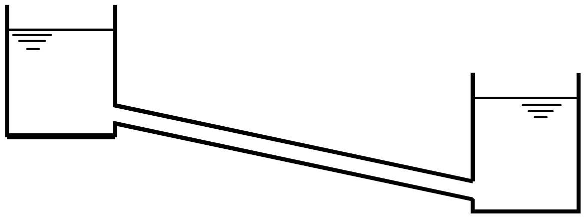


Fig. 2.4 - Two reservoirs with pipes at the bottom

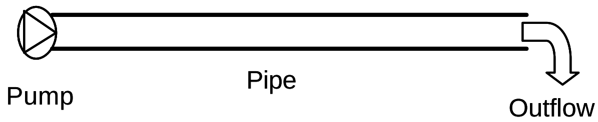


Fig. 2.5 - Pump and outflow

Closed and surcharged pipes don't have significant storage capabilities. The storage capability is for instance the stretching of the pipe and the compression of the water. This causes high pressures because of the stiffness of the pipe and the high compressibility of the water and is called water hammer and occurs when changes in flow velocity are relatively rapid. This type of flow is described separately. In closed pipe flow friction loss is the main describing factor for energy losses.

In this chapter we first deal with pressurised flow, than with open channel flow and conclude with the phenomenon water hammer.

2.1 Pressurised flow

In pressurised flow through pipes the pressure at the start of the pipe is higher than atmospheric. An open (high level) reservoir or a pump induces the pressure at the beginning of the pipe (see figure XX's). Flow through the pipe will cause an energy loss due to friction loss and local losses caused by release of flow lines (entrance and decelaration losses). Schematically all the losses and levels are summarized static pressure and dynamic pressure in figure XX.

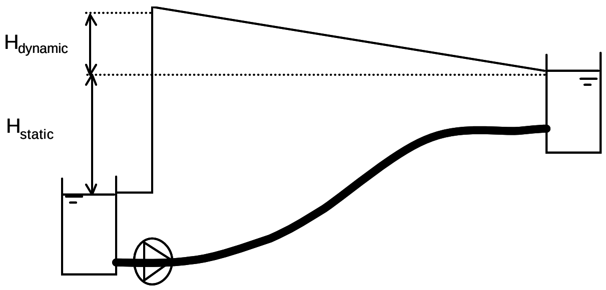


Fig. 2.6 - Two different energy levels

Mathematical description

Two equations describe the flow through pipes:

3 Continuity equation or mass balance

4 Motion equation or momentum balance

2.1.1 Mass balance/continuity equation

A control volume of pipe is considered with a cross section A and a length dx The mass balance states that ingoing mass equals outgoing mass. Incoming mass in a time frame dt is:

$$r Q_{in} dt = r u A dt \qquad \text{(eq. 2.1)}$$

with

Q_{in} : Incoming volume flow [m^3/s]

ρ : Specific mass [kg/m^3]

u : Mean velocity [m/s]

A : Cross section of the pipe [m^2]

Outgoing mass is analogue with the ingoing mass

$$r Q_{out} dt = r u A dt \qquad \text{(eq. 2.2)}$$

The mass balance over a time frame dt is

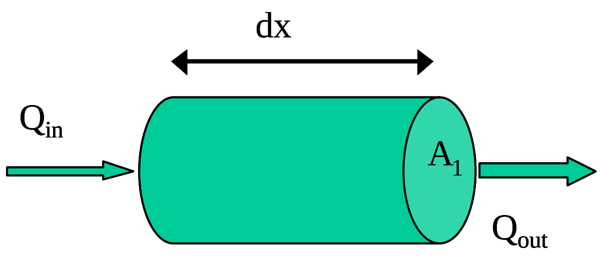


Fig. 2.7 -Mass balance

$$Q_{in} \partial t = Q_{out} \partial t + \partial A \partial x \qquad \text{(eq. 2.3)}$$

in words: ingoing mass equals outgoing mass plus storage within the control volume. The storage is the changing of the cross section dA.

Dividing the mass balance by δxδt gives

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad \text{(eq. 2.4)}$$

Momentum equation

Referring to Battjes (CT3310, chapter 2) the momentum equation is

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial p}{\partial x} + c_f \frac{|Q|Q}{AR} = 0 \qquad \text{(eq. 2.5)}$$

with

g : Gravitation

R : Wet perimeter

c_f : friction coefficient

The system of the continuity equation and the mass balance are known as the equations of De Saint-Venant (1871)

The dimensionless coefficient c_f can be expressed in the Chézy coefficient as $c_f = g/C^2$. With Q=uA the momentum equation becomes

$$A\frac{\partial u}{\partial t} + u\frac{\partial A}{\partial t} + 2Au\frac{\partial u}{\partial x} + u^2\frac{\partial A}{\partial x} + gA\frac{\partial p}{\partial x} + \frac{g}{C^2}\frac{A}{R}|u|u = 0$$

(eq. 2.6)

2.1.3 Rigid column approach

Water transport through pipes is characterised with slow changing boundary conditions. When the so-

Picture of de Saint-Venant

Jean Cleade de Saint-Venant (1797-1886) graduated at the Ecole Polytechnique in 1816. He had a fascinating career as a civil engineer and mathematics teacher at the Ecole des Ponts et Chaussées where he succeeded Coriolis. Seven years after Navier's death, Saint-Venant re-derived Navier's equations for a viscous flow, considering the internal viscous stresses, and eschewing completely Navier's molecular approach. That 1843 paper was the first to properly identify the coefficient of viscosity and its role as a multiplying factor for the velocity gradients in the flow. He further identified those products as viscous stresses acting within the fluid because of friction. Saint-Venant got it right and recorded it. Why his name never became associated with those equations is a mystery. certainly it is a miscarriage of technical attribution.
It should be remarked that Stokes, like de Saint-Venant, correctly derived the Navier-Stokes equations but he published the results two years after decSaint-Venant.
In 1868 de Saint-Venant was elected to succeed Poncelet in the mechanics section of the Académie des Sciences. By this time he was 71 years old, but he continued his research and lived for a further 18 years after this time. At age 86 he translated (with A Flamant) Clebsch's work on elasticity into French and published it as Theorie de l'élasticité des corps solides and Saint-Venant added notes to the text which he wrote himself.

called rigid column simplification is applied the presumptions are made:

- o Uniform and stationary flow
- o Prismatic pipe: The cross section of the pipe doesn't change over the length of the pipe resulting in $\frac{\partial A}{\partial x} = 0$
- o Water is incompressible
- o Elasticity of the pipe is negligible: $\frac{\partial A}{\partial t} = 0$
- o The fluid meets Newton's criteria being that viscosity is constant and only dependent on temperature.

The continuity equation then transforms in $\frac{\partial Q}{\partial x} = 0$ which becomes:

$$\frac{\partial Q}{\partial x} = 0 \rightarrow \frac{\partial uA}{\partial x} = 0 \rightarrow A\frac{\partial u}{\partial x} + u\frac{\partial A}{\partial x} = 0 \rightarrow \frac{\partial u}{\partial x} = 0$$

(eq. 2.7)

And the momentum balance than becomes:

$$A\frac{\partial u}{\partial t} + gA\frac{\partial p}{\partial x} + \frac{g}{C^2}\frac{A}{R}|u|u = 0 \qquad \text{(eq. 2.8)}$$

substituting $u = \frac{Q}{A}$ and dividing by gA gives

$$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{\partial p}{\partial x} + \frac{Q|Q|}{C^2A^2R} = 0 \qquad \text{(eq. 2.9)}$$

Considering a piece of pipe with length L and integrating the equations over this pipe length gives

$$\int_{x=0}^{x=L}\frac{\partial p}{\partial t}dx = -\int_{x=0}^{x=l}\frac{1}{gA}\frac{\partial Q}{\partial t}dx - \int_{x=0}^{x=l}\frac{Q|Q|}{C^2A^2R}dx \Rightarrow$$

$$p_2 - p_1 = -\frac{1}{gA}\frac{\partial Q}{\partial t}L - \frac{Q|Q|}{C^2A^2R}L$$

(eq. 2.10)

In stationary flow, the term $\frac{\partial Q}{\partial t}$ becomes zero or negligible: flow will only slowly change over time. For the roughness of the pipe wall in this equation the Chézy-coefficient is used. Often this is replaced by the Darcy-Weissbach friction coefficient $l = \frac{8g}{C^2}$. Different formulas are applied to calculate the friction coefficient λ, which is referred to in the next paragraph.
Combined with the substitution of the Darcy-Weissbach friction coefficient the socalled Darcy-Weissbach equation remains:

$$p_1 - p_2 = l \frac{8L}{p^2 g} \frac{Q|Q|}{D^5} = 0{,}0826 \frac{l\,L}{D^5}|Q|Q$$

(eq. 2.11)

Another popular representation of the Darcy Weissbach formula is:

$$\Delta H = \frac{l\,L}{D}\frac{u^2}{2g} \qquad \text{(eq. 2.12)}$$

2.3 Friction coefficients and local losses

In Battjes (CT2100, chapter 12.4) an extensive elaboration on different friction parameters is given, both theoretically and experimentally determined. The most used formulas are those of Manning, Chézy and White Colebrook.

White and Colebrook (1937) performed experiments to asses how ë varies in the transition from smooth to rough conditions as a function of the Reynolds number at a constant relative roughness. They found that λ in technical rough pipes much smoother varies than in experiments of Nikuradse. Colebrook developed an expression with which λ can be deterimend as a function of the relative roughness in Nikuradse's coefficient k/D and the Reynolds number:

$$\frac{1}{\sqrt{l}} = -2\log\left(0{,}27\frac{k_N}{D} + \frac{2{,}5}{\mathrm{Re}\sqrt{l}}\right) \qquad \text{(eq. 2.13)}$$

This formula can only iteratively be solved. The variable determining the friction coefficient is the Nikuradse roughness of the pipe wall. This can vary between 0,05 mm for smooth pipes like PVC or PE pipes and 20 mm for old cast iron pipes with large encrustations.

In network calculation programs the λ-value is calculated automatically as a function of the hydraulic circumstances expressed in the Reynolds number and the relative roughness of the pipe expressed in k/D.

For convenience diagrams are developed to quickly determine the value of λ. The Moody-diagram is the most commonly used diagram. This diagram is shown in figure 2.8.

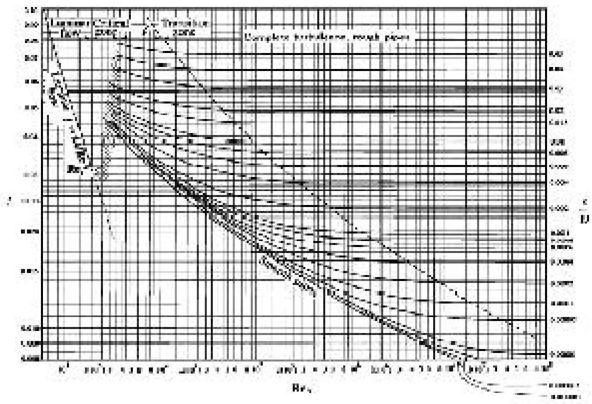


Fig. 2.8 - Moody diagram

Local losses

Local losses are caused by sudden deceleration of flows combined with release of flow lines from the pipe wall. Examples are given in figure 2.9.

In fact local losses are separately addressed because this is a violation of the assumption that the pipes are prismatic $\frac{\partial A}{\partial x} = 0$.

Local losses depend on the velocity in the pipe and are expressed in an analogue formula as the Darcy Weissbach equation.

$$\Delta H_{local} = x\frac{u^2}{2g} \qquad \text{(eq. 2.14)}$$

For the value of ξ a lot of experiments have been

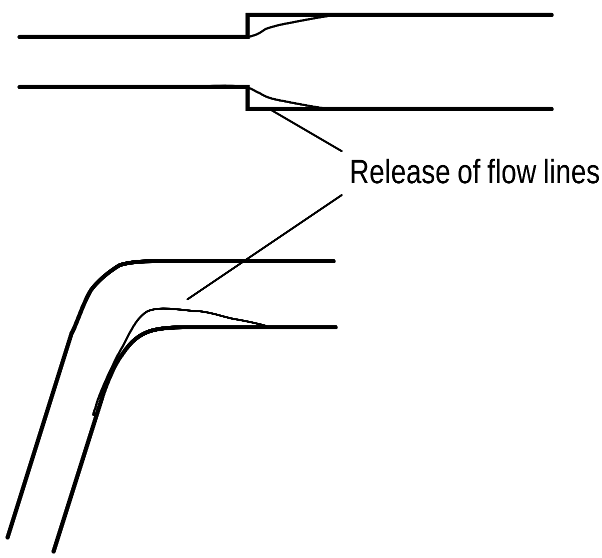


Fig. 2.9 - Release of flow lines in sudden increase in diameter and in bends

performed resulting in large tables with all kind of hydraulic situations and the resulting local loss coefficient. An extensive handbook is made by I'del cik, which is used all over the world.

Some examples of such tables are in appendix 2.1.

2.4 Summary of pressurized transport

The factors determining the energy losses during pressurized flow are schematically drawn in figure 2.10.

Energy losses are mainly friction losses and deceleration losses. The formula of Darcy Weisbach is one dimensional and there is no time dependency. Network calculations based on the one-dimensional formulas are so-called steady state calculations.

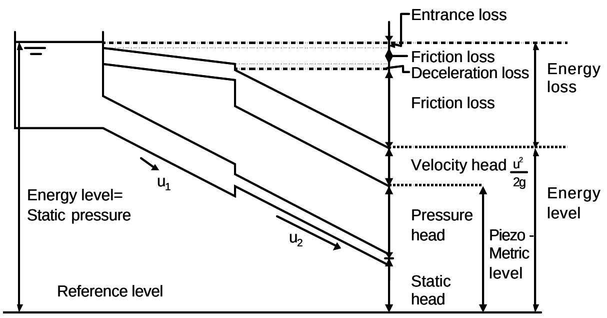


Fig. 2.10 - Losses and energy levels

2.5 Open channel flow

When turning to open channel flow, occuring in sanitary engineering in urban drainage networks, exactly the same principles are applied as in pressurised flow. In this also a 1-dimensional approach is usually adopted. This means that the (3 dimensional) velocity vector in the momentum and mass-balance equation are integrated into a one-dimensional form. The dependent variables are integrated quantities. In order to get a closed system of equations simplifying assumptions are needed. Local effects such as caused by weirs, are taken into account by estimating their effect on the integrated equations. The reduction into one space dimension is defensible; when studying the geometry of a conduit in an urban drainage system the flow is to a large extent 1-dimensional, except for the velocity field at the direct entrance. In manholes the velocity field is 3-dimensional, but this can be circumvented in a 1-D model by applying extra terms to account for frictional losses.

The equations applied in a 1-dimensional approach are the well-known De Saint-Venant equations (De Saint-Venant (1871)):

Momentum balance:

$$\underbrace{\frac{\partial Q}{\partial t}}_{I}+\underbrace{\frac{\partial}{\partial x}\left[b\,\frac{Q^2}{A}\right]}_{II}+\underbrace{gA\frac{\partial h}{\partial x}}_{III}+\underbrace{c_f\frac{Q|Q|}{R_h A}}_{IV}=0 \qquad \text{(eq. 2.15)}$$

Mass balance:

$$\frac{\partial Q}{\partial x}+\frac{\partial A(h)}{\partial t}=\frac{\partial Q}{\partial x}+B(h)\frac{\partial h}{\partial t}=0 \qquad \text{(eq. 2.16)}$$

In which:

Q	discharge	(m^3/s)
A	cross-sectional area	(m^2)
B	width of the free water surface	(m)
g	gravitational acceleration	(≈9.813 m/s^2)
R_h	hydraulic radius	(m)
c_f	resistance constant	(-)
h	water level	(m)
x	location along x-axis	(m)
t	time	(s)
β	Boussinesq's number	(-)

The assumptions applied are:

- Hydrostatic pressure.
- Velocity components in y and z direction are negligible compared to the velocity component in x direction ($u_y = u_z << u_x$).

The individual terms in the momentum balance equations (eq. 2.15) are:

I	acceleration term
II	convective term
III	gravitational term
IV	friction term

The third term may also be written as:

$$gA\frac{\partial h}{\partial x}=gA\left[\frac{\partial a}{\partial x}+\frac{\partial z_b}{\partial x}\right]=gA\left[\underbrace{\frac{\partial a}{\partial x}}_{IIIa}-\underbrace{i_b}_{IIIb}\right] \qquad \text{(eq. 2.17)}$$

In which:

a	water depth	(m)
z_b	bottom level	(m)
i_b	bottom slope	(-)

Term IIIa is the pressure term and term IIIb is the gravity term.

The De Saint-Venant equations (using the dynamic wave approach) form a hyperbolic system of partial differential equations. This implies that in order to obtain a well-posed problem the initial condition (Q and h at t=0) and two boundary conditions have to be defined.

Several possible simplifications of eq. 2.15 and 2.16 are applied depending on the possibility of neglecting terms in the momentum balance equation. For instance when terms I and II (the inertia terms) can be neglected, which is the case when the flow varies only slowly with time and space, either the kinematic or diffusion wave equation is obtained. In Table 1 the possible simplifications for the momentum equations are summarised along with the identification normally used.

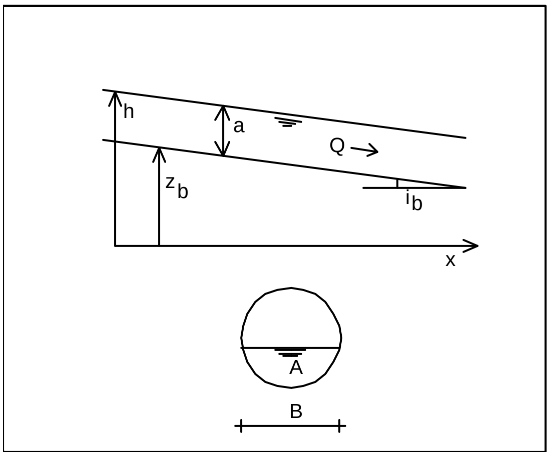


Fig. 2.11 - Definition sketch

The simplified versions of the momentum equation are mostly used to drive analytical solutions for special (academic) cases, or to simplify the numerical calculation for practical cases. In fact the first computer programs applied in the field of urban drainage (end of the 1970's and the early 1980´s) were based on the kinematic wave simplification. The main reason for this was that the inertia terms (especially the convective term) are more difficult to handle in terms of numerical stability, computational effort and RAM. Nowadays there is a very high degree of acciability to computational power which has led to a situation in which any network can be dealt with using the full dynamic wave equations.

Due to the integration of two space dimensions, several new parameters enter the equations. For instance, the geometry of the problem like hydraulic radius, wetted area and hydraulic depth are such parameters. The parameter β in the convective term (term II) is the Boussinesq number (Boussinesq, (1897)), defined by:

$$b = \frac{\int_0^A u^2 dA}{\overline{u}^2 A} \qquad \text{(eq. 2.18)}$$

The Boussinesq number accounts for the fact that the velocity is not uniformly distributed over the cross section of the flow, in fact it is a correction factor due to simplifying the 3-dimensional flow equations.

The value for β is >1 and can eventually reach values >1.2 in extreme cases, e.g. when a sediment bed is present in a channel with a circular cross section at shallow water depths, see Kleywegt (1992). In practice, however a value of 1 is applied since the effect is rather limited when compared to the uncertainty in other parameters introduce in the equations (for instance the value for the wall roughness en local loss coefficients).

Table 1 - Possible simplification for the momentum equation

Terms taken into account	Identification of the simplified equation	main assumptions
I+II+III+IV	Dynamic wave	no additional assumptions
I+II+III	Gravity wave	friction is sma ll compared to gravity and inertia terms
III+IV	Diffusion wave	inertia terms are small compared to gravity and friction
IIIb+IV	Kinematic wave	inertia terms are small compared to gravity and friction and $\frac{\partial a}{\partial x} << i_b$

Transition between open channel flow and surcharged transport

A problem is encountered when pipes starts to flow full, instability-like phenomena evolve. It should be stated however, that in reality when pipes tend to flow full a kind of chaotic behaviour may occur (see Fig 2.12). This is because air escaping between the water level and the pipe wall influences the water flow.

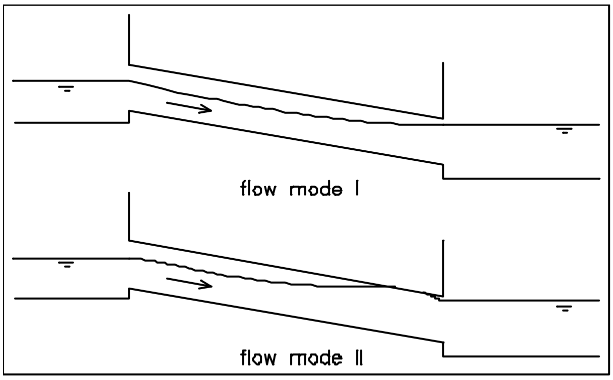


Fig. 2.12 - Physical instability. Due to air entrainment the flow may change between modes I and II in an unpredictable pattern in time.

It was observed in laboratory experiments (see e.g. de Somer (1984)) that even at steady state flow under certain geometrical conditions unpredictable transitions in flow mode can occur. The origin of numerical instabilities at nearly full flow can be characterised as a boundary condition problem. Essentially, it is so that nowhere in the system the water level is fixed, under pressurised conditions this implies that the flow is defined by a difference in pressure only. Therefore, the absolute water level or, equivalently, the local pressure is undefined by lack of a boundary condition in this respect.

It is accepted practice to circumvent this by applying the so-called Preissmann slot[1]. In fact the cross-sectional geometry of, e.g,. a circular pipe is slightly

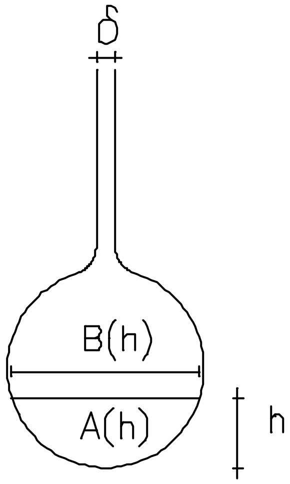


Fig. 2.13 - The Preissmann slot

changed in order to avoid the occurrence of a transition between free surface flow and pressurised flow by adding a slot with a small width (d) on top of the pipe. In this manner the water level directly follows from the mass-balance equation. In fig 2.13 this slot is depicted, the width of the slot d is defined theoretically by setting the celerity of a disturbance at the free surface in the slot equal to the celerity of a disturbance in the full-pressurised case.

The celerity of a surface wave is defined by:

$c = u \pm \sqrt{gH_h}$ in which H_h is the hydraulic depth:

$H_h = \dfrac{A(h)}{B(h)}$. Taking a circular cross-section as an example, A en B are functions of the water depth (see annexe I); when the conduit is almost completely filled it is easily seen that the value of c becomes undefined [2]:

$$\lim_{h\to 2R} B(h) = 0 \Rightarrow \lim_{h\to 2R} h_h(h) \to \infty \Rightarrow \lim_{h\to 2R} c(h) \to \infty \qquad \text{eq. 2.19}$$

Applying the Preissmann slot the width of this slot (d) is defined by stating: $c_{free\ surface} = c_{pressurised}$

(1) The idea of this piezometric slot originates from A. Preissmann and was further developed by Cunge & Wagner (1964) for practical application.

(2) In practice, this celerity is limited to $c = \sqrt{\dfrac{1}{\dfrac{r}{K} + \dfrac{rD}{Ed}}}$ representing the velocity of a pressure wave travelling through the system. In which r is the density of water in kg/m^3, d is the wall thickness of the pipe in m, D is the diameter of the pipe in m, E is elasticity modulus of the pipe material in N/m^2 and K is the compressibility of water in N/m^2.

$$u+\sqrt{g\frac{p\,R^2}{d}} = u+\sqrt{\frac{1}{\frac{r}{K}+\frac{r\,D}{Ed}}} \Rightarrow d = \frac{r\left[\frac{1}{K}+\frac{D}{Ed}\right]}{gp\,R^2}$$

(eq. 2.20)

The resulting values for the slot width d though, may become very small resulting in high values for the Courant number (i.e. >3 to 4). Therefore, in practice, the slot width is set to a fixed value of e.g. 0.1-5% of the diameter of the conduit. This does introduce a discrepancy with reality. As long as no water hammer problems are studied these, discrepancies however, are insignificant.

2.6 Water Hammer

Water hammer is a special flow condition in surcharged pipes. As stated in section XX the changing of boundary conditions during normal situation are so slow that a rigid column approach is allowed. Certain changes in boundary conditions can be so fast that the flow doesn't meet the requirements for a rigid column approach and the flow must be analysed in a different way. Most important features are that the compressibility of the water and the elasticity of the pipe is not neglected, but taken into account.

Main effect is that storage within the filled pipe is possible due to the compressibility of the water (more water in the cross-section) and the elasticity of the pipe (variation in the cross-section).

A schematic picture of the phenomenon Water Hammer is given in figure 2.14. A stream of water is suddenly stopped, resulting in a positive pressure at the upstream side and a negative pressure at the down stream side.

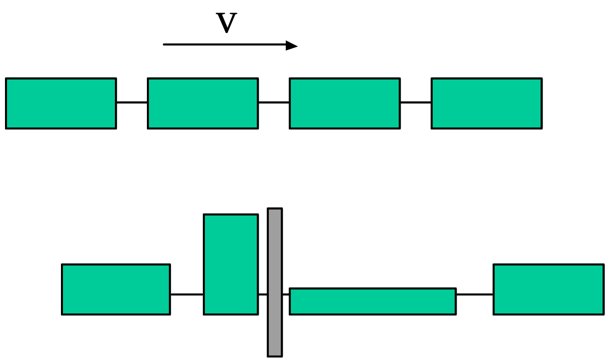


Fig. 2.14 - Schematic of water hammer

Mathematical description

The original constitution equations (De Saint-Venant are:

Mass balance: $\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0$ (eq. 2.21)

Continuity equation:

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial p}{\partial x}+c\frac{|Q|Q}{AR}=0 \quad \text{(eq. 2.22)}$$

We now have to find relations between the pressure p and the fluid density ρ and the cross section A.

Relation between pressure and fluid density

The compression modulus of a fluid is defined as:

$$\frac{\mathrm{d}d}{\mathrm{dp}}=\frac{d}{\mathrm{K}} \quad \text{(eq. 2.23)}$$

Through the partial differentials of ρ to t and x we find:

$$\frac{\partial r}{\partial t}=\frac{d\,r}{dp}\frac{\partial p}{\partial t}=\frac{r}{K}\frac{\partial p}{\partial t}$$
$$\frac{\partial r}{\partial x}=\frac{d\,r}{dp}\frac{\partial p}{\partial x}=\frac{r}{K}\frac{\partial p}{\partial x} \quad \text{(eq. 2.24)}$$

Relation between cross section A and pressure p

Consider a circle shaped cross section of a pipe with an internal diameter D and a uniform wall thickness δ. The wall thickness is relatively small compared to the diameter (δ << D) (See figure XX). A small pressure increase in the pipe (dp) will increase the positive tension of dσ. The increase will meet the equation 2 δ* dσ = D*dp

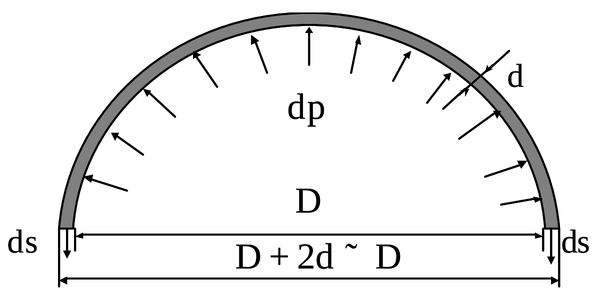


Fig. 2.15 - Cross section pressurised pipe

The increase in the wall tension dó will induce an increase of the length of the pipe wall P = πD and thus the diameter D. According to Hooke's law this can be expressed as:

$$\frac{dD}{D}=\frac{dP}{P}=\frac{ds}{E} \qquad \text{(eq. 2.25)}$$

This leads to

$$\frac{dD}{D}=\frac{D}{2d}\frac{dp}{E} \qquad \text{(eq. 2.26)}$$

Because A is proportional to D^2

$$\frac{dA}{A}=2\frac{dD}{D}=\frac{D}{d}\frac{dp}{E} \qquad \text{(eq. 2.27)}$$

and

$$\frac{dA}{dp}=\frac{D}{dE}A \qquad \text{(eq. 2.28)}$$

With this the partial differentials of A to t and x can be expressed in $\frac{\partial p}{\partial t}$ and $\frac{\partial p}{\partial x}$:

$$\frac{\partial A}{\partial t}=\frac{dA}{dp}\frac{\partial p}{\partial t}=\frac{D}{dE}A\frac{\partial p}{\partial t}$$
$$\frac{\partial A}{\partial x}=\frac{dA}{dp}\frac{\partial p}{\partial x}=\frac{D}{dE}A\frac{\partial p}{\partial x} \qquad \text{(eq. 2.29)}$$

Mass balance

The mass balance of a pipe is

$$\frac{\partial}{\partial t}(rA)+\frac{\partial}{\partial x}(rAu)=0 \qquad \text{(eq. 2.30)}$$

Complete elaboration of this equation gives:

$$A\frac{\partial r}{\partial t}+r\frac{\partial A}{\partial t}+ru\frac{\partial A}{\partial x}+rA\frac{\partial u}{\partial x}+uA\frac{\partial r}{\partial x}=0 \qquad \text{(eq. 2.31)}$$

Substituting all equations describing the variation of ρ and A with p followed by a division by A gives:

$$\left\{\frac{r}{K}+\frac{rD}{Ed}\right\}\frac{\partial p}{\partial t}+\left\{\frac{r}{K}+\frac{rD}{Ed}\right\}u\frac{\partial p}{\partial x}+r\frac{\partial u}{\partial x}=0 \qquad \text{(eq. 2.32)}$$

If we define c like: $\frac{1}{c^2}=\frac{r}{K}+\frac{rD}{Ed}$ this will become

$$\frac{1}{c^2}\frac{\partial p}{\partial t}+\frac{u}{c^2}\frac{\partial p}{\partial x}+r\frac{\partial u}{\partial x}=0 \qquad \text{(eq. 2.33)}$$

The value of c is equal to the speed of pressure wave in the specific circumstances.
In a non-elastic pipe E becomes ∞. If the fluid is non-compressible than also the value k becomes ∞, leaving c to go to infinity as well. Meaning that the continuity equation derives to $\frac{\partial u}{\partial x}=0$, as is valid when the rigid column approach is applied. The boundaries towards E and K imply a rigid column, both for the fluid and the pipe wall.

The formula can also be written as

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+rc^2\frac{\partial u}{\partial x}=0$$

With $\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}=\frac{dp}{dt}$ this becomes:

$$\frac{dp}{dt}+rc^2\frac{\partial u}{\partial x}=0 \qquad \text{(eq. 2.34)}$$

Rewriting this using the piezometric level

$$j=\frac{p}{rg}+z$$

$$\frac{dp}{dt}=rg\frac{\partial j}{\partial t}=rg\left(\frac{\partial j}{\partial t}+u\frac{\partial j}{\partial x}\right)$$

gives

$$\frac{\partial j}{\partial t}+\frac{c^2}{g}\frac{\partial u}{\partial x}=0 \qquad \text{(eq. 2.35)}$$

Equation of movement

The equation of movement is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}-g\sin\beta+g\frac{u|u|}{C^2R}=0$$

(eq. 2.36)

with β the slope of the pipe. This can be rewritten using the piezometric level instead of a pressure level: φ = z + h = z + p/ρg:

$$\frac{\partial \varphi}{\partial t}=\frac{1}{\rho g}\frac{\partial p}{\partial t}$$

$$\frac{\partial \varphi}{\partial x}=\frac{1}{\rho g}\frac{\partial p}{\partial x}-\sin\beta$$

leading to

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial \varphi}{\partial x}+g\frac{u|u|}{C^2R}=0 \qquad \text{(eq. 2.37)}$$

In the case of water hammer the friction can be neglected as well as the convective term $\frac{\partial u}{\partial x}$, which leaves the equation of movement to

$$\frac{\partial u}{\partial t}+g\frac{\partial \varphi}{\partial x}=0 \text{ or } \frac{1}{g}\frac{\partial u}{\partial t}+\frac{\partial \varphi}{\partial x}=0 \qquad \text{(eq. 2.38)}$$

Examples of water hammer

The water hammer formulas describe what happens when the pressure and/or volume flow boundaries vary so quickly that the rigid column approach is not valid any more. This is the case when valves are closed, pumps are started or stopped (planned or unplanned in the case of pump trip) or any other sudden change in pressure or flow.

The set of equations is:

$$\frac{1}{g}\frac{\partial u}{\partial t}+\frac{\partial \varphi}{\partial x}=0 \text{ and } \frac{\partial \varphi}{\partial t}+\frac{c^2}{g}\frac{\partial u}{\partial x}=0 \qquad \text{(eq. 2.39)}$$

The general solution of this set of equations is

$$\varphi=F(x+ct)+f(x-ct)+\varphi_0 \qquad \text{(eq. 2.40)}$$

$$u=-\frac{g}{c}\left[F(x+ct)+f(x-ct)\right]+u_0 \qquad \text{(eq. 2.41)}$$

with φ_0 and u_0 are the initial pressure level and velocity at t = 0.

These equations are called the characteristic equations. Basically these equations describe the movement of the pressure wave during the time step Δt over a distance c* Δt along the pipe. The function φ=f(x-ct) moves in positive direction and the function φ=F(x+ct) moves in negative direction.

These formulas can be used to estimate the maximum and minimum pressures as a result of water hammer and give an indication whether special measures are necessary to avoid too high or too low pressures. Too low pressure might induces cavitation or when thin wall pipes are used a implosion of the pipe. An example of this is given in figure 2.16.

An example for calculation of the maximum and minimum pressure in a pipe as result of closure of a valve will be elaborated

Starting point are the characteristic equations:

$$\varphi=F(x+ct)+f(x-ct)+\varphi_0 \qquad \text{(eq. 2.42)}$$

Fig. 2.16 - Imploded tank

$$u = -\frac{g}{c}\left[F\left(x+ct\right)+f\left(x-ct\right)\right]+u_0 \quad \text{(eq. 2.43)}$$

At the moment the valve closes the forward moving function becomes zero: f(x-ct) = 0. This transforms the equation to:

$$j = F\left(x+ct\right)+j_0 \quad \text{(eq. 2.44)}$$

$$u = 0 = -\frac{g}{c}\left[F\left(x+ct\right)\right]+u_0 \quad \text{(eq. 2.45)}$$

or

$$j - j_0 = \frac{c \cdot u_0}{g} \quad \text{(eq. 2.46)}$$

If the pump would have been stopped, meaning that the backward moving function becomes zero would have resulted in

$$j - j_0 = -\frac{c \cdot u_0}{g} \quad \text{(eq. 2.47)}$$

Both these functions are called the Joukovsky equations. The values for the pressure that are calculated using this function is the absolute maximum or minimum that can occur during valve closure or pump trip. In practise these pressures will be lower, because valves don't close instantaneously and pumps have an inertia causing them to continue working during some seconds. Another aspect is the value of c, the speed of pressure wave movement. When the water contains gas as is possible in sewerage, the value will be much lower. Also the characteristics of the pipe determine the value of c. A less flexible pipe will induce a higher maximum pressure.

Example

The maximum pressure for two types of pipe will be considered at the event of quick closure of a valve. Assume a pipe with a length of 5000 meter a diameter of 1000 mm, made of steel with a wall thickness δ of 15 mm. The elasticity of steel is $2{,}2*10^5$ N/m^2 and the compressibility of water $K_{water} = 2{,}05 * 10^5$ N/m^2.

$$\frac{1}{c^2} = \frac{r}{K} + \frac{rD}{Ed} \Rightarrow c = \frac{1}{\sqrt{\frac{1}{r}\left(\frac{1}{K}+\frac{D}{Ed}\right)}} = 1390 \text{ m/s}$$

With an initial velocity of the water of 1 m/s the additional pressure will be

$$\frac{c \cdot u_0}{g} = \frac{1390*1}{10} = 139 \text{ mWc}$$

Probably this pressure is inadmissible and counter measures have to be taken. One of the most simple measures is to avoid the quick closure of the valve. The deceleration of the fluid is smoother and water hammer can be avoided. To quantify this the following calculation can be made. The pressure wave is formed during the last 20% of the actual closing of the valve. In this period the deceleration is largest. High pressures (water hammer) can be avoided if the pressure wave is allowed to travel from the valve to the upstream boundary and back, during the last 20% of the closure interval of the valve. In the last 20 % the actual deceleration will take place, causing the possible water hammer.

With a length of 5000 meter between the valve and an upstream boundary this will take 2* 5000/1390 = 7 seconds. An upstream boundary is for instance a water tower or a connection to a more looped network. Essential is that the pressure can be relieved by relatively small storage capacity, a pressure relieve valve (figure 2.17) or a larger network.

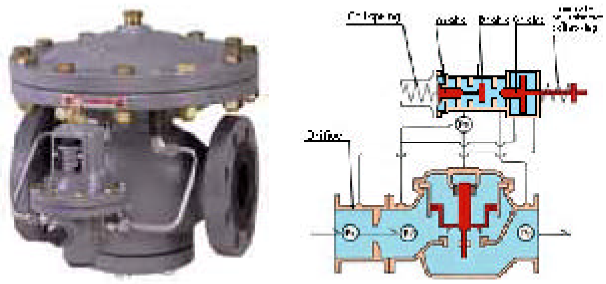


Fig. 2.17 - Pressure relieve valves

Consider another pipe of a more flexible material like PVC. The length is 5000 meter; the diameter is 630 mm (largest common available diameter) and a wall thickness of 25 mm (PN10, see also design exercise). The elasticity of the material is $2{,}0*10^9$ N/m^2 and the compressibility of the water is $2{,}05*10^5$ N/m^2. The pressure wave speed is in this case 270 m/s and the maximum extra pressure is 27 mWc with an initial velocity of 1 m/s.

Although this extra pressure is probably allowable in this situation, also the negative pressure wave at the downstream side of the valve has to be considered.

This negative pressure is larger than atmospheric and can cause cavitation. In this case the damp tension is sub seeded and the water 'boils' with all negative implications of cavitation (see paragraph XX). Closing the last 20% of the pipe in a time the pressure wave can travel from the valve to the upstream boundary ands back again will be a solution as well. This will take 2*5000/270 = 37 seconds.

Water hammer prevention

To prevent the high prssure as result of water hammer for instance a buffer tower can be used. The principle is the same as the wind kettel. Figure 2.18 shows the effect of a buffer tower in combination with a pump trip. Often water towers are used like this. Figure 2.19 shows the effect of a buffer tower when a valve is quickly closed.

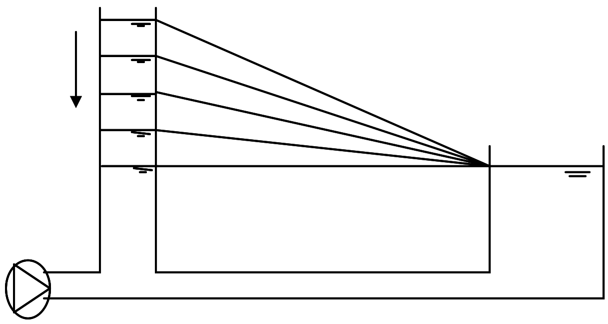


Fig. 2.18 - Buffer tower and pump trip

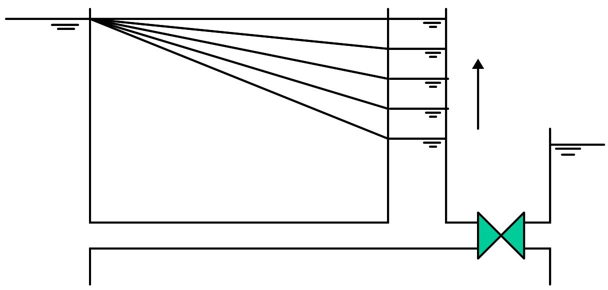


Fig. 2.19 - Buffer tower and valve

2.7 Summary

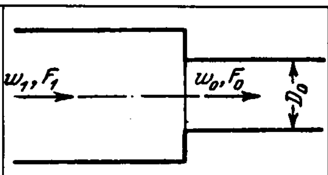
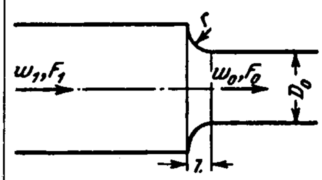
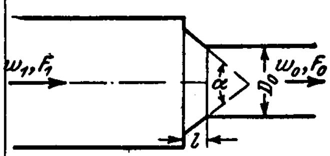
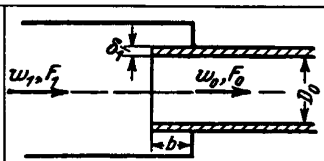
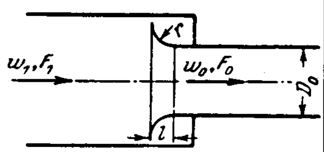
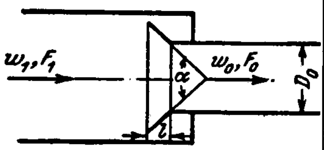
Three types of one-dimensional flow can be distinguished within the field of water transport through pipes:

- o Surcharged pressurised flow:
  - Rigid column approach
  - Darcy Weissbach
  - Time independent
  - Friction loss dominant
  - Drinking water transport, sewerage water
- o Open channel flow
  - Free water surface
  - Time dependant
  - Storage dominant
  - Storm water, urban drainage and sewerage
- o Water hammer
  - Special case of pressurised flow
  - Fast changing boundaries, time dependent
  - Special analysis
  - Risk approach

Entrées avec retrécissement brusque
$Re = w_0 D_H / \nu > 10^4$

Chapitre III

Diagramme 3.9

Conditions à l'entrée	Schéma	Coefficient de perte de charge $\zeta = \dfrac{\Delta H}{\gamma w_0^2 / 2g}$
A. Section d'entrée dans une paroi frontale ($b/D_H = 0$) $D_H = 4F_0/\Pi_0$; Π_0 : périmètre		
Bord d'entrée à angle droit		$\zeta = 0{,}5\,(1 - F_0/F_1)$
Bord d'entrée arrondi		$\zeta = \zeta'(1 - F_0/F_1)$ où ζ' est déterminé suivant la courbe $\zeta = f\,(b/D_H)$ sur le diagramme 3.3 (graphique *c*).
Bord d'entrée de forme conique		$\zeta = \zeta'(1 - F_0/F_1)$ où ζ' est déterminé suivant la courbe $\zeta = f\,(\alpha^0, l/D_H)$ sur le diagramme 3.6.
B. Section d'entrée en avant de la paroi frontale ($b/D_H > 0$)		
Bord d'entrée effilé ou non		$\zeta = \zeta'(1 - F_0/F_1)$ où ζ' est déterminé suivant la courbe $\zeta = f\,(\delta_1/D_H, b/D_H)$ sur le diagramme 3.1.
Bord d'entrée arrondi		$\zeta = \zeta'(1 - F_0/F_1)$ où ζ' est déterminé suivant la courbe $\zeta = f\,(r/D_H)$ sur le diagramme 3.3 (graphiques *a* et *b*).
Bord d'entrée de forme conique		$\zeta = \zeta'(1 - F_0/F_1)$ où ζ' est déterminé suivant la courbe $\zeta = f\,(\alpha^0, l/D_H)$ sur le diagramme 3.5 ; les valeurs de ν sont données dans le § 1.3, *b*).

Elargissement brusque en aval d'un tronçon long et rectiligne, un diffuseur, etc, avec une répartition des vitesses suivant la loi exponentielle
Section circulaire ou rectangulaire Re = $w_0 D_H/\nu > 3{,}5 \cdot 10^3$

Chapitre IV

Diagramme 4.2

$$\frac{w}{w_{max}} = \left(1 - \frac{y}{R_0}\right)^{\frac{1}{m}} \; ; \; m \geqslant 1$$

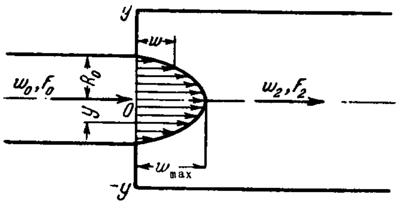
$$\zeta = \frac{\Delta H}{\frac{\gamma w_0^2}{2g}} = \frac{1}{n^2} + N - \frac{2M}{n}$$: est déterminé sur le graphique *a*).

$$M = \frac{(2m+1)^2 (m+1)}{4m^2 (m+2)}$$

$$N = \frac{(2m+1)^3 (m+1)^3}{4m^4 (2m+3)(m+3)}$$

sont déterminés sur sur le graphique *b*) ;

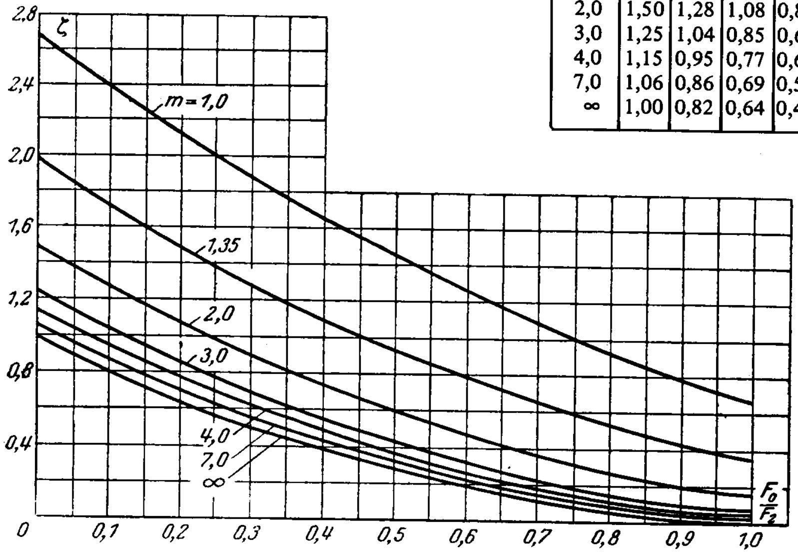
Les valeurs de ν sont données dans le paragraphe 1.3, *b*).



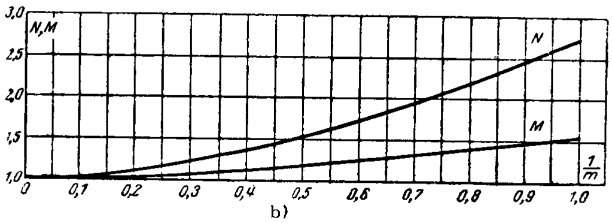
$D_H = 4F_0/\Pi_0$; Π_0 : périmètre ; $n = F_2/F_0$

Valeurs de ζ

m	F_0/F_2									
	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	1,0
1,0	2,70	2,42	2,14	1,90	1,66	1,45	1,26	1,09	0,94	0,70
1,35	2,00	1,74	1,51	1,29	1,00	0,93	0,77	0,65	0,53	0,36
2,0	1,50	1,28	1,08	0,89	0,72	0,59	0,46	0,35	0,27	0,16
3,0	1,25	1,04	0,85	0,68	0,53	0,41	0,30	0,20	0,14	0,07
4,0	1,15	0,95	0,77	0,62	0,47	0,35	0,25	0,17	0,11	0,05
7,0	1,06	0,86	0,69	0,53	0,41	0,29	0,19	0,12	0,06	0,02
∞	1,00	0,82	0,64	0,48	0,36	0,25	0,16	0,09	0,04	0



a)



b)

m	1,0	1,35	2,0	3,0	4,0	7,0	∞
N	2,70	2,00	1,50	1,25	1,15	1,06	1,0
M	1,50	1,32	1,17	1,09	1,05	1,02	1,0