Scattering in metals

- Metals without scattering
- Impurity scattering
- Screening
- Localization
- Doped semiconductors
- Quasiparticles and Fermi liquid theory
- Electron-electron scattering

Electron as a wave packet

Bloch waves: fixed (quasi)momentum; coordinate absolutely uncertain

Useless for most transport phenomena!

Let us build a wave packet $\Delta\vec{p}\Delta\vec{r} \sim \hbar^3$

Collect Bloch waves with quasimomenta $|\vec{p} - \vec{p}_0| \le |\Delta\vec{p}|$

$I \sim \int d\vec{p} e^{i\vec{p}\vec{r}/\hbar - i\varepsilon(\vec{p})t/\hbar} u_{\vec{p}}(\vec{r})$ Expand for $\vec{p}$ around $\vec{p}_0$:

$$I \sim u_{\vec{p}_0}(\vec{r}) e^{i\vec{p}_0\vec{r}/\hbar - i\varepsilon(\vec{p}_0)t/\hbar} \int_{\Delta\vec{p}} d\delta\vec{p} e^{i\delta\vec{p}(\vec{r} - t\partial\varepsilon(\vec{p})/\partial\vec{p})/\hbar}$$

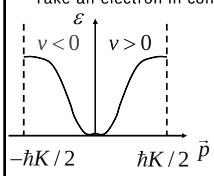
The integral is non-zero provided

$\vec{r} \approx \frac{\partial\varepsilon}{\partial\vec{p}} t$ ⇒ **Group velocity:** $\vec{v} \approx \frac{\partial\varepsilon}{\partial\vec{p}}$

Bloch oscillations

How conduction in an ideal metal would look like?

Take an electron in constant electric field $\frac{d\vec{p}}{dt} = e\vec{E}$



Quasimomentum increases: does the velocity increase?

No!

Electrons perform oscillating motion: **Bloch oscillations** No conductance!

Only have been observed in VERY clean semiconductors. Why do they never occur in real systems?

Because of the scattering!!

Scattering

Scattering of electrons

Elastic

Very complicated potential

Momentum changed
Energy not changed

- Bulk impurities
- Surface impurities
- Dislocations and other defects
- Interfaces
- …

Leads to: conductance, localization, …

Inelastic

Momentum changed
Energy changed

- Other electrons
- Lattice vibrations (phonons)
- Collective oscillations
- External radiation
- …

Leads to: conductance, dephasing…

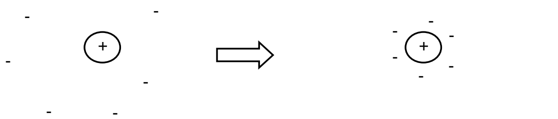
Impurity scattering

Neutral impurities: impenetrable spheres $R \sim R_b = \hbar^2/me^2 \sim 0.1\mathrm{nm}$

(same order of the lattice period)

Charged impurities: Long-range Coulomb potential $\varphi \sim e/r$

In metals, also behave like impenetrable spheres due to screening



Screening

Unscreened charge: creates Coulomb potential

$\Delta\varphi = 4\pi e\delta(\vec{r})$ ⇒ $\varphi = e/r$

If there are free charges around: Screened charge

Potential obeys Poisson equation: $\Delta\varphi = 4\pi e\delta n_e$

δn_e - shift of the electron density due to the potential

Thomas-Fermi approximation:

$$\delta n_e = n_e(\mu + e\varphi) - n_e(\mu) \approx \frac{\partial n_e}{\partial\mu}\varphi$$

Screening radius: $a_s = (4\pi e^2 \partial n_e/\partial\mu)^{-1/2}$

(only makes sense for $a_s \ge \lambda_F$)

Screening

Poisson equation: $\Delta\varphi = a_s^{-2}\varphi$

Solution: $\boxed{\varphi = -(e/r)e^{-r/a_s}}$

Decays at the distances of the order of $a_s = (4\pi e^2 \partial n_e / \partial\mu)^{-1/2}$!

"Impenetrable sphere" of the radius a_s

Estimates for the free electron gas

$$n_e(\mu) = \frac{1}{3\pi^2\hbar^3}(2m\mu)^{3/2} \Longrightarrow a_s \propto \mu^{-1/4} \propto n^{-1/6}$$

$$\lambda_F \propto n^{-1/3}$$

TF approximation is good for high density!

Screening

Estimates for the free electron gas: Cu

$E_F = 7eV \approx 10^{-18}\,J$

$m = 10^{-30}\,kg \Longrightarrow \boxed{\lambda_F = \hbar/\sqrt{2mE_F} \approx 10^{-10}\,m = 0.1nm}$ $\boxed{v_F \approx 10^6\,m/s}$

$\hbar = 10^{-34}\,J\cdot s \Longrightarrow \boxed{n_e = \frac{1}{3\pi^2\hbar^3}\left(2mE_F\right)^{3/2} = 3\cdot 10^{28}\,m^{-3}}$

$\boxed{e^2/\hbar c = 1/137 \Rightarrow e^2 = 3\cdot 10^{-28}\,J\cdot m}$

$\Longrightarrow \boxed{a_s = \frac{1}{\sqrt{4\pi e^2(2n/3E_F)}} = 10^{-11}\,m}$

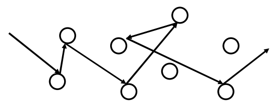
$\boxed{a_s \sim \lambda_F}$ **in metals**

Impurity scattering

Elastic: Momentum relaxation; no energy relaxation

Effective scattering cross-section: $a_s^2 \sim 10^{-20}\,m^{-2}$

Density of impurities: n_i



If an electron moves a distance L it collides with $n_i L a_s^2$ impurities

Mean free path l - distance it passes without collisions

$l \sim 1/(n_i a_s^2)$ Example: pure Cu $c_i \sim 10^{-5} \Rightarrow l \sim 10^5 n_e^{-1} a_s^{-2} \sim 300\mu m$

Momentum relaxation time: $\tau = l/v$ Temperature independent!!

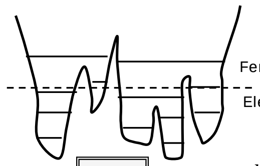
Anderson localization

Intermediate conclusion: The more impurities the higher is the conductance.

Wrong and contradicts the common sense!

Fermi level here: weak disorder; metal

Ohm's law: $\boxed{R \propto L}$



Fermi level here: strong disorder; insulator

Electron states are localized $\boxed{R \propto e^L}$

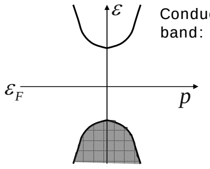
Need really dirty metals

Criterion: $\boxed{k_F l \sim 1}$ Recollect $l \sim 1/(n_i a_s^2) \Longrightarrow c_i \sim 1$

Doped semiconductors

Localization: for metals exotic possibility and almost never observed. It is a pity, since physics is interesting. Let us look at semiconductors.

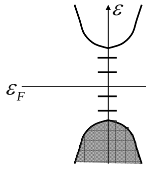
Clean semiconductor, zero temperature: a good insulator with a gap



Conduction band: empty

Valence band: fully occupied

Doped semiconductor: impurity states



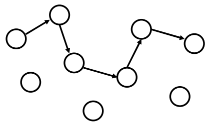
Donor (electron) states

Acceptor (hole) states

Impurity states: randomly located in space and energy

Doped semiconductors

Transport: hopping between localized states: Insulator!!



Probability of a single hop:

$$P \propto \exp(-r/a - \Delta E/k_B T)$$

Conduction: electrons optimize the hopping path to increase the total probability.

Even more impurities: states start to overlap: Percolation
Metallic state

Electron-electron scattering

Typical energy of electron-electron interaction: $U \sim e^2 n_e^{1/3}$

Take again copper: $n_e = 3 \cdot 10^{28} m^{-3}$ $\boxed{U = 9 \cdot 10^{-18} J \approx E_F}$

Strong interactions — too bad! What should we do?

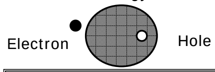
Landau theory of Fermi liquid:

Elementary excitations (quasiparticles) in a system of interacting electrons have properties similar to elementary excitations in a free electron gas

Quasiparticles in a free electron gas

Ground state: states below the Fermi energy are occupied; others are empty

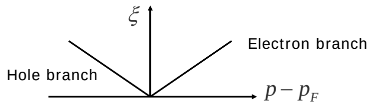
Elementary excitations:



Electron energy: $\boxed{\xi_e = \frac{p^2 - p_F^2}{2m} \approx \frac{p_F}{m}(p - p_F) = v_F(p - p_F)}$

Hole energy: $\boxed{\xi_h = \frac{p_F^2 - p^2}{2m} \approx \frac{p_F}{m}(p_F - p) = v_F(p_F - p)}$

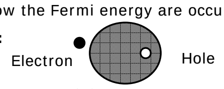
$\boxed{\xi = v_F |p_F - p|}$



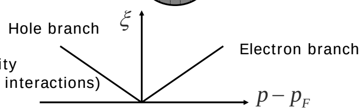
Quasiparticles in Fermi liquid

Ground state: states below the Fermi energy are occupied; others are empty

Elementary excitations:



$\boxed{\xi = v|p_F - p|}$



v -effective velocity (determined by interactions)

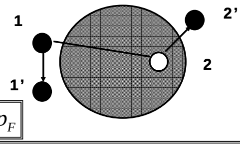
Wave function of a quasiparticle: $\boxed{\psi \propto \exp(i\xi t/\hbar - \gamma t/\hbar)}$

γ - damping due to inelastic interactions (scattering of quasiparticles)

Quasiparticles are well-defined provided $\boxed{\gamma \ll \xi}$

Damping in weakly interacting Fermi gas

1 interacts with 2



Momentum conservation:

$\boxed{\vec{p}_1 + \vec{p}_2 = \vec{p}'_1 + \vec{p}'_2}$

Pauli principle: $\boxed{p_1, p'_1, p'_2 > p_F;\ p_2 < p_F}$

Energy conservation: Golden rule $\boxed{\gamma_{p_1} \propto \int \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon'_1 - \varepsilon'_2) d\vec{p}_2 d\vec{p}'_1}$

Result: $\boxed{\gamma_{\varepsilon_1} \approx \varepsilon^2 / \varepsilon_F}$

Must be also valid in Fermi liquid (phase space argument)

For $\boxed{\varepsilon \ll \varepsilon_F}$, $\boxed{\gamma \ll \varepsilon}$ - quasiparticles are well-defined

Electron-electron scattering

$\boxed{\gamma_{\varepsilon_1} \approx \varepsilon^2 / \varepsilon_F}$ Relevant energies: $\varepsilon \sim k_B T$

Energy relaxation time: $\boxed{\tau_\varepsilon \sim \hbar \varepsilon_F / (k_B T)^2}$

Let us compare this to the momentum relaxation time: $\boxed{\tau \sim l / v}$

$\tau \ll \tau_\varepsilon \Rightarrow k_B T \ll \sqrt{\hbar v \varepsilon_F / l}$

Again 5-nines copper: $l = 300 \mu m,\ \varepsilon_F = 7eV$

$T = 500K$ - does not occur in real life