Macroscopic theory of superconductivity

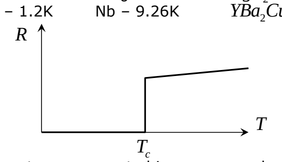
- Zero resistance
- Meissner effect
- Specific heat
- Intermediate state
- London equations
- Penetration depth

Resistance

Experimental facts: there are metals and metallic compounds with the following properties:

- Below certain critical temperature T_c the resistance vanishes. Kamerlingh-Onnes, 1914
  Examples:

W – 0.012K   α-*Hg* – 4.15K MgB_2 - 39K
Al – 1.2K   Nb – 9.26K $YBa_2Cu_3O_{7-x}$ - 95K



If currents are generated in a superconductor, they do not decay.

Meissner effect

- Superconductors are ideal diamagnets. Weak magnetic fields not penetrate superconductors (Meissner effect). Fields greater than H_c destroy superconductivity.

Examples: $W:1G=10^{-4}T$ $\alpha-Hg:411G$
$Al:99G$ $Nb:1980G$

The external field generates surface currents that screen the field.

Meissner effect

Magnetic field disappears in the bulk of a superconductor

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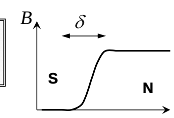
Must be a characteristic scale of the magnetic field decay: Penetration depth

Phenomenologically:
$$\delta(T)=\frac{\delta(0)}{\sqrt{1-(T/T_c)^4}}$$



Phase transition:

- Zero magnetic field – second order (at the transition point, there is no Meissner effect, and phases are the same)
- Finite magnetic field – first order phase transition

Specific heat

Experimentally: activation benaviour in the superconducting phase

$$c(T)\propto \exp(-\Delta/k_BT), T\ll T_c$$

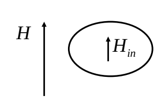
At the transition point: jump of the specific heat

Conclusion: new state of electrons in a superconductor, different from the normal metal

Avenues to proceed:
- Phenomenological theory: London equations, Ginzburg-Landau equations
- Microscopic theory: Bardeen – Cooper - Schrieffer

Meissner effect

Consider a metallic ellipsoid in an external magnetic field



A tiny bit of electrodynamics

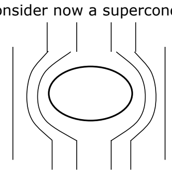
Outside: $\vec{H}=\vec{B}, \vec{M}=0$

Inside:
$$\vec{H}_{in}=\vec{H}-4\pi n\vec{M}$$
$$\vec{B}_{in}=\vec{H}_{in}+4\pi\vec{M}$$

H – magnetic field
B – magnetic induction
M – magnetization
0 < *n* < 1 – demagnetization factor (geometry dependent)

Meissner effect

Consider now a superconducting ellipsoid in an external magnetic field



Inside:

$$\vec{H}_{in} = \vec{H} - 4\pi n\vec{M}$$
$$\vec{B}_{in} = \vec{H}_{in} + 4\pi\vec{M} = 0$$

$$H_{in} = H/(1-n) > H$$

$H > H_c$ - normal state

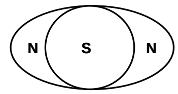
$H < H_c(1-n)$ - superconducting state

$H_c(1-n) < H < H_c$ - a problem!

Intermediate state

$H_c(1-n) < H < H_c$

Coexistence of normal and superconducting states?

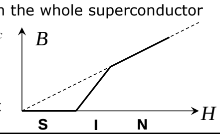


Does not work: the field penetrates the normal parts, but then the field must decay away from **S** – no normal part is possible

Solution: "Intermediate state"

$$H_{in} = H_c \Rightarrow B = \frac{H}{n} - \frac{H_c(1-n)}{n}$$

The field in the whole superconductor equals H_c



Normal and superconducting domains coexist

London equations

Phenomenological theory based on the experimental facts:
Zero resistance and Meissner effect.

No resistance: The electrons are accelerated by the field

$$m\frac{d\vec{v}}{dt} = e\vec{E},\ \vec{j} = ne\vec{v} \Rightarrow \frac{d}{dt}\frac{m}{ne^2}\vec{j} = \vec{E} \quad \Rightarrow \quad \frac{\partial}{\partial t}\frac{m}{ne^2}\vec{j} = \vec{E}$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}$$

$$\frac{\partial}{\partial t}\left(\vec{\nabla}\times\frac{m}{ne^2}\vec{j} + \frac{1}{c}\vec{H}\right) = 0$$

$$\vec{\nabla}\times\frac{m}{ne^2}\vec{j} + \frac{1}{c}\vec{H} = 0$$

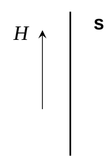
$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}\vec{\nabla}$$

related to Fermi velocity

related to the current

Penetration depth

$$\frac{\partial}{\partial t}\frac{m}{ne^2}\vec{j} = \vec{E} \qquad \vec{\nabla}\times\frac{m}{ne^2}\vec{j} + \frac{1}{c}\vec{H} = 0$$



Maxwell equation: $\vec{\nabla}\times\vec{H} = \frac{4\pi}{c}\vec{j}$

$$\Delta\vec{H} = \delta^{-2}\vec{H}$$

$$\delta = \sqrt{\frac{mc^2}{4\pi ne^2}}$$ - penetration depth

Decay of the magnetic field in the bulk of S: $H\exp(-x/\delta)$
(also the current)

Estimates: $\delta \sim 3\cdot 10^{-6} m$ -reasonably agrees with experiments. Should we substitute density of all electrons???