## Strongly correlated systems

- Wigner crystal
- Hall effect
- Integer quantum Hall effect
- Fractional quantum Hall effect
- Kondo effect

## Wigner crystal

Consider electrons at low density

Intuition from classical physics:

Low density ⟹ Long distances between electrons ⟹ Weak interactions

What do we get from quantum mechanics?

Kinetic energy: $E_{kin} \sim p^2/2m \sim \hbar^2/ma^2$

Potential energy (no screening): $E_{pot} \sim e^2/a$

Low density of electrons: $a > \hbar^2/me^2 = a_B$

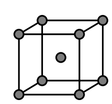
Potential energy dominates!

## Wigner crystal

What is the ground state of the electron system at low density?

We need to minimize the potential energy!

Wigner crystal Bulk-centered cubic lattice



$a > \hbar^2/me^2 = a_B$

Problems with observation:

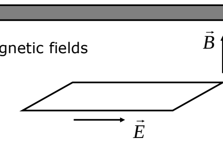
- Difficult to create a low density
- No uniform positive background
- Destroyed by disorder

Realisations:

- Electrons at helium surface
- Semiconducting heterostructures

## Hall effect

Electron moving in crossed electric and magnetic fields




Lorentz force: $\vec{F} = e\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$

y-direction: no force $E_y = \frac{1}{c}Bv_x$ ⟹ voltage in the transverse direction Hall effect

$$\begin{matrix} I = env_x W \\ V_H = E_y W \end{matrix} \Rightarrow R_H = \frac{V_H}{I} = \frac{B}{enc}$$ - (classical) Hall resistance

Impurity independent $\omega_c \gg 1/\tau$

Proportional to the magnetic field

## Landau quantization in 2D

A free electron in magnetic field: $\vec{B} \parallel \hat{z}$

Schrödinger equation: $-\frac{\hbar^2}{2m}\left(\vec{\nabla} + \frac{ie\vec{A}}{\hbar c}\right)^2 \psi = \varepsilon\psi$ $A_y = Bx; A_x = A_z = 0$

Solutions: labeled by one index *n*

$\psi_n(\vec{r}) = \exp(ik_y y)\varphi_n(x - \hbar c k_y/eB)$

$\varphi_n$ - wave functions of a harmonic oscillator

Energies: $\varepsilon_n = (e\hbar B/mc)(n+1/2) \equiv \hbar\omega_c(n+1/2)$ - strongly degenerate!!

(Landau levels)

$\omega_c = eB/mc$ - cyclotron frequency

## Filling factor

$\varepsilon_n = \hbar\omega_c(n+1/2)$ How many states are there in the area *A* on one LL?

$\psi_n(\vec{r}) = \exp(ik_y y)\varphi_n(x - \hbar c k_y/eB)$

Quantize $k_y$ $k_y = 2\pi m_y/W$

Condition for the maximal *x*: $L = \hbar c k_y/eB = k_y l^2 = 2\pi l^2 m_{\max}/W$

$l = \sqrt{\hbar c/eB}$ - magnetic lenth

\# of states: $m_{\max} = A/2\pi l^2 \equiv n_B A$

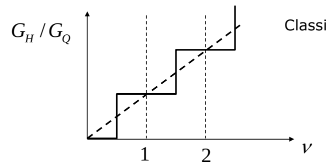
Filling factor: $\nu = \frac{\#electrons}{\#states} = \frac{n}{n_B} = \frac{2\pi\hbar c n}{eB}$

Fix electron concentration: Higher magnetic fields correspond to lower filling factors

## Integer Quantum Hall effect

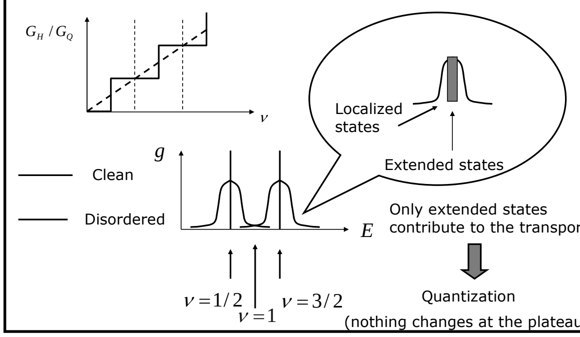
$G_H = \frac{enc}{B}$  Integer filling factor: $\nu = m \in \mathbb{N}$  $G_H = G_Q m$  $G_Q \equiv e^2/2\pi\hbar$

Experiment (Von Klitzing '80)




At plateaus: no longitudinal resistance

## Integer Quantum Hall effect



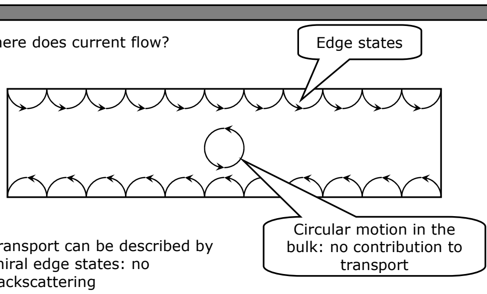

Only extended states contribute to the transport

Quantization

(nothing changes at the plateau)

## Edge states

Where does current flow?



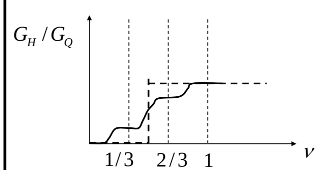

Transport can be described by chiral edge states: no backscattering

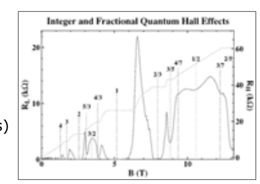
## Fractional Quantum Hall effect

Experiment (Tsui, Störmer, Gossard '81) : additional plateaus at

$\nu = p/(2m+1)$

$G_H = \frac{p}{2m+1} G_Q$




New states of matter (Laughlin states)

Interactions are important!

## Laughlin wave function

Consider a state with $\nu = 1/(2m+1)$

Wave function: $z_n = x_n + iy_n$

$$\Psi(z_1 ... z_n) \propto \prod_{ij} (z_i - z_j)^{(2m+1)} \prod_i e^{-|z_i|^2/2l}$$

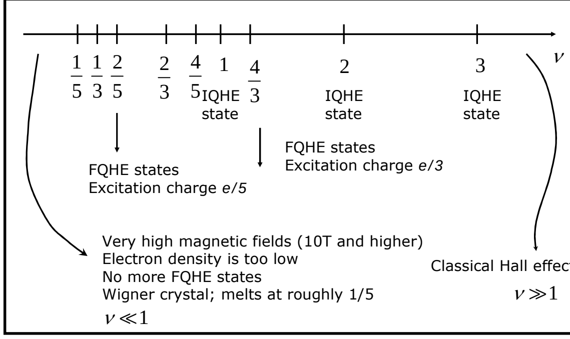
Fermionic!

Properties:

- Equivalent to a system of Coulomb charges in a confining potential;
- Excitations: fractional charge *e/(2m+1)*, can be probed in experiments
- Excitations: fractional statistics (not bosonic nor fermionic)
- Edge states: *(2m+1)* channels; not very well understood

$\nu = p/(2m+1)$  Excitations still have fractional charge *e/(2m+1)* and fractional statistics

## Electrons in magnetic field



## Kondo effect

Experimentally: resistance minimum in some very clean metals (Cu, Au)

Explanation: spin-flip scattering

$$V = -J\sum_i \vec{\sigma}\vec{S}_i \delta(\vec{r}-\vec{R}_i)$$

σ - electron spin (vector of Pauli matrices)

S - impurity spin

What if we treat it in the perturbation theory?

Amplitude of electron scattering from σ to σ':

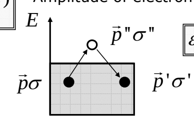
First order: $-J\left(\vec{\sigma}\vec{S}\right)_{\sigma'\sigma}$

## Kondo effect

$$V = -J\sum_i \vec{\sigma}\vec{S}_i \delta(\vec{r}-\vec{R}_i)$$

Amplitude of electron scattering from σ to σ':

Second order:




$$\varepsilon(\vec{p}) = \varepsilon(\vec{p}\,')$$

$$J^2 \sum_{\vec{p}\,''\sigma''} \frac{(\sigma S)_{\sigma'\sigma''}(\sigma S)_{\sigma''\sigma}}{\varepsilon(\vec{p})-\varepsilon(\vec{p}\,'')}\left(1-f(\vec{p}\,'')\right)$$

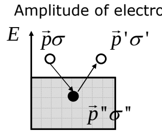
Makes sure the intermediate state is empty

## Kondo effect

$$V = -J\sum_i \vec{\sigma}\vec{S}_i \delta(\vec{r}-\vec{R}_i)$$

Amplitude of electron scattering from σ to σ':

More second order:




$$\varepsilon(\vec{p}) = \varepsilon(\vec{p}\,')$$

$$-J^2 \sum_{\vec{p}\,''\sigma''} \frac{(\sigma S)_{\sigma''\sigma}(\sigma S)_{\sigma'\sigma''}}{\varepsilon(\vec{p}\,'')-\varepsilon(\vec{p}\,')} f(\vec{p}\,'')$$

Makes sure the intermediate state is occupied

## Kondo effect

Summing up: Amplitude of electron scattering from σ to σ':

$$-J\left(\sigma S\right)_{\sigma'\sigma}\left[1 - Jg\ln\frac{E_F}{\max(T,\xi)}\right]$$

First order

Second order

J > 0: ferromagnetic; the total spin of electron and impurity S+1/2

But the total spin of electron and impurity is conserved

→ Spin-flip impossible; amplitude goes down

J < 0: antiferromagnetic: amplitude goes up (spin-flip scattering)

## Resistance minimum

Resistance: proportional to the amplitude squared

$$R \sim R_{non-Kondo} + R_K(1 - 2Jg\ln E_F / k_B T)$$

J < 0: increases at low temperatures

Other contributions (usually phonons):
always decrease at low tempertures

↓

Resistance minimum

## Kondo resonance

Amplitude of electron scattering from σ to σ':

$$-J\left(\sigma S\right)_{\sigma'\sigma}\left[1 - Jg\ln\frac{E_F}{\max(T,\xi)}\right]$$

Kondo temperature:

$$T_K = E_F \exp(-1/g\left|J\right|)$$

For antiferromagnetic interaction, $T < T_K$
the perturbation theory breaks down

- Exact solution (one impurity; no solution for several impurities)
- Summation of the leading terms

$$R \sim R_{non-Kondo} + \frac{R_K}{(1 + Jg\ln E_F / k_B T)^2}$$

J<0: diverges!!!

Resonance corresponding to a bound state of an impurity and an electron cloud: Collective effect