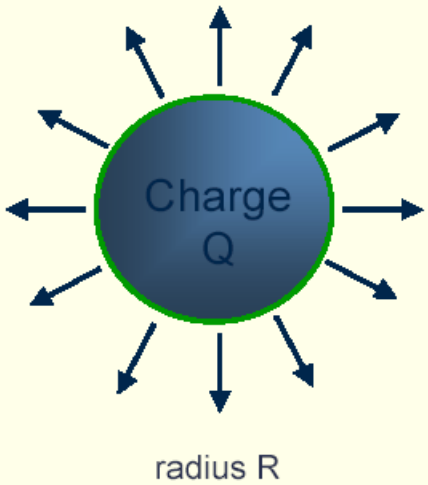
charging energy: $E_c = e^2/2C$

- What is the capacitance of an isolated piece of metal (for example a sphere)?



Electric field:

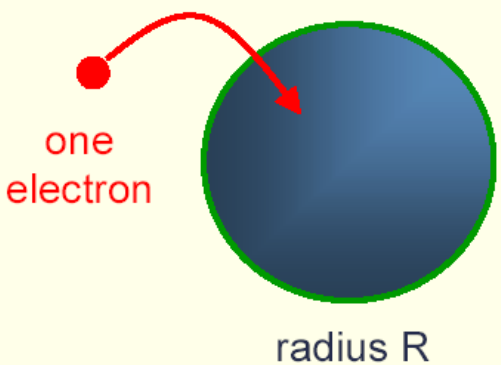
$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \quad (r > R)$$

Voltage:

$$V(R) = -\int_{R}^{\infty} \vec{E}(\vec{r}) \cdot d\vec{r} = \frac{Q}{4\pi\varepsilon_0 R}$$

$$C = Q/V = 4\pi\varepsilon_0 R$$

- What is the energy needed to charge the sphere with one electron (1/2QV with Q = e)?



R	C	E/k_B
10 μm	1.1 x 10^{-15} F	0.84 K (^{3}He)
1 μm	1.1 x 10^{-16} F	8.4 K (LHe)
0.1 μm	1.1 x 10^{-17} F	84 K (LN_2)
0.01 μm	1.1 x 10^{-18} F	840 K (spa)

$C = 4\pi\varepsilon_0 R$

overview energy scales

k_BT	thermal energy scale; at room temperature 26 meV.
eV	the energy an electron gains when crossing a potential difference V
μ	chemical potential; the energy required to add a particle to the system
$\mu_B B$	Zeeman energy of an electron in a magnetic field
E_F	Fermi energy (it is the energy of the highest occupied electron state); for metals it is on the order of a few eV. For semiconductors, it can be much smaller.
ΔE	level spacing; energy difference between single-particle states calculated from the Schrödinger Equation
E_C	charging energy $E_C=e^2/2C$ where C is the total capacitance of the system (we come back to this when discussing Coulomb blockade)
E_T	Thouless energy: the energy scale of coherence effects (see lecture notes)

In discussing band bending and semi-conductors: the work function (the energy requires to an electron from the Fermi level into the continuum), the valence and conduction bands

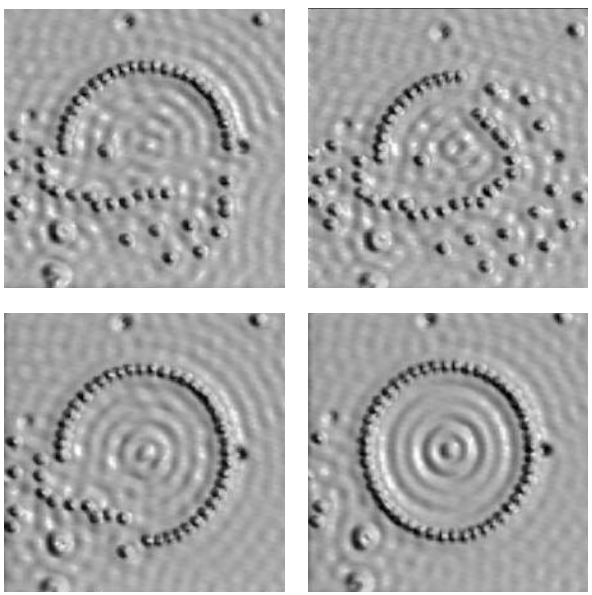
length scales: quantum regime

An important feature of mesoscopic and nanoscale structures is that their dimensions are COMPARABLE to the fundamental "size" of the electron causing their electrical properties to be strongly influenced by QUANTUM-MECHANICAL transport effects

In solid-state systems the size of the electron is essentially given by the FERMI WAVELENGTH λ_F (What is a typical value for a metal?)

⇒ In most METALS the electron density is very large (10^{21} cm^{-3}) and the value of the Fermi wavelength is consequently of order a FEW nanometers

⇒ In semiconductors however the LOWER carrier density ($<<10^{21}$ cm^{-3}) gives rise to wavelengths of several TENS of nanometers



• IN THE FIGURES SHOWN HERE A SCANNING TUNNELING MICROSCOPE IS USED TO MOVE IRON ATOMS ON THE SURFACE OF COPPER CREATING A SO-CALLED QUANTUM CORRAL

• THE IRON ATOMS TRAP SURFACE ELECTRONS OF COPPER INSIDE THE RING AND INTERFERENCE OF THE REFLECTED ELECTRON WAVES PRODUCES THE RIPPLES INSIDE THE CORRAL

• THIS FIGURE SHOWS VERY NICELY THE WAVE-MECHANICAL PROPERTIES OF THE ELECTRONS ON A SURFACE

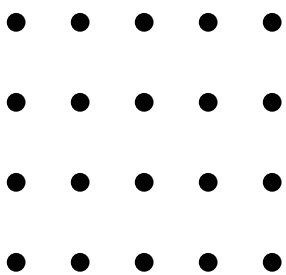
Eigler Group, IBM
www.almaden.ibm.com/vis/stm/gallery.html

scattering

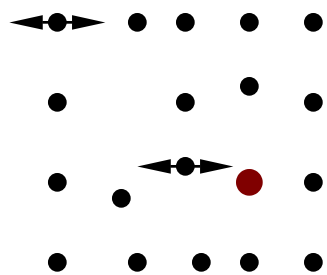
Electron propagation in real materials is NOT an uninterrupted process but is instead DISRUPTED by electron SCATTERING from a number of different sources

The origin of such scattering can be ANY source of DISORDER that destroys the perfect symmetry of the crystal structure

⇒ **Examples of such disorder include DEFECTS and IMPURITIES in the crystal structure but scattering from other ELECTRONS as well as from the quantized LATTICE VIBRATIONS (phonons) is also possible**



ELECTRONS IN A PERFECTLY PERIODIC POTENTIAL PROPAGATE WITHOUT BEING SCATTERED … THIS WELL KNOWN RESULT IS REFERRED TO AS BLOCH'S THEOREM



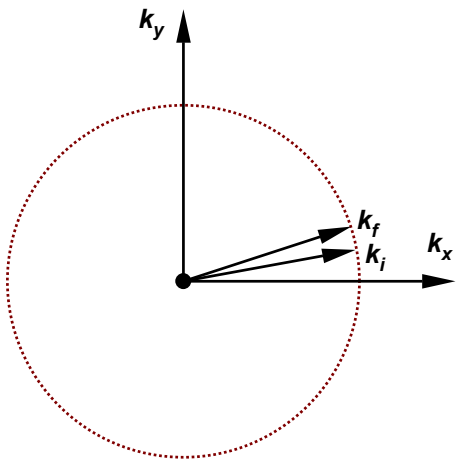
IN REAL CRYSTALS HOWEVER THE PRESENCE OF INEVITABLE DISORDER DISRUPTS ELECTRON PROPAGATION THROUGH THE CRYSTAL STRUCTURE

elastic and inelastic scattering

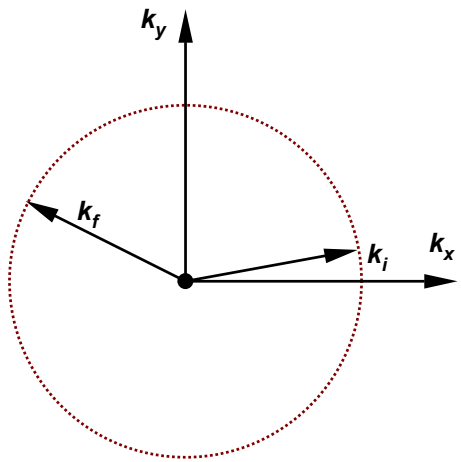
To characterize electron transport in materials we therefore need to introduce various materials-dependent LENGTH SCALES

*** These length scales define the distance over which properties such as the electron MOMENTUM, ENERGY, and the PHASE of the wavefunction are RANDOMIZED by scattering in the crystal**

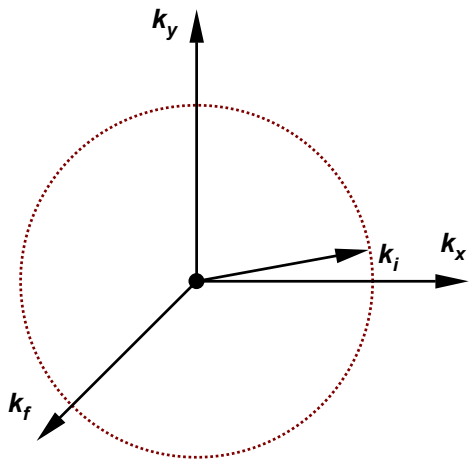
*** The values of these length scales are strongly MATERIAL-DEPENDENT and can also vary significantly with TEMPERATURE (WHY?)**



ELASTIC SCATTERING EVENT WITH LITTLE MOMENTUM CHANGE



ELASTIC SCATTERING EVENT WITH LARGE MOMENTUM CHANGE



INELASTIC SCATTERING EVENT WITH LARGE MOMENTUM CHANGE

From the GOLDEN RULE in quantum mechanics we know that scattering from a STATIC potential does NOT change the energy of the electron $\Rightarrow$ We therefore expect that scattering from FIXED impurities in the crystal will be ELASTIC while that from PHONONS and other ELECTRONS will be INELASTIC

mean free path (ℓ)

An important time scale for electron transport is the RELAXATION TIME (τ) which is the average time over which the initial momentum of the electron is REVERSED through a series of scattering events in the crystal

*** Using the relaxation time we may introduce the concept of the MEAN FREE PATH which may be defined the average DISTANCE electrons travel before backscattering**

$$l = v_F \tau$$

*** Some IMPORTANT QUANTITIES related to the relaxation time and mean free path include**

$$Mobility\ \mu = \frac{e\tau}{m^*} = \frac{el}{\hbar k_F}$$

$$2D\ Diffusion\ Constant\ D = \frac{1}{2} v_F^2 \tau$$

$$Conductivity\ \sigma = \frac{ne^2\tau}{m^*} = ne\mu$$

$$1D\ Diffusion\ Constant\ D = v_F^2 \tau$$

*** There is an elastic mean free path (ℓ_e; scattering fixed impurities and boundaries) and an inelastic mean free path (ℓ_i; scattering off phonons and other electrons).**

coherent versus incoherent transport

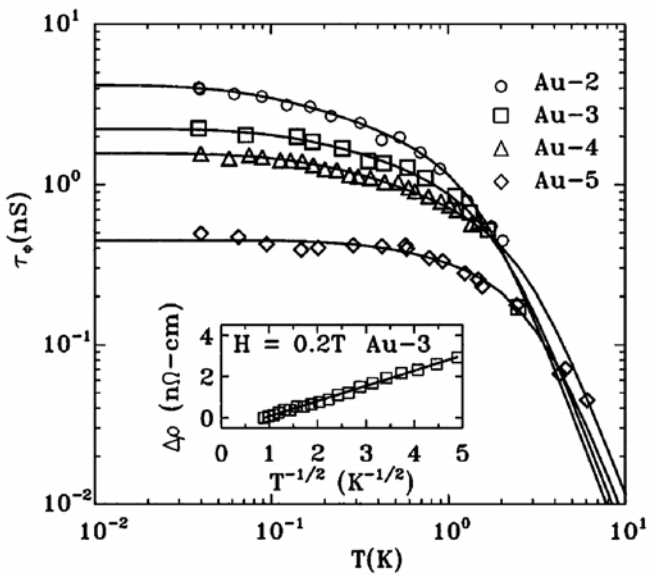
phase breaking length ℓ_φ

To account for the disruption of interference effects in real materials we introduce the electron PHASE-BREAKING TIME (τ_φ) which can be thought of as the average time that elapses between dephasing events

*** The PHASE-BREAKING LENGTH (l_φ) can be defined as the average distance that electrons DIFFUSE in the material before their phase is disrupted through scattering**

$$l_\varphi = \sqrt{D\tau_\varphi} \qquad (2.4)$$

*** To observe clear interference effects it is necessary that this length is COMPARABLE to the device sizes which often requires that experiments be performed at LOW TEMPERATUES**



• THE MEASURED VARIATION OF THE PHASE-BREAKING TIME WITH TEMPERATURE IN SMALL GOLD WIRES

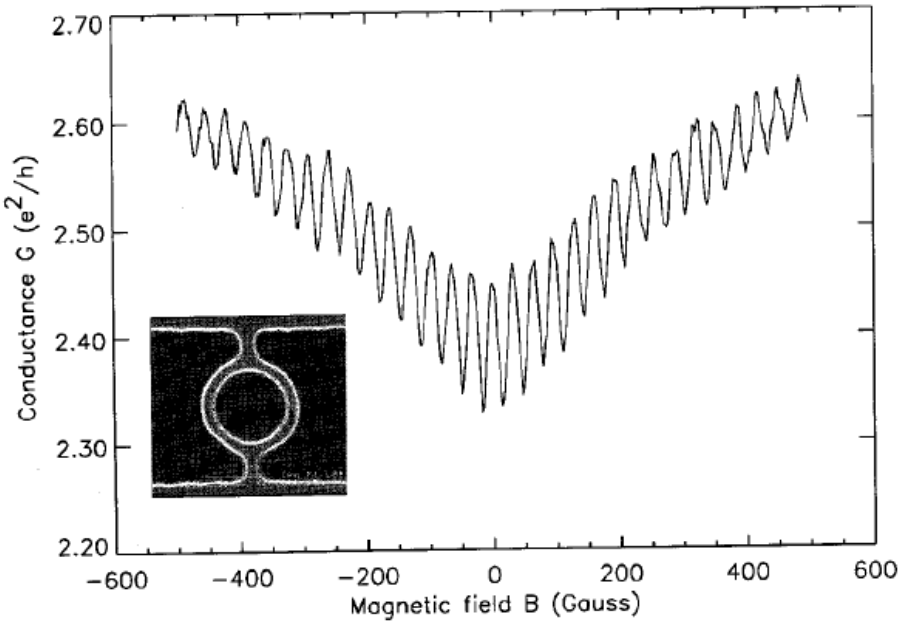
• OFTEN WE DOT DISTINGUISH BETWEEN INELASTIC MEAN FREE PATH AND THE PHASE BREAKING LENGTH. THEY ARE DIFFERENT THOUGH. WHICH ONE IS LARGER?

P. Mohanty *et al.*
Phys. Rev. Lett. 78, 3366 (1997)

phase coherence

at low temperatures

$\ell_\phi \geq 100\ \mu m$



The resistance of a small ring with a diameter of about 1 micron (the light gray areas in the inset) as a function of a magnetic field applied perpendicular to the ring plane shows periodic oscillations, known as *Aharonov-Bohm oscillations*.

They indicate that a significant fraction of the electrons traverse the ring phase coherently.

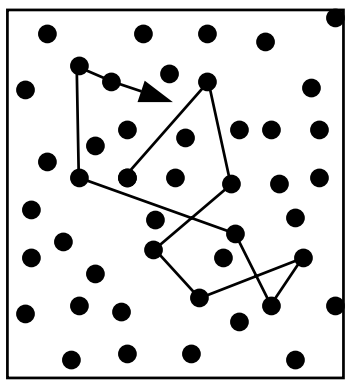
transport regimes

Since submicron structures can now be fabricated on length scales SMALLER than the average impurity spacing in semiconductors it is possible to study electron transport in a number of different REGIMES

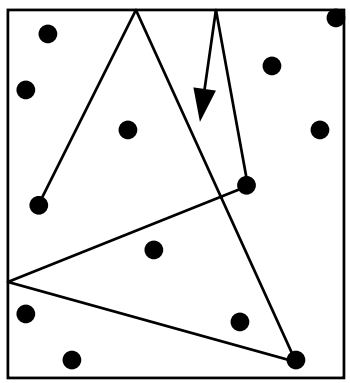
*** In DIFFUSIVE conductors the mean free path is much SMALLER than the sample dimensions and DISORDER scattering dominates**

*** In a QUASI-BALLISTIC conductor the mean free path and device size are COMPARABLE**

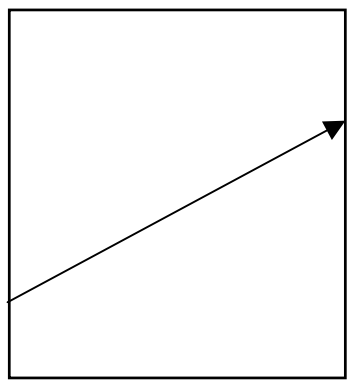
*** A BALLISTIC conductor contains NO impurities and so the dominant source of electron scattering is at the device BOUNDARIES (Is the resistance zero?)**



DIFFUSIVE TRANSPORT



QUASI-BALLISTIC TRANSPORT



BALLISTIC TRANSPORT

When is transport diffusive/ballistic?

When is transport classical/quantum?

diffusive

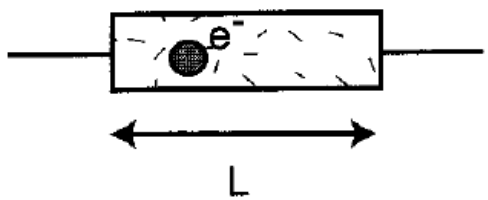
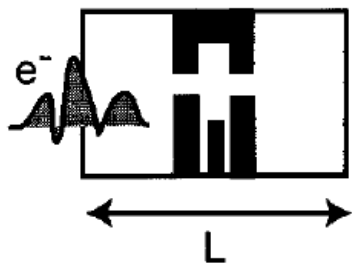
classical: $\lambda_F, \ell_i, \ell_e \ll L$

quantum: $\lambda_F, \ell_e \ll L, \ell_i$

ballistic

classical: $\lambda_F \ll L < \ell_e, \ell_i$

quantum: $\lambda_F, L < \ell_e < \ell_l$

conventional device: 		mesoscopic device: 	
$L \gg l_e$	diffusive	$L \lesssim l_e$	ballistic
$L \gg l_\phi$	incoherent	$L \lesssim l_\phi$	phase coherent
$L \gg \lambda_F$	no size quantization	$L \lesssim \lambda_F$	size quantization
$e^2/C < k_B\Theta$	no single electron charging	$e^2/C \gtrsim k_B\Theta$	single electron charging effects

		GaAs(100)	Si (100)	UNITS
Effective Mass	m	0.067	0.19	$m_e = 9.1 \times 10^{-28}$ g
Spin Degeneracy	g_s	2	2	
Valley Degeneracy	g_v	1	2	
Dielectric Constant	ε	13.1	11.9	$\varepsilon_0 = 8.9 \times 10^{-12}$ F m^{-1}
Density of States	$\rho(E) = g_s g_v (m/2\pi\hbar^2)$	0.28	1.59	10^{11} cm^{-2} meV^{-1}
Electronic Sheet Density[a]	n_s	4	1-10	10^{11} cm^{-2}
Fermi Wave Vector	$k_F = (4\pi n_s/g_s g_v)^{1/2}$	1.58	0.56–1.77	10^6 cm^{-1}
Fermi Velocity	$v_F = \hbar k_F/m$	2.7	0.34–1.1	10^7 cm/s
Fermi Energy	$E_F = (\hbar k_F)^2/2m$	14	0.63–6.3	meV
Electron Mobility[a]	μ_e	10^4–10^6	10^4	cm^2/V·s
Scattering Time	$\tau = m\mu_e/e$	0.38–38	1.1	ps
Diffusion Constant	$D = v_F^2\tau/2$	140–14000	6.4–64	cm^2/s
Resistivity	$\rho = (n_s e \mu_e)^{-1}$	1.6–0.016	6.3–0.63	kΩ
Fermi Wavelength	$\lambda_F = 2\pi/k_F$	40	112–35	nm
Mean Free Path	$l = v_F\tau$	10^2–10^4	37–118	nm
Phase Coherence Length[b]	$l_\phi = (D\tau_\phi)^{1/2}$	200–⋯	40–400	nm$(T/\mathrm{K})^{-1/2}$
Thermal Length	$l_T = (\hbar D/k_B T)^{1/2}$	330–3300	70–220	nm$(T/\mathrm{K})^{-1/2}$
Cyclotron Radius	$l_{cycl} = \hbar k_F/eB$	100	37–116	nm$(B/\mathrm{T})^{-1}$
Magnetic Length	$l_m = (\hbar/eB)^{1/2}$	26	26	nm$(B/\mathrm{T})^{-1/2}$