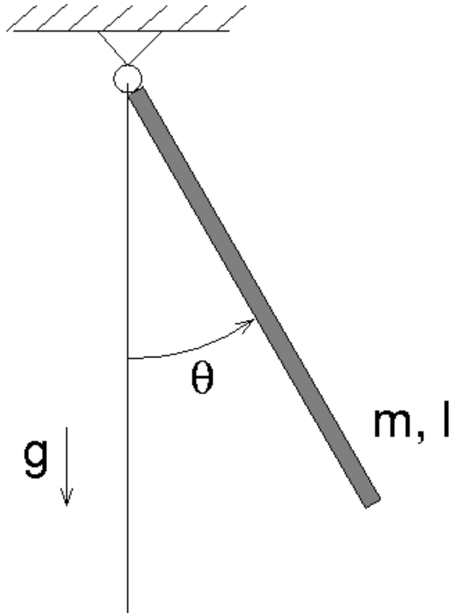
D & S
AE3-914

February 18, 2010

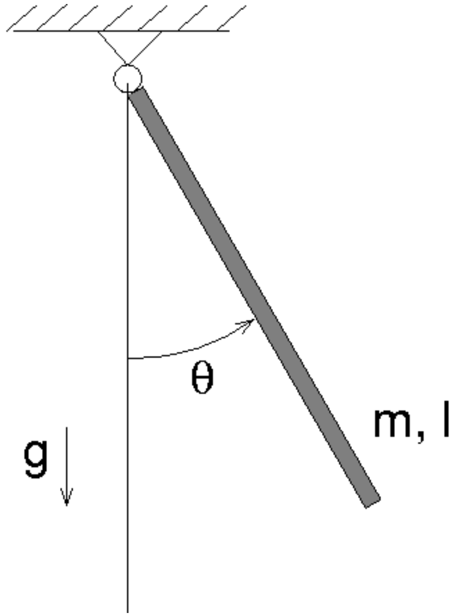
[redacted]

Lagrangian dynamics

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k^{nc} \quad k = 1, \ldots, ndof$$

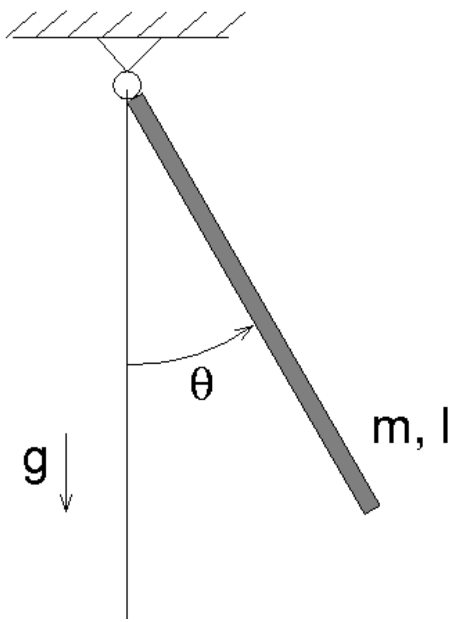


Equations of motion?



$$\ddot{\theta} + \frac{3g}{2l}\sin\theta = 0$$

Solution to the equations of motion?



Conservative system:

$$\frac{1}{6}ml^2\dot{\theta}^2 - \frac{1}{2}mgl\cos\theta = \mathbb{E}$$

Phase portrait:

$$\theta(0), \dot{\theta}(0) \rightarrow \mathbb{E}$$

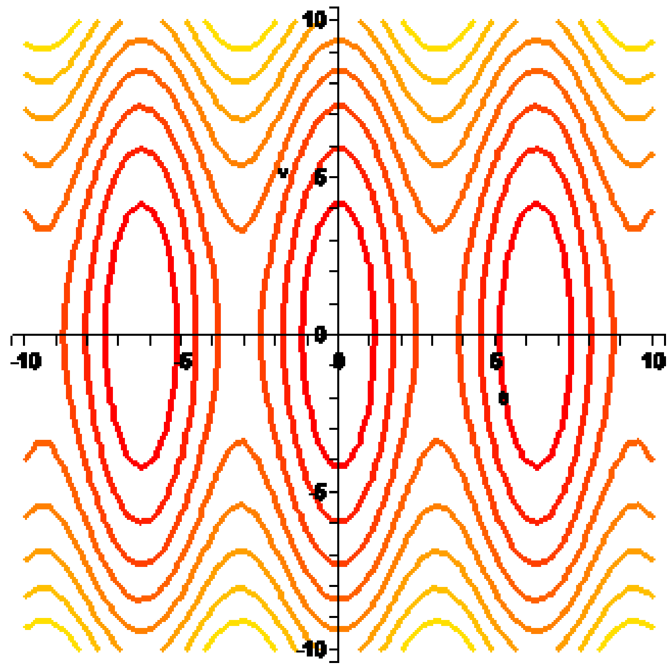
$$\dot{\theta} = \sqrt{\frac{6}{ml^2}\left(\mathbb{E} + \frac{1}{2}mgl\cos\theta\right)}$$

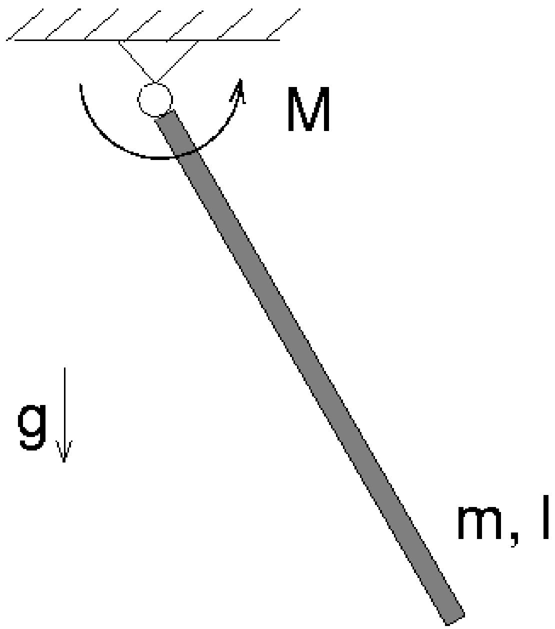
$$m = 1\,\mathrm{kg}$$

$$l = 1\,\mathrm{m}$$

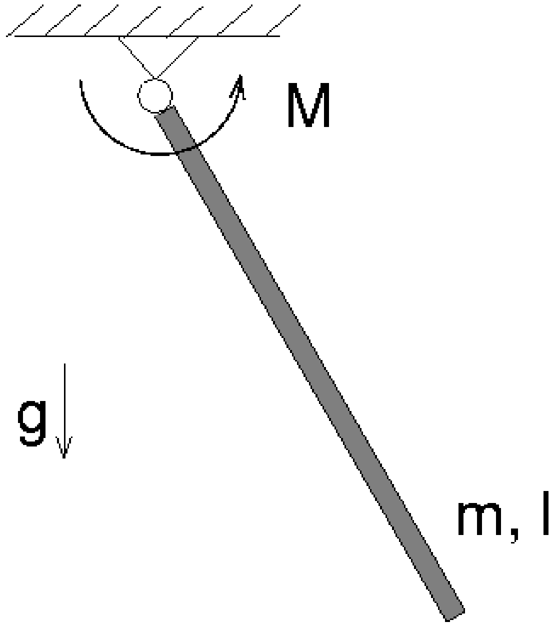
$$g = 10\,\mathrm{m/s^2}$$

Phase portrait:



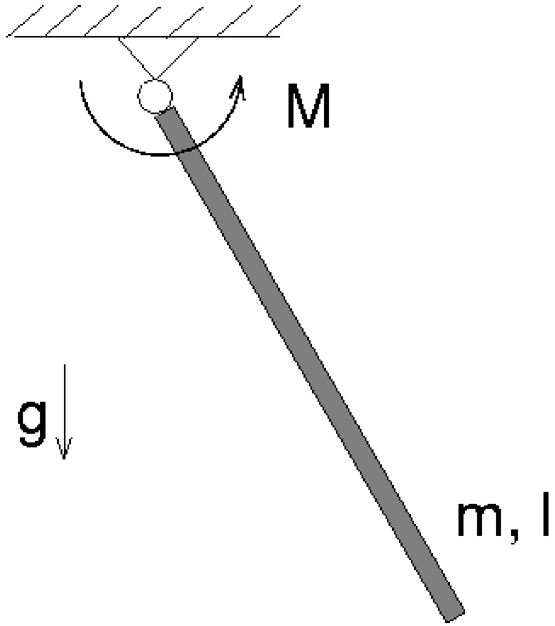


Equations of motion?



Is energy conserved?

a) Yes
b) No
c) No idea



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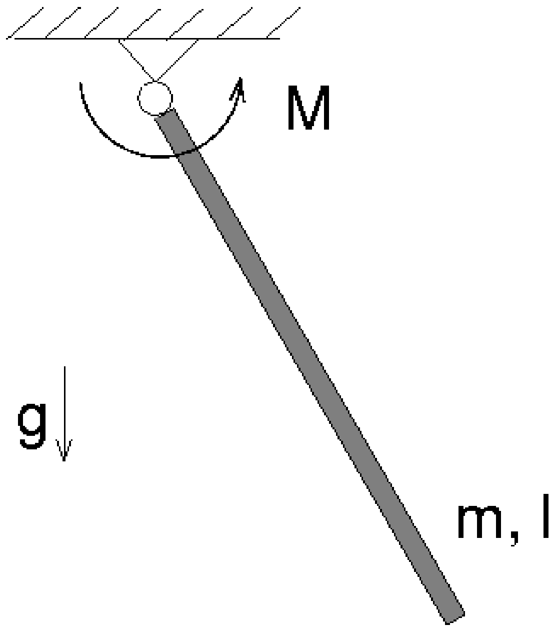


Is anything conserved?

a) Yes
b) No
c) No idea

$$L = L(\mathbf{q}, \dot{\mathbf{q}}, t)$$

$$\frac{dL}{dt} = \sum \frac{\partial L}{\partial q_k} \dot{q}_k + \sum \frac{\partial L}{\partial \dot{q}_k} \ddot{q}_k + \frac{\partial L}{\partial t}$$

$$\frac{d}{dt}\left[\frac{\partial L(\mathbf{q},\dot{\mathbf{q}},t)}{\partial \dot{q}_k}\right]-\frac{\partial L(\mathbf{q},\dot{\mathbf{q}},t)}{\partial q_k}=0$$

$$\frac{\partial L}{\partial q_k}=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)$$

$$\frac{dL}{dt} = \sum \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)\dot{q}_k + \sum \frac{\partial L}{\partial \dot{q}_k}\ddot{q}_k + \frac{\partial L}{\partial t}$$

$$= \frac{d}{dt}\left(\sum \frac{\partial L}{\partial \dot{q}_k}\dot{q}_k\right) + \frac{\partial L}{\partial t}$$

$$\frac{d}{dt}\left(\sum \frac{\partial L}{\partial \dot{q}_k}\dot{q}_k - L\right) = -\frac{\partial L}{\partial t}$$

If $L \neq L(t)$ then $\sum \frac{\partial L}{\partial \dot{q}_k}\dot{q}_k - L =$ constant

Jacobi energy integral

$$h(\mathbf{q}, \dot{\mathbf{q}}, t) = \sum \frac{\partial L}{\partial \dot{q}_k} \dot{q}_k - L$$

If $L \neq L(t)$ then h is conserved

Any quantity remaining constant in time is called an **Integral of motion**

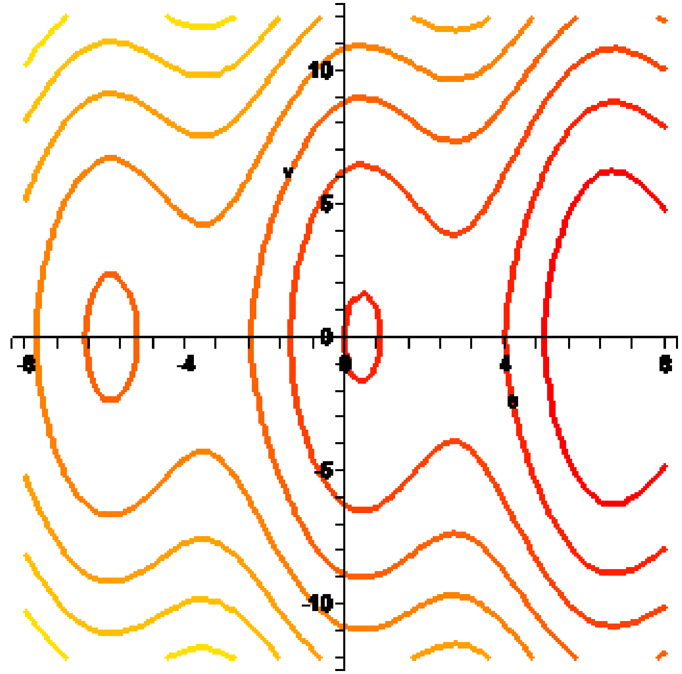
Phase portrait:

$$\theta(0), \dot{\theta}(0) \rightarrow h$$

$$\dot{\theta} = \sqrt{\frac{6}{ml^2}\left(h + \frac{1}{2}mgl\cos\theta + M\theta\right)}$$

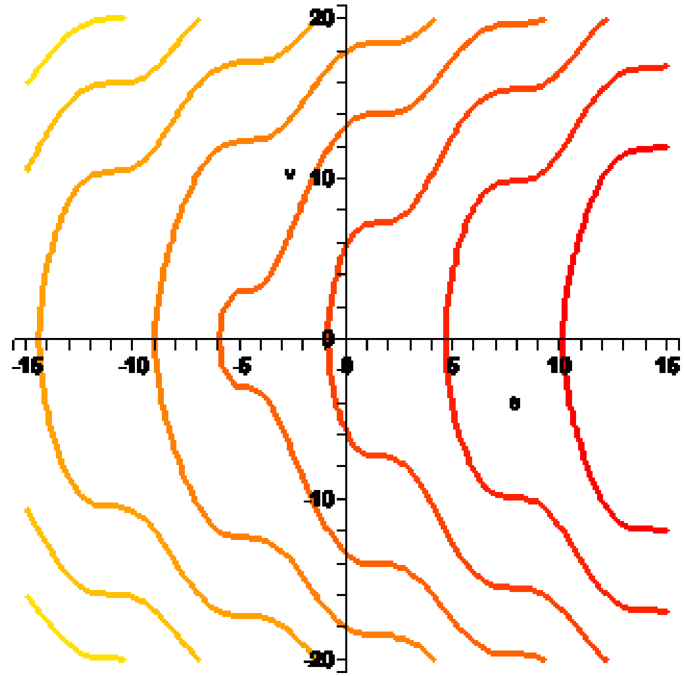
Phase portrait:

$$M = 2\,\mathrm{Nm}$$



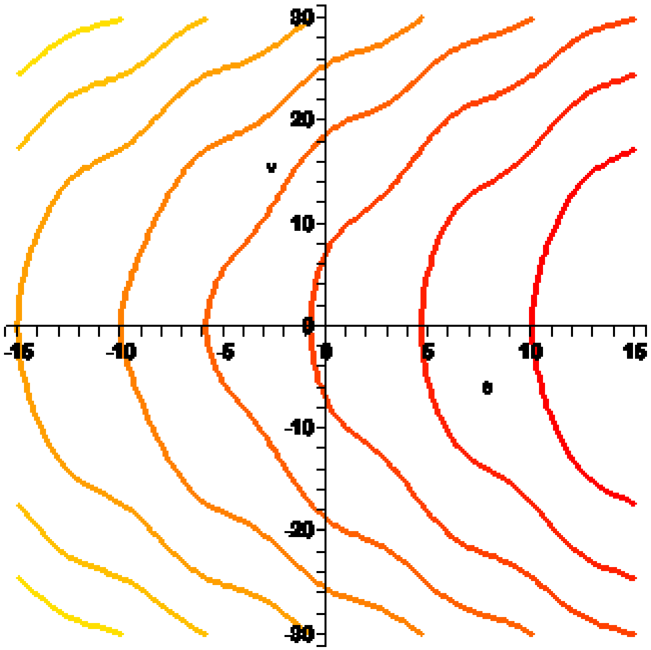
Phase portrait:

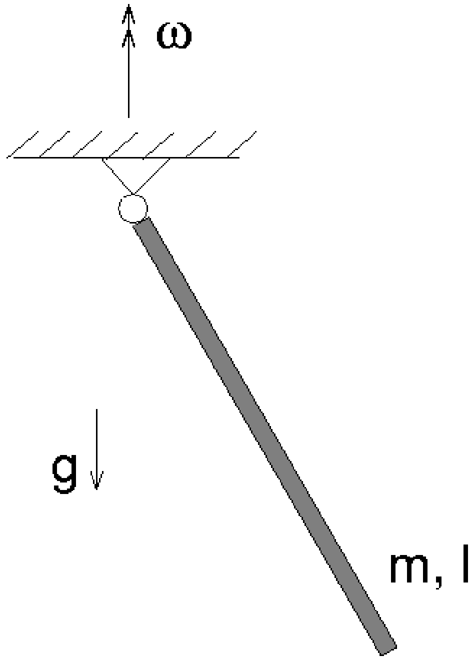
$$M = 5\,\mathrm{Nm}$$



Phase portrait:

$$M = 10\,\mathrm{Nm}$$



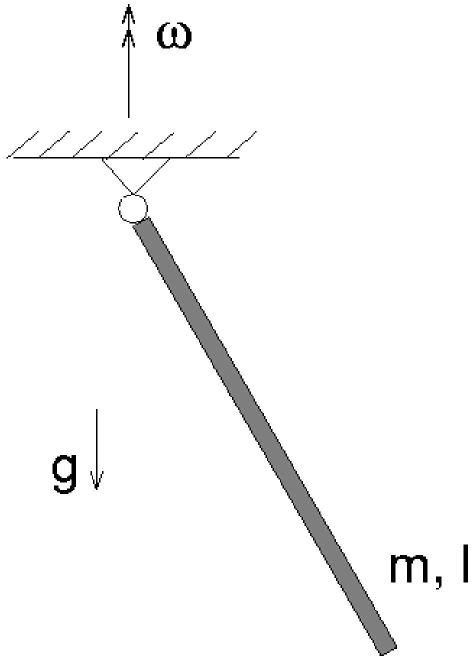


ω is constant

Lagrangian function L?

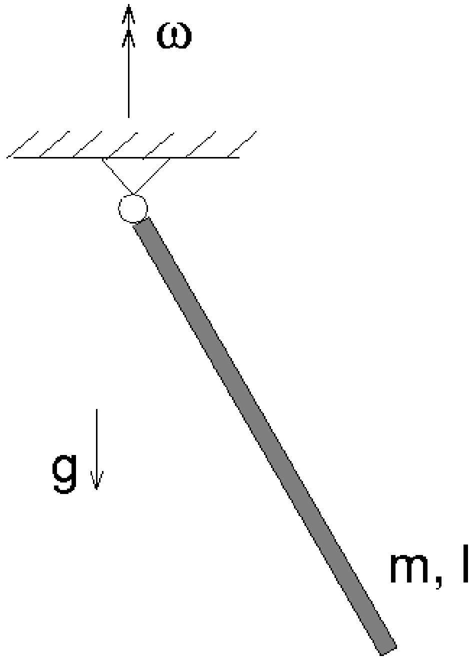
Jacobi energy integral h?

Equation of motion?



Is energy conserved?

a) Yes
b) No
c) No idea



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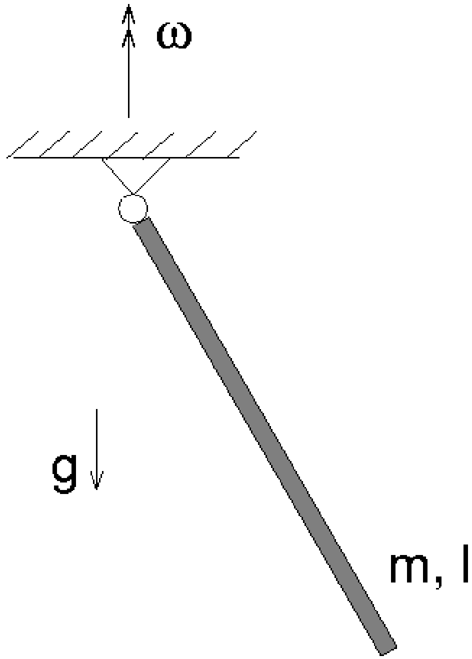
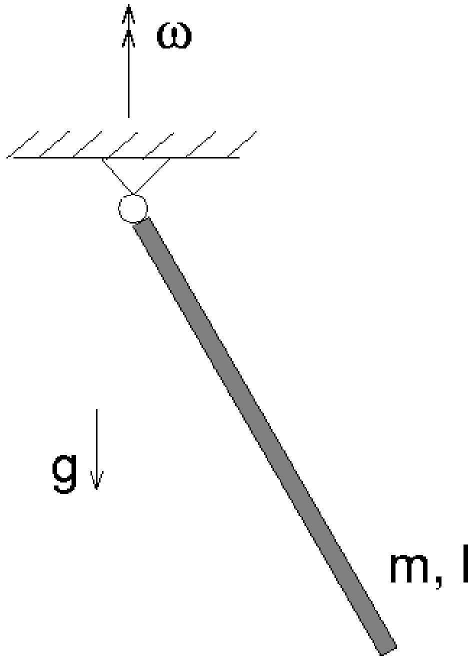


Is anything conserved?

a) **Yes:** h

b) No

c) No idea



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When all forces are conservative we have a

conservative system

This typically means that the total energy is conserved, and it *equals* a (constant) Jacobi energy integral.

If we have a generalised coordinate *q* of a Lagrangian system such that

$$L = L(\dot{q}) \qquad \text{but} \qquad L \neq L(q)$$

then *q* is an **ignorable coordinate**

If q is an ignorable coordinate

$$\frac{\partial L}{\partial q} = 0$$ and consequently

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

$$\frac{\partial L}{\partial \dot{q}} = C_q$$

the generalised momentum associated with *q* is an **integral of motion**

Satellite system

$$V = -GM_e \frac{m}{r}$$