lecture 3: diffusive transport

- classical regime (Drude)
- quantum corrections (WL, UCF)

See also

Thomas Schaepers, Phase-coherent transport (on Blackboard)

S. Datta, *Electronic Transport in Mesoscopic Systems,*
(Cambridge University Press, Cambridge, UK, 1995)

Drude model

Based on free-electron gas picture
Electrical resistance results from scattering (all information is lost upon scattering)

How fast does an electron move in an electric field?

$$\overrightarrow{v_d} \equiv \left\langle \vec{v}(t) \right\rangle = -\frac{e\vec{E}\tau}{m^*} \equiv -\mu\vec{E}$$

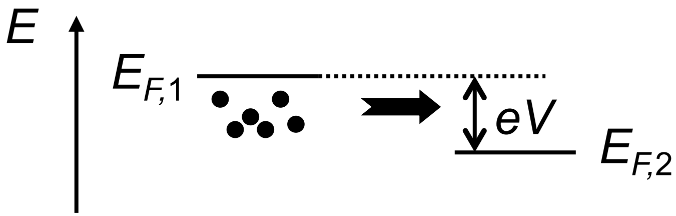
mobility $\mu \equiv \frac{e\tau}{m^*}$

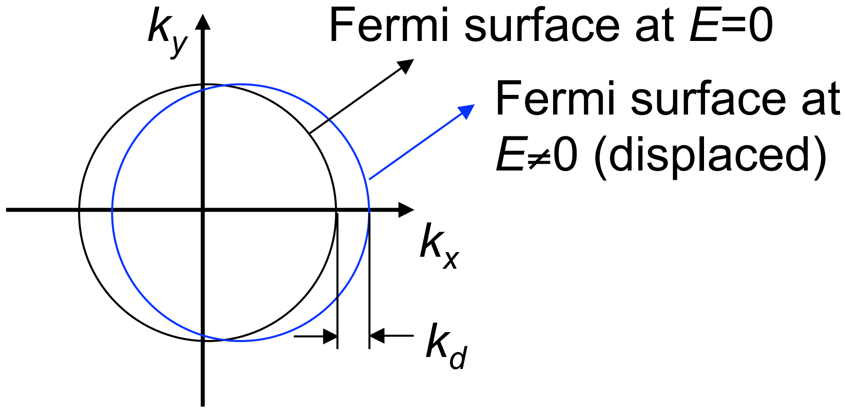
Drude conductivity

$$\sigma = \frac{\vec{j}}{\vec{E}} = \frac{-en\left\langle \vec{v} \right\rangle}{\vec{E}} = ne\mu = \frac{e^2 n\tau}{m^*}$$

Einstein relation

Electrons near E_F diffuse due to density gradient (in energy range $E_{F,1}$-$E_{F,2}$)





Einstein relation $\sigma = e^2 \rho(E_F) D$

Diffusion constant $D \equiv \int_0^\infty \langle \vec{v}_x(t)\vec{v}_x(0) \rangle dt$ Kubo formula

$$D = \frac{1}{d} v_F^2 \tau \quad (d \text{ dimensions})$$

Recall $l = \sqrt{Dt}$ for for t greater than τ

Quantum correction: weak localization

(blackboard)

weak localization in a 2DEG

mobility 1000 V/cm^2

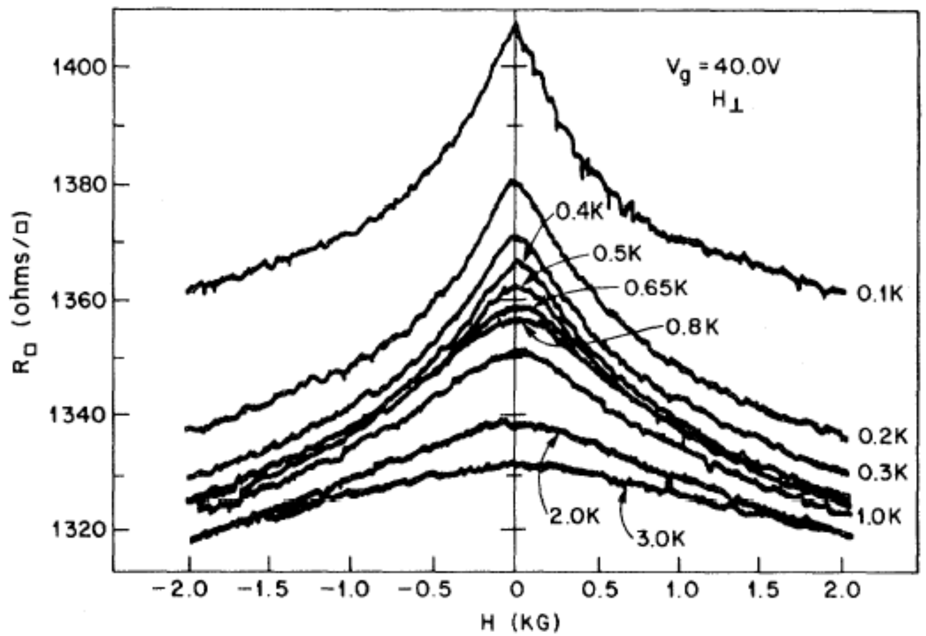


FIG. 2. Low-field magnetoresistance of a Si(111) MOSFET in a perpendicular field for various temperatures. Electron density is 4.52×10^{12} cm^{-2}.

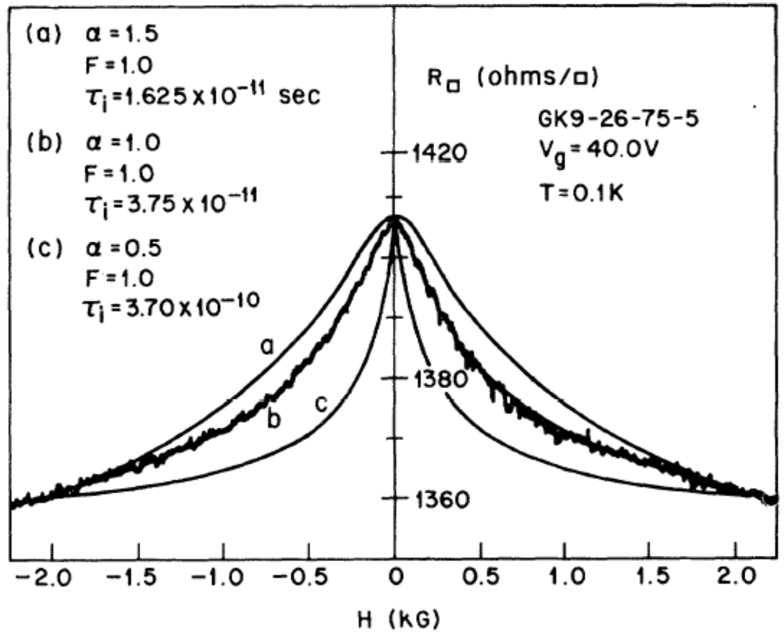


FIG. 3. Same data as in Fig. 2 at 0.1 K with various fits to Eq. (5). The data is fitted at $H = 0$ and 2.0 kG and the resultant parameters are listed on the figure. The fit is relatively insensitive to the choice of F.

The devices were 1.0-mm long and 0.25-mm wide with potential probes separated by 0.25 mm.

Bishop, Dynes and Tsui
PRB 26 (1982) 773

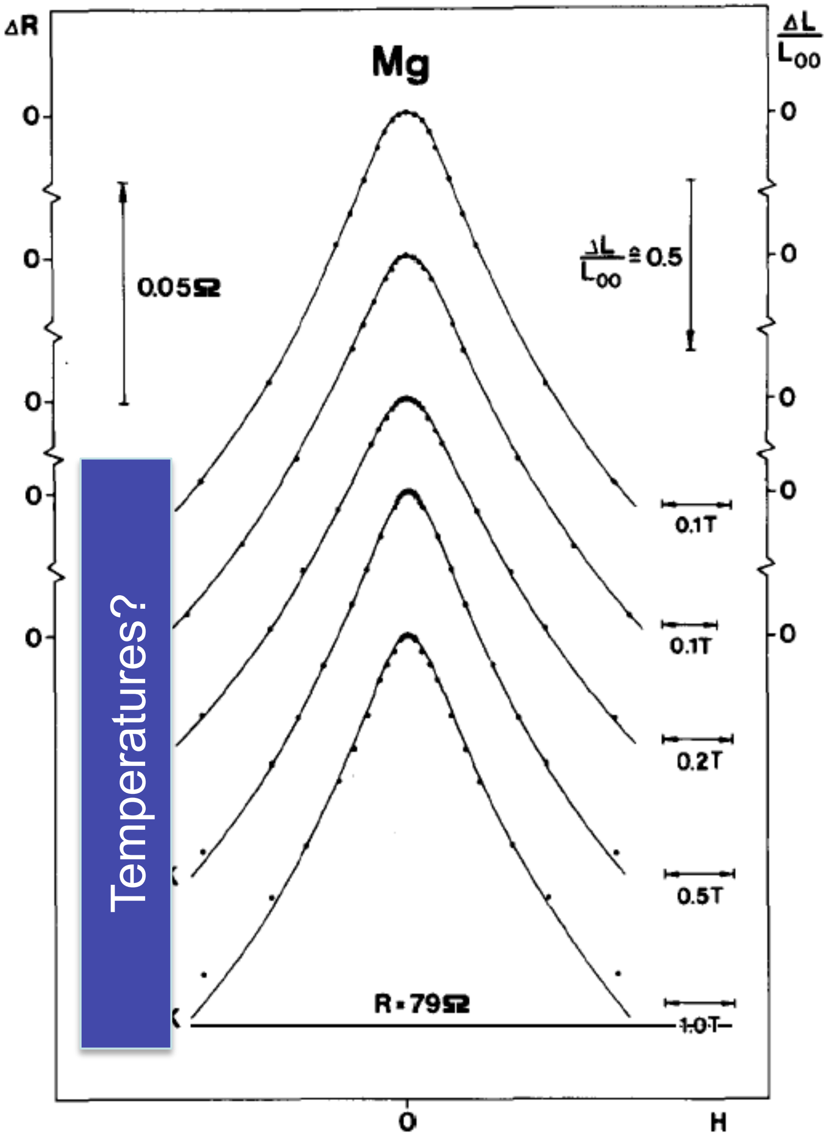
Weak localization

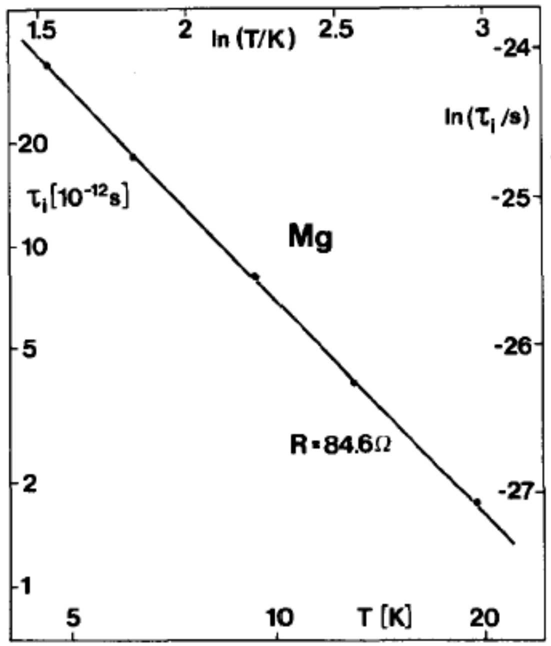
$$\Delta\sigma_{2D} = -\frac{1}{4\pi}\frac{2e^2}{h}\ln\left[\frac{\tau_\varphi}{\tau_e}\right]$$

quenched condensed metal films: evaporation of films on substrates cooled at helium temperatures results in more homogeneous films and makes it possible to prepare high-Ohmic films (why needed?)

ℓ_e = 0.5 – 1 nm
t = 7-12 nm
L ~ 1 mm (?)
R_{square} = 50-200 Ω/square
(What is the value of τ_e?)

$$B_c = \frac{h}{|e|\,l_\phi^2}$$

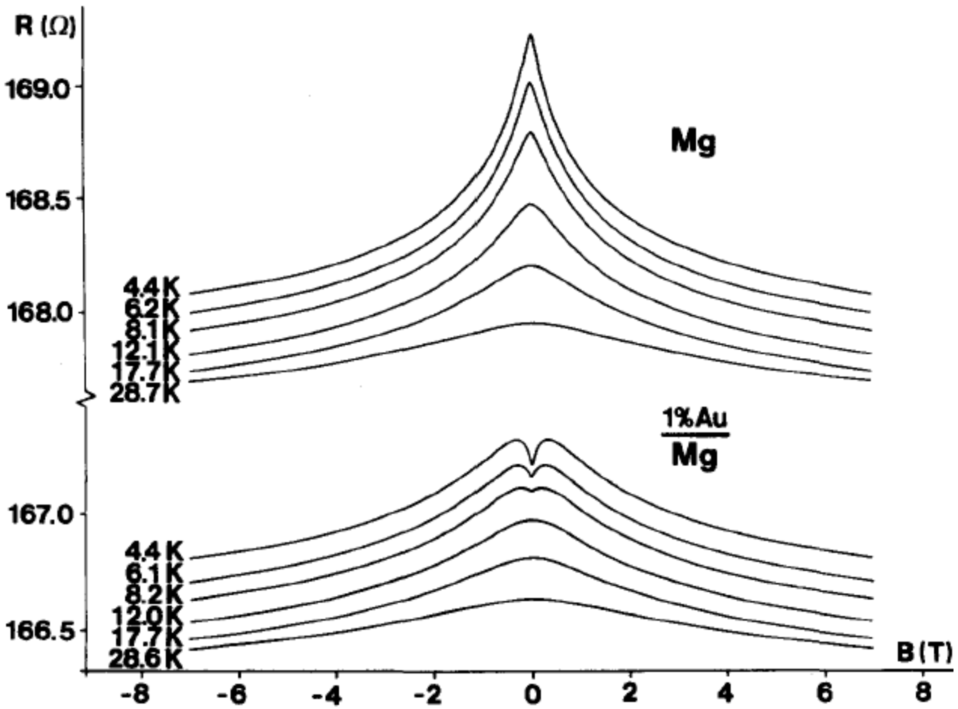


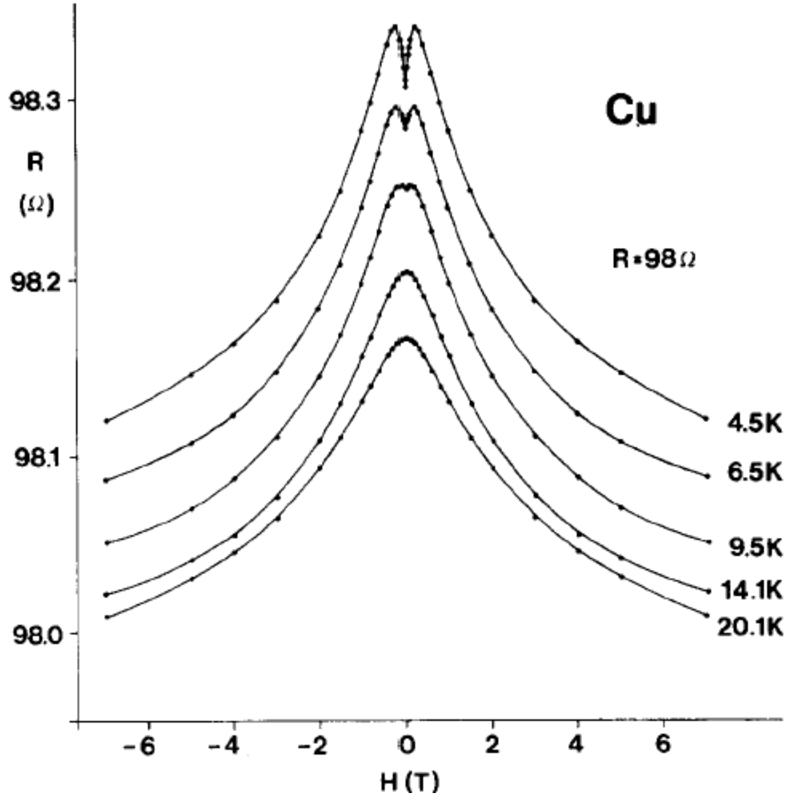


Bergmann
Physics Reports 107 (1984)

weak anti-localization

Spin-flip scattering breaks time-reversal symmetry, so it destroys (or even reverses) WL.
A small magnetic field can suppress spin-flip scattering.
The result is a combination of enhanced and suppressed backscattering (peak and dip)



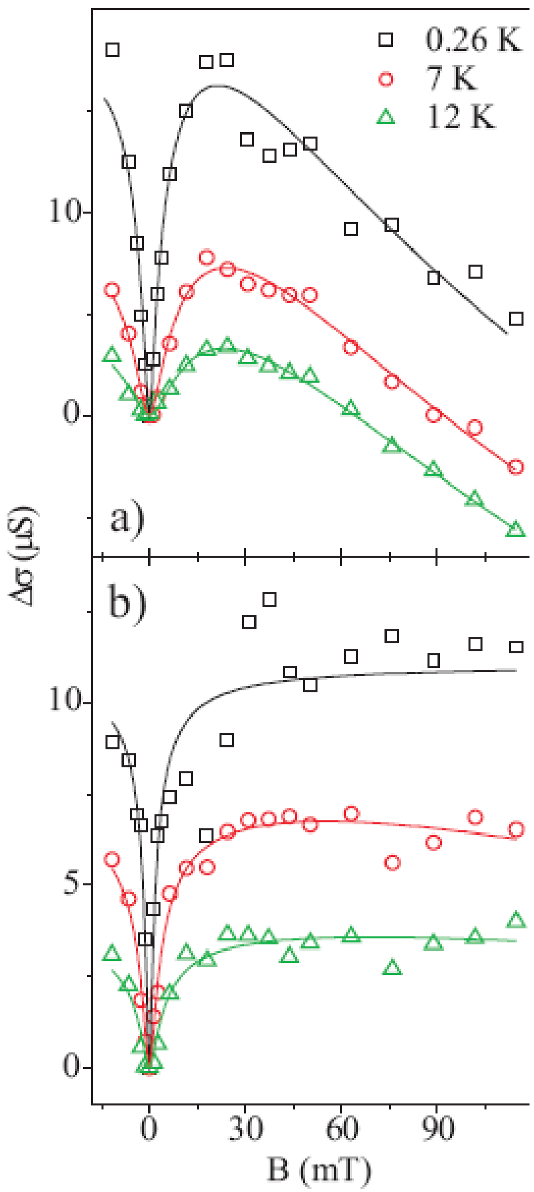


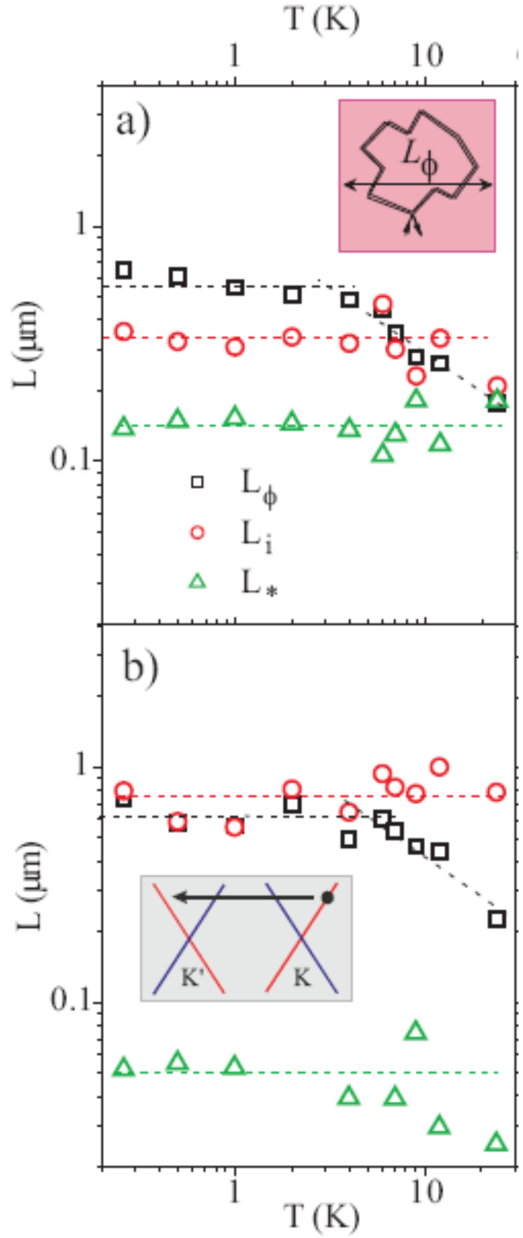
Bergmann
Physics Reports 107 (1984) 1-58

weak (anti)-localization in graphene

Magnetoresistance remains a very useful tool for characterizing the phase coherence properties of new materials.

In graphene, a new mechanism for weak anti-localization was found.





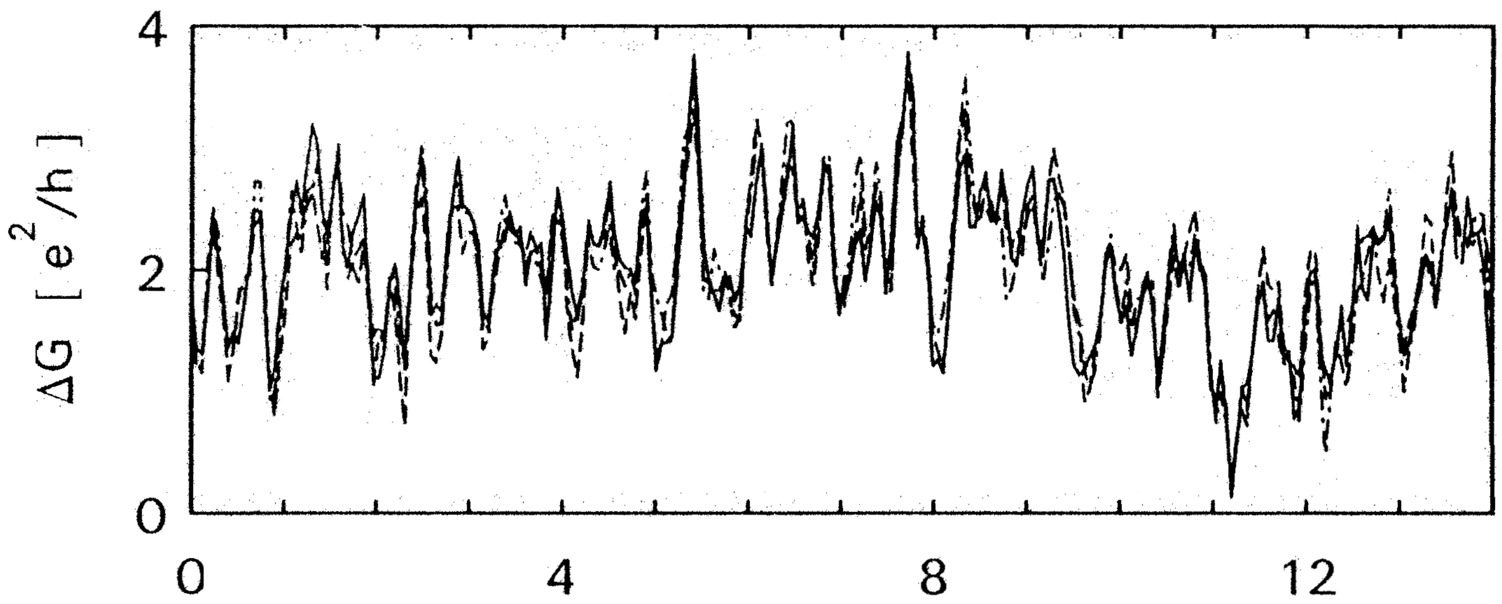
Tikhonenko et al, PRL 2008

Universal Conductance Fluctuations (UCF)

Since the detailed pattern of the fluctuations is determined by the SPECIFIC paths that electrons follow as they diffuse through the wire the fluctuations may be viewed as a MAGNETO-FINGERPRINT of its SPECIFIC impurity configuration

- **The fluctuations are highly REPRODUCIBLE as long as the sample is maintained at low temperatures (< 1 K)**

- **Their pattern is changed COMPLETELY however if the sample is warmed to room temperature and then cooled back down again since this gives rise to a NEW microscopic disorder configuration in the wire**



REPRODUCIBILITY OF THE CONDUCTANCE FLUCTUATIONS MEASURED IN A GOLD RING

S. Washburn and R. A. Webb
Adv. Phys. 35, 375 (1986)

Universal Conductance Fluctuations (UCF)

Experiments performed on a variety of different material systems have shown that at low temperatures the conductance fluctuations exhibit a UNIVERSAL amplitude of order e^2/h

The universal amplitude has been confirmed by THEORETICAL studies which suggest that their zero-temperature amplitude should be INDEPENDENT of the system size or the degree of disorder

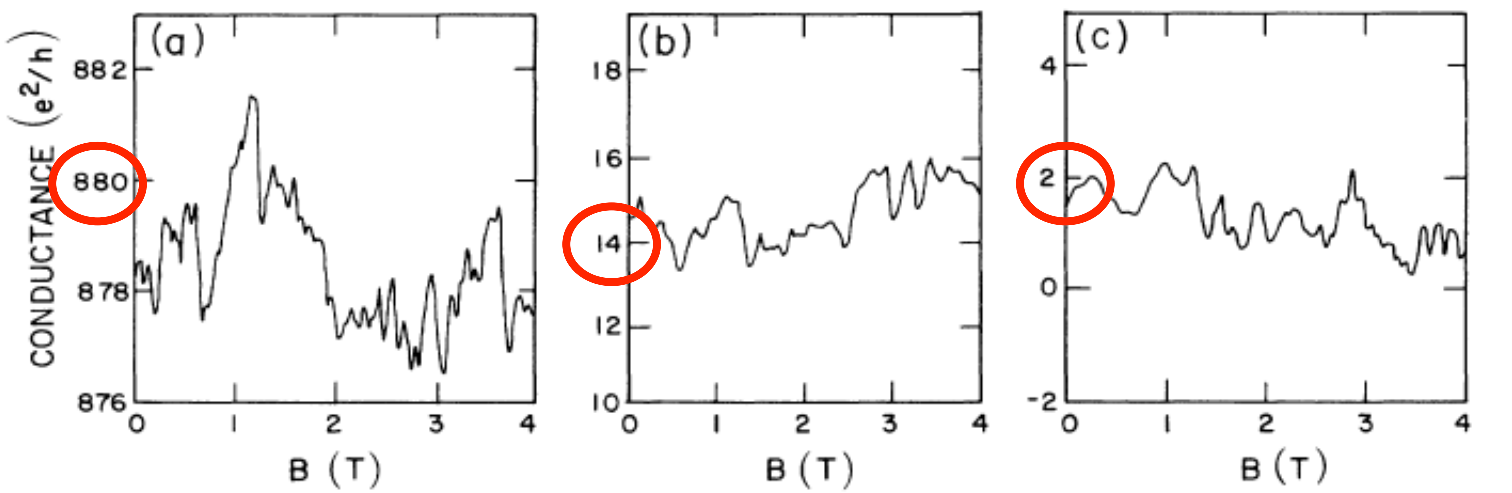


FIG. 1. Comparison of aperiodic magnetoconductance fluctuations in three different systems. (a) $g(B)$ in 0.8-μm-diam gold ring, analysis of data from Refs. 3 and 4, reprinted with the permission of Webb *et al.* (the rapid Aharonov-Bohm oscillations have been filtered out). (b) $g(B)$ for a quasi-1D silicon MOSFET, data from Ref. 9, reprinted with the permission of Skocpol *et al.* (c) Numerical calculation of $g(B)$ for an Anderson model using the technique of Ref. 11. Conductance is measured in units of e^2/h, magnetic field in tesla. Note the 3 order-of-magnitude variation in the background conductance while the fluctuations remain order unity.

P. A. Lee *et al.*
Phys. Rev. B 35, 1039 (1987)

Interference and Interaction in multi-wall carbon nanotubes

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Appl. Phys. A 69, 283–295 (1999)

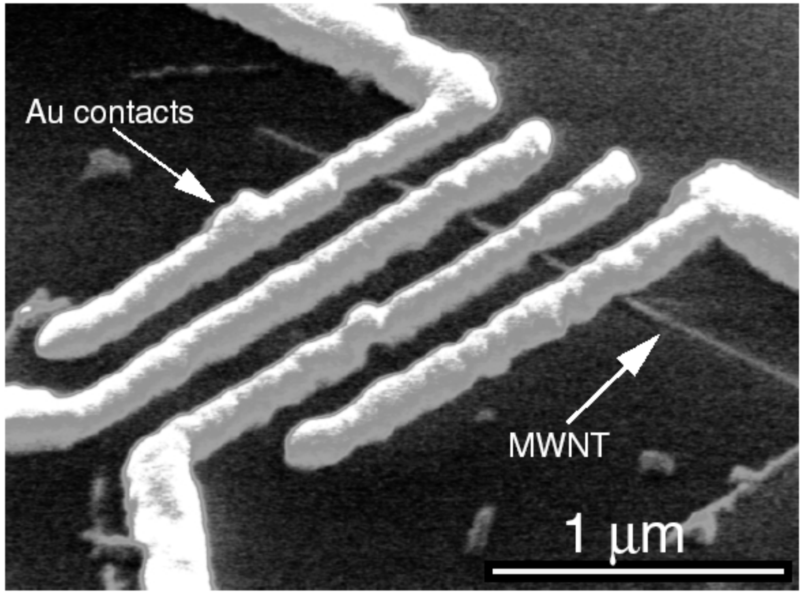


Fig. 1. Scanning electron microscopy image of a single multi-wall nanotube (MWNT) electrically contacted by four Au fingers from above. The separation between the contacts is 350 nm center to center

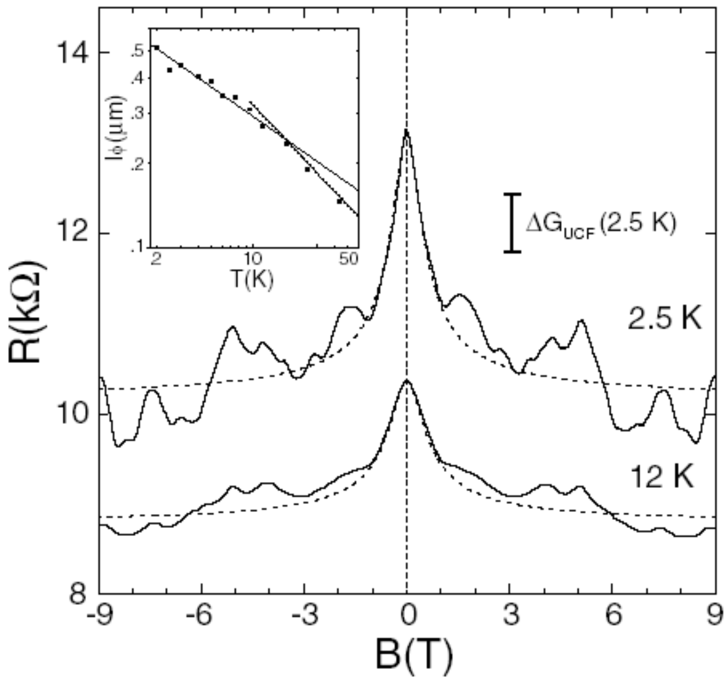


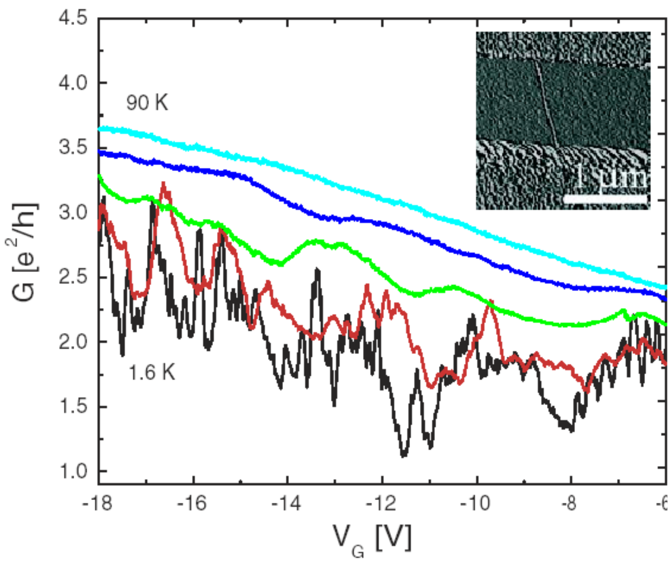
Fig. 5. Four-terminal magnetoresistance of a MWNT in perpendicular field for two temperatures. The voltage probes are separated by 1.9 μm. *Dashed curves* show fits using one-dimensional weak-localization theory. *Inset:* deduced phase-coherence length l_ϕ as a function of temperature T

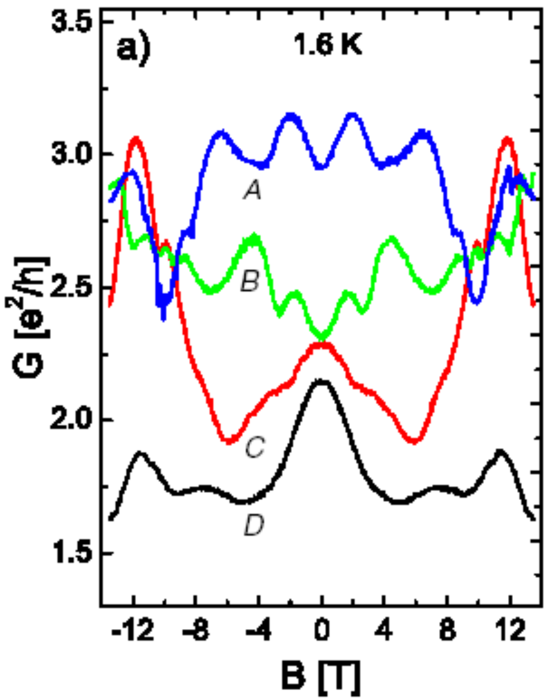
$$\Delta G = -0.62\frac{e^2}{\hbar L}\left(\frac{1}{l_\phi^2}+\frac{w^2}{3l_{\mathrm{m}}^4}\right)^{-1/2}$$

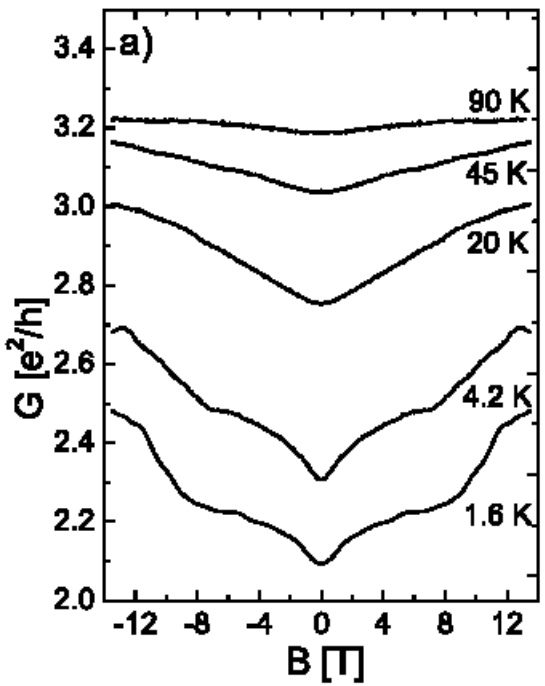
Quasi-ballistic transport
ℓ_e = 90-180 nm
d = 25 nm
L = 350 nm

Sample-Specific and Ensemble-Averaged Magnetoconductance of Individual Single-Wall Carbon Nanotubes

H. T. Man and A. F. Morpurgo
Kavli Institute of NanoScience Delft, Faculty of Applied Sciences, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands







non-averaged at different gate-voltages

averaged over different gate-voltages

weak localization

Important points to remember:

- **phase coherent summation of time-reversed trajectories (closed loops) leads to an increased probability for electrons to return to their initial position (increase of the resistance). We call this coherent back scattering.**
- **Only a few paths contribute: weak localization**
- **Measurements are done in diffusive samples at low temperatures and allow the determination of the phase-coherence length (you should be able to give an estimate of the phase-coherence length from measurement).**
- **A magnetic field breaks time-reversal symmetry and kills weak localization.**
- **The correction is typical in the order of 0.1 % (2D) to a few percent in 1D (check yourself).**

$$\Delta\sigma_{2D} = -\frac{1}{4\pi}\frac{2e^2}{h}\ln\left[\frac{\tau_\varphi}{\tau_e}\right] \qquad \Delta\sigma_{1D} = -\frac{2e^2}{h}\frac{\ell_\varphi}{W}\ln\left[1-\left(1+\frac{\tau_\varphi}{\tau_e}\right)^{-1/2}\right]$$

Course schedule

1	9 sep		introduction, overview, material systems	1.1
2	16 sep		DOS, energy & length scales, dimensionality, transport regimes	1.2 and 1.3
3	23 sep		conduction in the classical regime (Drude) phase-coherent transport 1 (WL,UCF)	2.1, 2.2 and 2.3
4	30 sep	**Herre vdZant**	phase-coherent transport 2 (AB, AAS, pers. current)	5.1 and 5.2
5	7 oct		ballistic transport (Landauer, focussing)	3.1, 3.2, 3.3
6	14 oct	**MOVE, TBD**	ballistic transport (quantized conductance)	4.1, 4.2
7	21 oct		quantum Hall effect	4.3, 4.4
8	15 nov		charging, CB, electron box	7.1, 7.2
10	22 nov		quantum dots	7.2
11	29 nov		superconductivity, NS, Andreev reflection	6.1, 6.2
12	6 dec		SNS, MAR, Josephson junction	6.3
13	13dec		quantized mechanical motion, phonons	
14	20 dec		summary /illustration of concepts - graphene	

four questions on weak localization and UCF

- What is the connection between weak localization and UCF? What happens to WL and UCF when I make the sample larger?
- How do I distinguish UCF from noise?
- How does the resistance vary with magnetic field in weak localization? Why? What about UCF?
- How can one estimate the phase-coherence length from a typical measurement of a weak-localization curve? Does the WL peak peak get larger or smaller when I increase the phase coherence length?