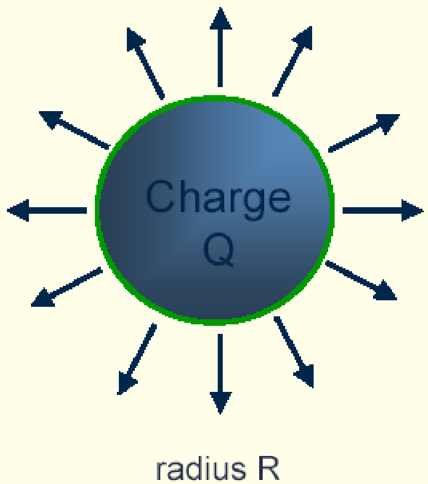
single-electron tunneling (SET)

- classical dots (SET islands): level spacing is NOT important; only the charging energy (=classical effect, many electrons on the island)

- quantum dots: level spacing (quantum confinement) AND charging energy important (few electrons on the dot)

charging energy: $E_C = e^2/2C$

• What is the capacitance of an isolated piece of metal (for example a sphere)?



Electric field:

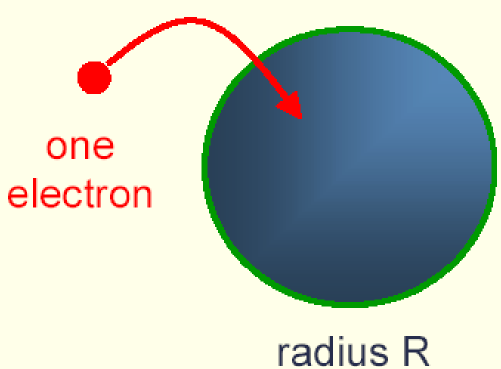
$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \quad (r > R)$$

Voltage:

$$V(R) = -\int_R^\infty \vec{E}(\vec{r}) \cdot d\vec{r} = \frac{Q}{4\pi\varepsilon_0 R}$$

$$C = Q/V = 4\pi\varepsilon_0 R$$

• What is the energy needed to charge the sphere with one electron (1/2QV with Q = e)?



R	C	E/k_B
10 μm	1.1×10^{-15} F	0.84 K (^3He)
1 μm	1.1×10^{-16} F	8.4 K (LHe)
0.1 μm	1.1×10^{-17} F	84 K (LN_2)
0.01 μm	1.1×10^{-18} F	840 K (spa)

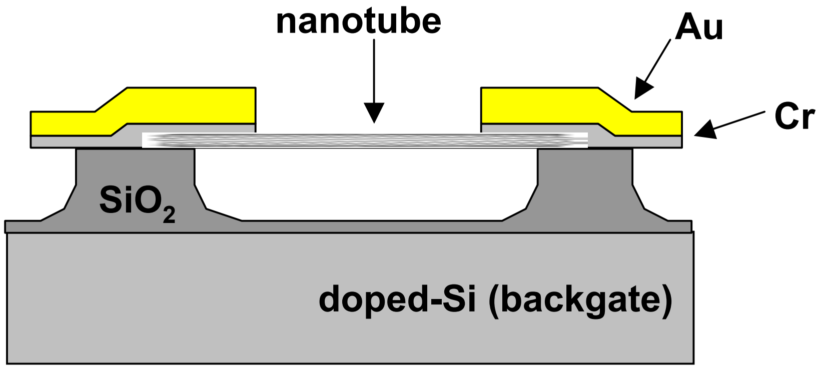
$C = 4\pi\varepsilon_0 R$

capacitances

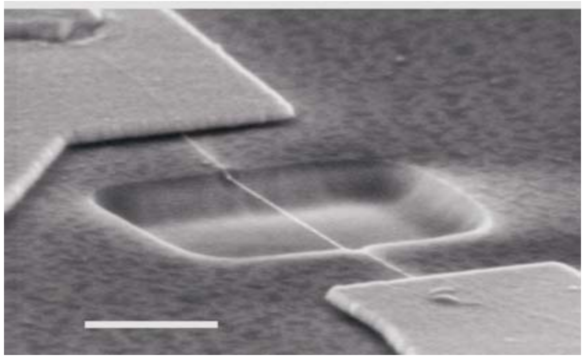
isolated sphere (dot): $C_{sphere} = \varepsilon_0\varepsilon_r\, 2\pi d$

isolated disk: $C_{disk} = \varepsilon_0\varepsilon_r\, 4d$

parallel plate: $C_{parallel\ plate} = \varepsilon_0\varepsilon_r\, A/d$

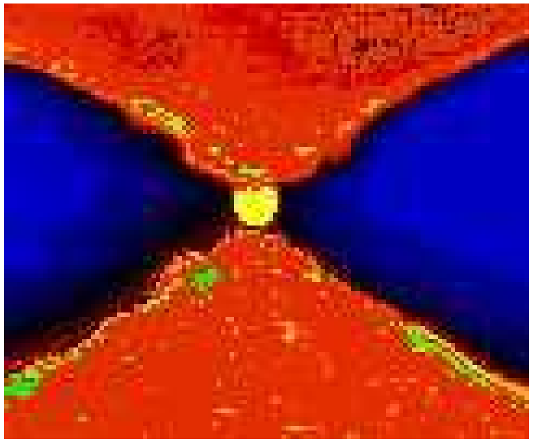


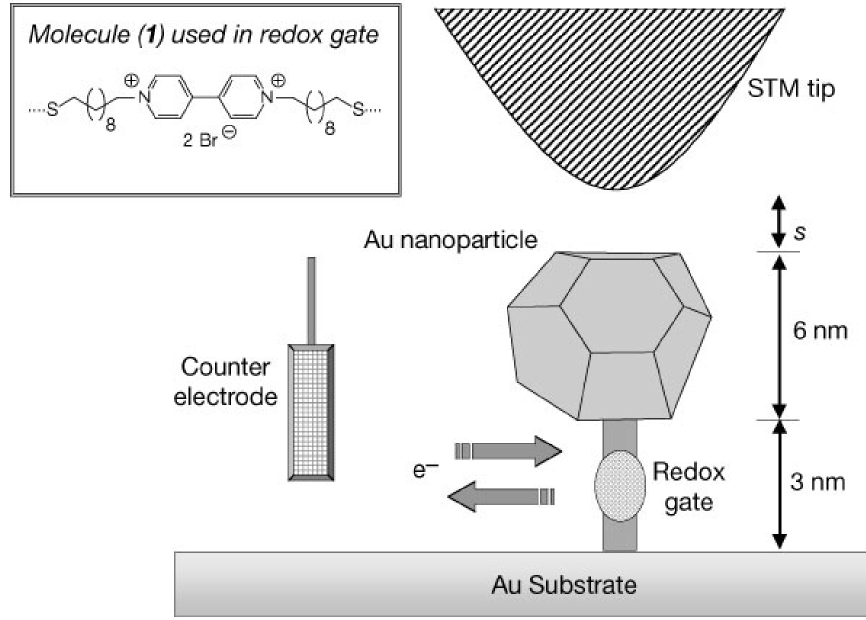
nanotube with diameter, r, above a ground plane at distance h: $C_{NT} = \varepsilon_0\varepsilon_r\, 2\,\pi\, L\, /\, \ln(2h/r)$

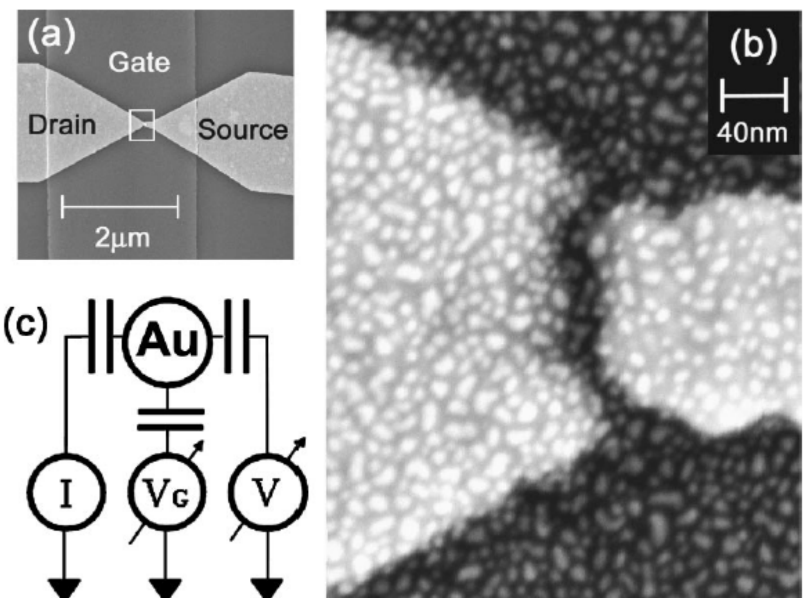


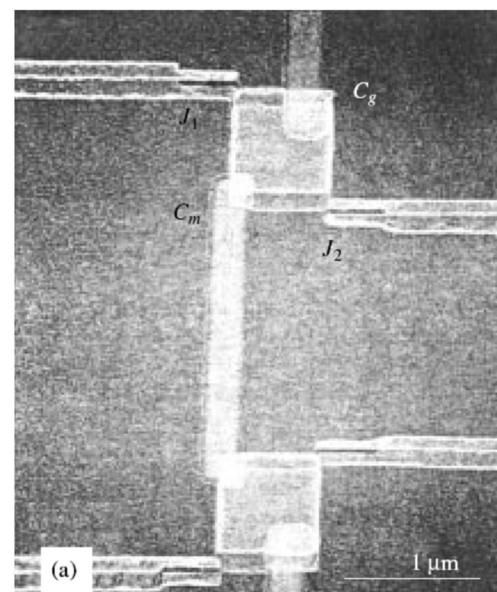
quick estimate: capacitance per unit length: $C' = \varepsilon_0\varepsilon_r = \varepsilon_r\, 10\ \mathrm{aF/\mu m}$

contacting (metallic) nano-scale objects





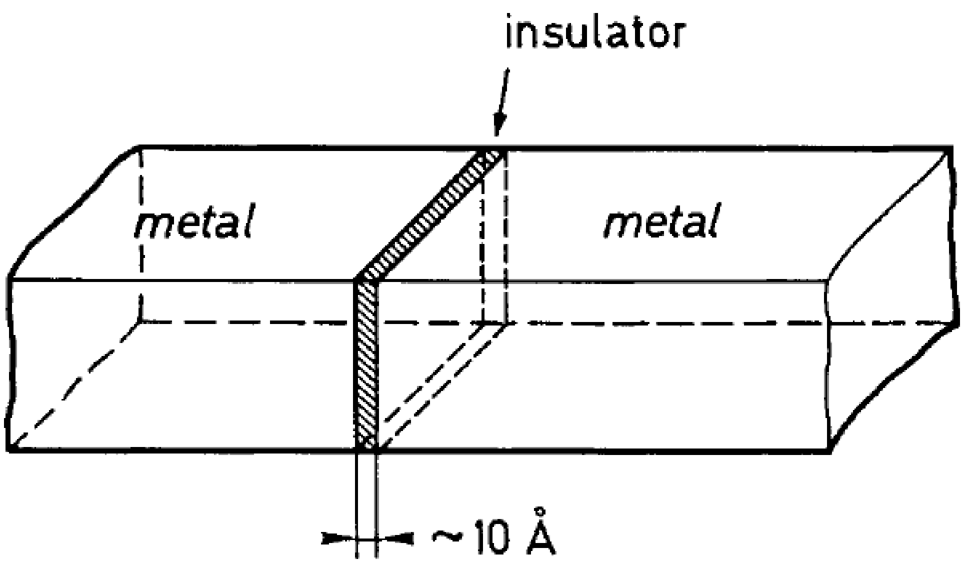


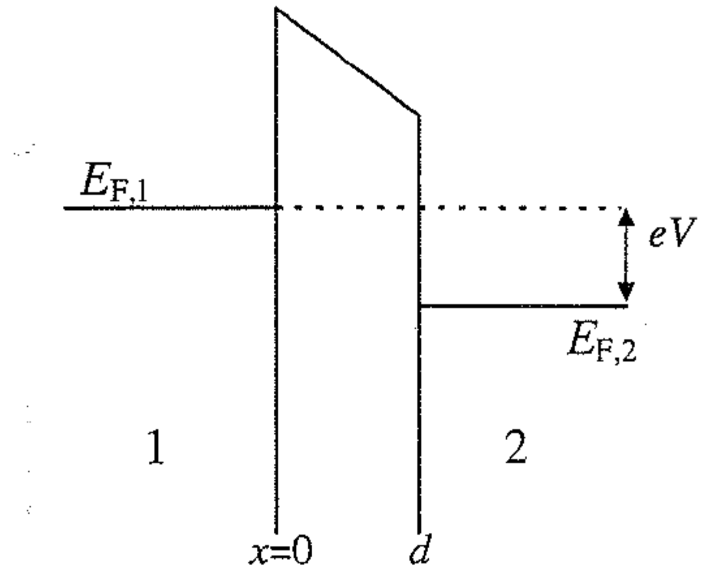


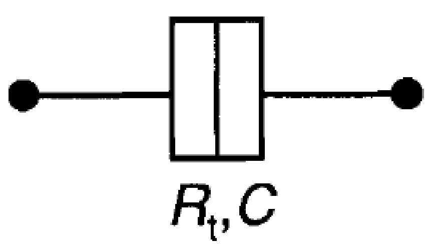
tunnel junction

$$\Gamma_{12} = \frac{2\pi}{\hbar} \int_{-\infty}^{\infty} |t_{12}(E)|^2 \rho_1(E - E_1) f(E - E_1) \rho_2(E - E_2)\{1 - f(E - E_2)\} dE$$

$$\Gamma_{12} = \frac{\Delta U}{e^2} G_t \frac{1}{\exp(\Delta U / kT) - 1}$$





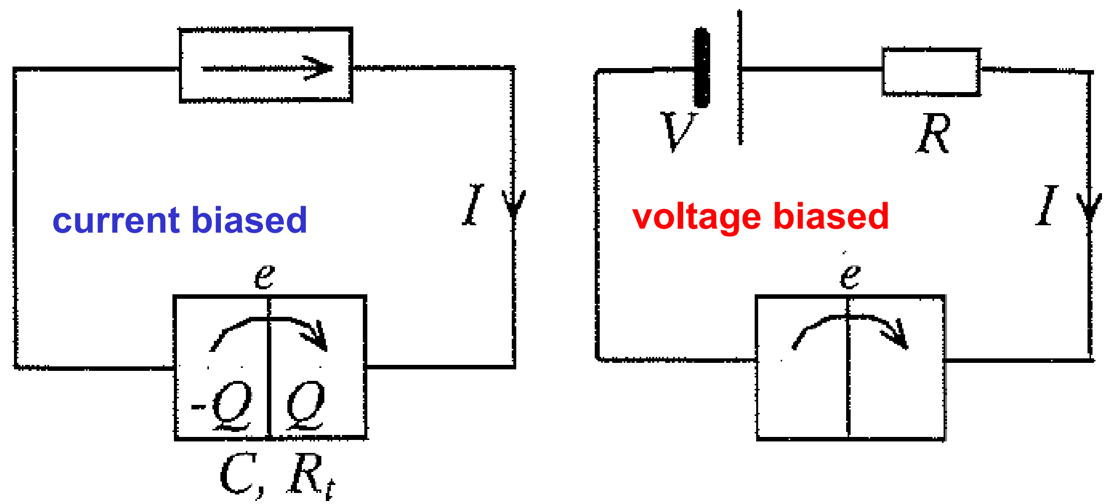


$$\Gamma = \begin{cases} -\frac{\Delta U}{e^2} G_t & \text{for } \Delta U < 0 \\ 0 & \text{for } \Delta U > 0 \end{cases}$$

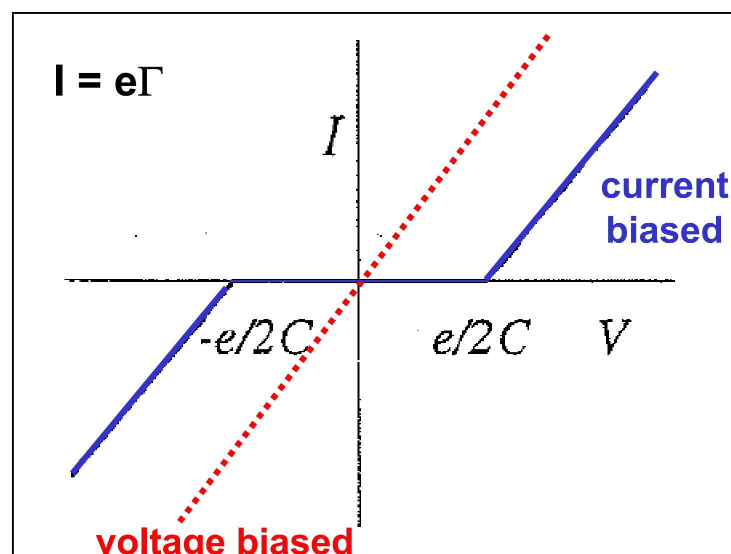
current versus voltage biased

$$\Delta U \equiv U_{final} - U_{initial} = \frac{(Q-|e|)^2}{2C} - \frac{Q^2}{2C} = -\frac{|e|}{C}\left(Q - \frac{|e|}{2}\right)$$

$$\Delta U = -|e|V$$



In practice, it is very difficult to establish current biasing in a single junction circuit. The impedance of air is of the order of hundred Ohms, thereby shunting the high resistance of the current source. One needs to create high resistances within a micron distance (otherwise shunting at high frequencies) of the junction to prevent this. One can use other tunnel junctions (SET transistor) or very thin disordered metals as leads.



single tunnel junction connected to high-resistance leads

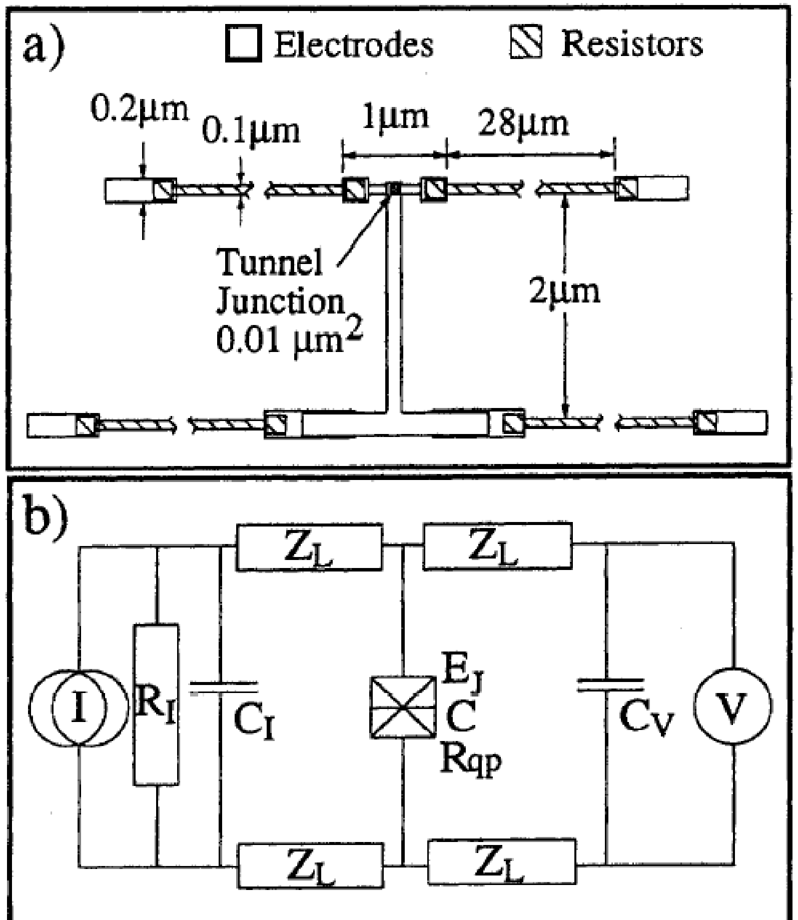
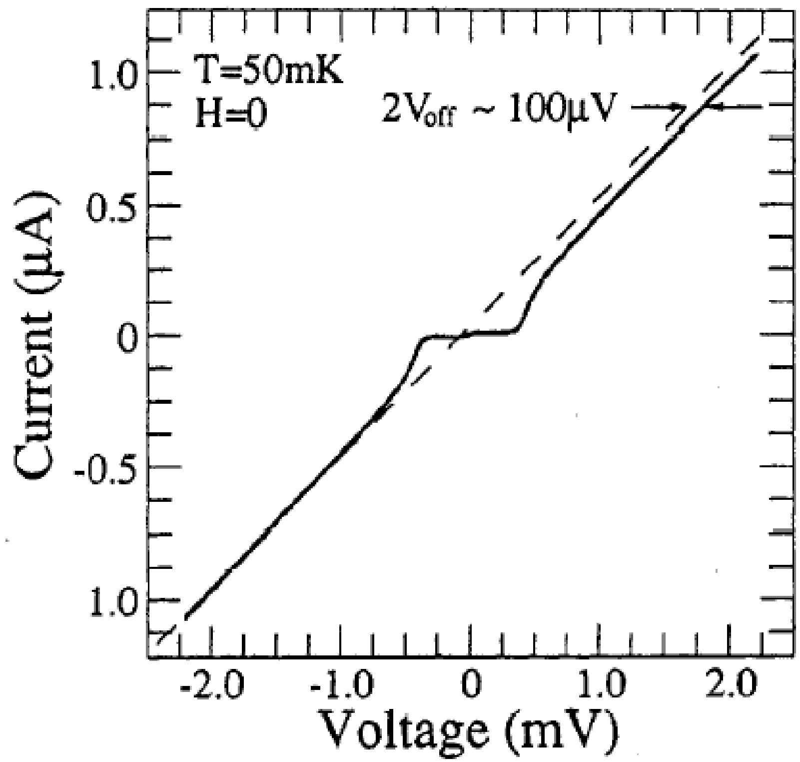


Fig. 1. (a) A schematic of the junction and resistor construction and (b) the equivalent circuit of the measurements

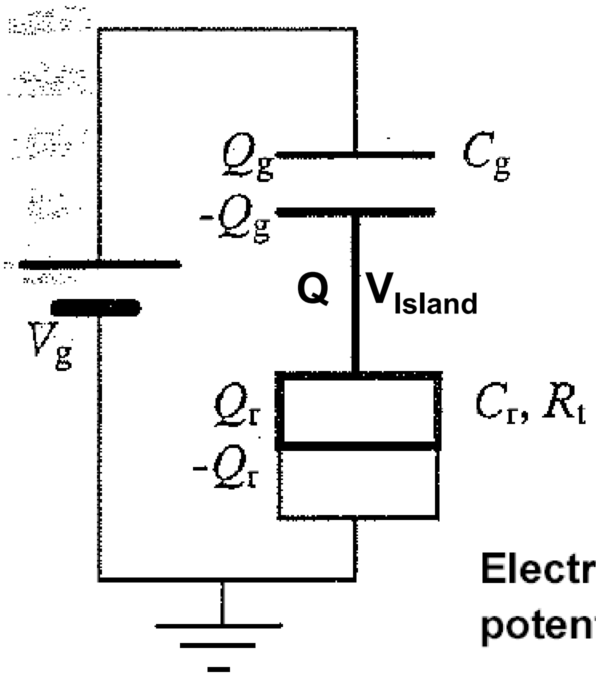


resistance leads: Cr 5-10 nm thick

*Kuzmin and Haviland, Physica Scripta **T42** (1992) 171*

the island: single-electron box

No current flows



Electrostatics gives the following relation between the different potentials and the charge Q on the island ($V_D = 0$):

$$CV_I - C_G V_G = Q\ ,$$

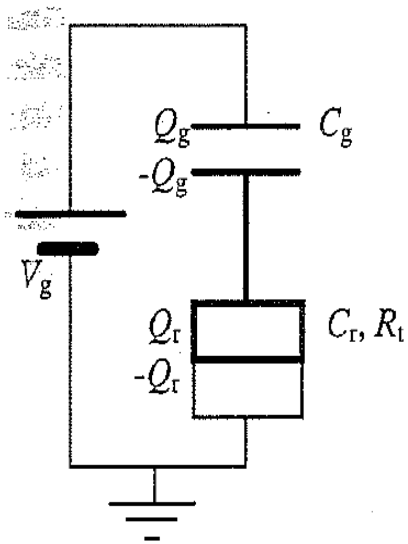
where the total charge is $C = C_R + C_G$.

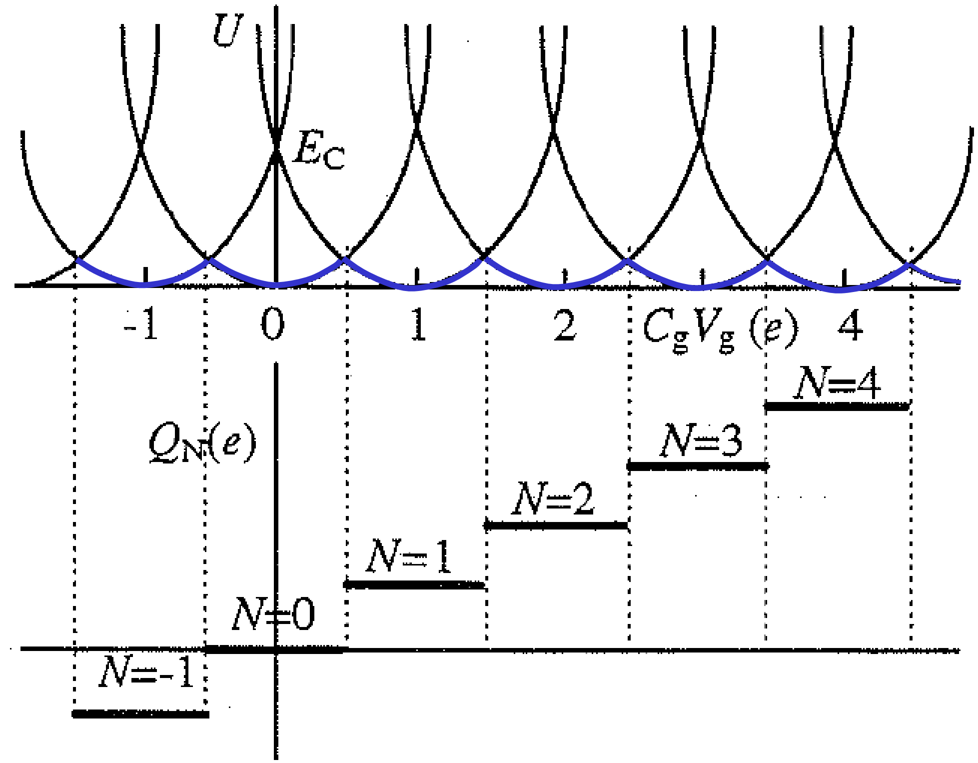
This equation can be written in the form:

$$V_I = V_{ext} + \frac{Q}{C} \quad \text{with} \quad V_{ext} = C_G V_G / C\ ,$$

i.e., the potential on the dot is determined by the charge residing on it and by the induced potential V_{ext} of the source, drain and gate.

the island: single-electron box

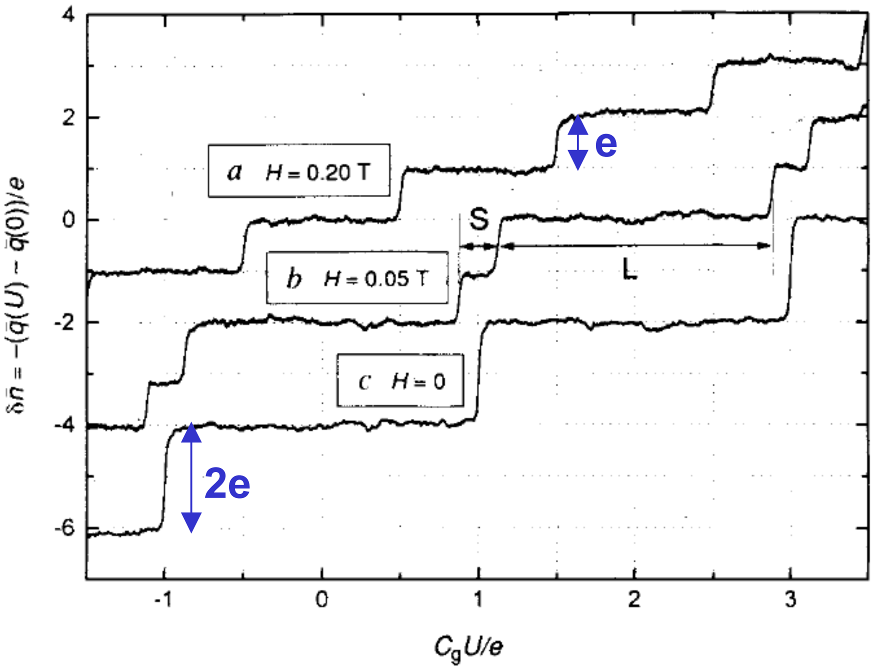




We take as a reference configuration the one for which all voltages and the charge are zero. The electrostatic energy with respect to this reference configuration after changing the source, drain and gate potentials and putting N electrons (of charge $-e$) on the island is then found as the work needed to put this extra charge on the island and the energy cost involved in changing the external potential when a charge Q is present:

$$U_{ES}(N) = \int_{Q=0,V_{ext}=0}^{-Ne,V_{ext}} (V_I dQ + Q dV_{ext}) = \frac{(Ne)^2}{2C} - NeV_{ext} = \frac{(Ne - C_G V_G)^2}{2C} + const \, .$$

measured charge quantization in a normal and superconducting single-electron box



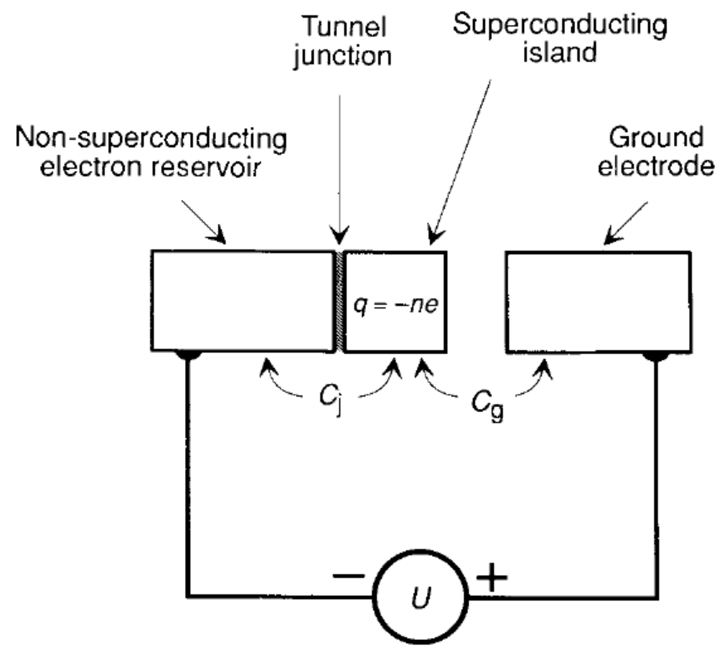


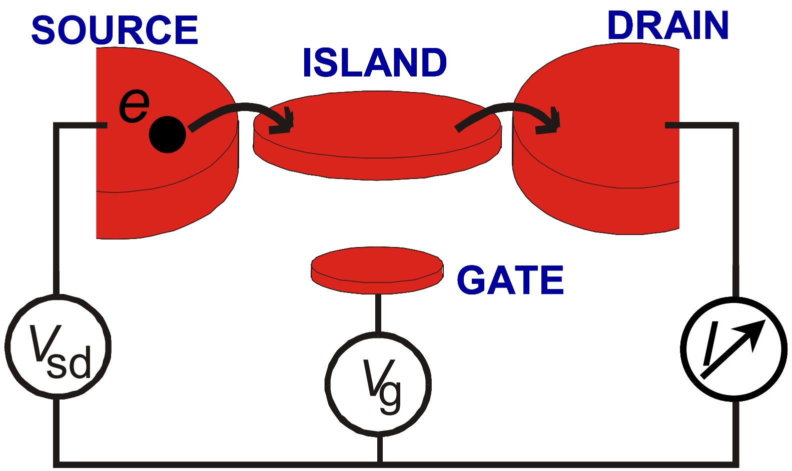
FIG. 1 Schematic diagram of the experiment. The superconducting island is a $30 \times 110 \times 2{,}260$ nm Al strip containing $\sim 10^9$ atoms. Its dimensions are such that the electrostatic energy of one extra electron is much larger than the energy $k_B T$ of thermal fluctuations at temperature $T \sim 30$ mK. The island can exchange electrons with a Cu (3 wt% Al) thin-film electrode (which acts as an electron reservoir) through a tunnel junction[17]. The total charge q of the island varies under the influence of the externally controlled voltage source U connected between the electron reservoir and a ground electrode. The variation with U of the time average $\bar{q}$ of the island charge is measured by a Coulomb blockade electrometer (not shown) which is weakly capacitively coupled to the island. The nanofabrication and low-noise measurement techniques involved in this type of experiment have been described in refs 5 and 18.

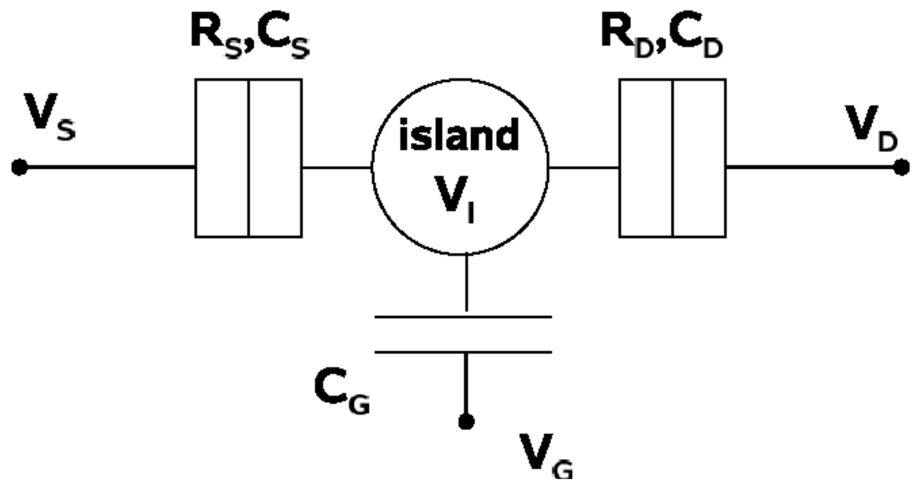
Two-electron quantization of the charge on a superconductor

P. Lafarge, P. Joyez, D. Esteve, C. Urbina & M. H. Devoret

NATURE · VOL 365 · 30 SEPTEMBER 1993

double-barrier circuit: single-electron transistor





Following the same analysis as for the electron box:

The charge Q **on the island is given by:**

$$CV_I - C_S V_S - C_D V_D - C_G V_G = Q \quad \textbf{and} \quad C = C_S + C_D + C_G \, .$$

$$V_I = V_{ext} + \frac{Q}{C} \quad \textbf{with} \quad V_{ext} = (C_S V_S + C_D V_D + C_G V_G)/C \, .$$

The electrostatic energy is: $U_{ES}(N) = \int_{Q=0, V_{ext}=0}^{-Ne, V_{ext}} (V_I dQ + Q dV_{ext}) = \frac{(Ne)^2}{2C} - NeV_{ext} \, .$

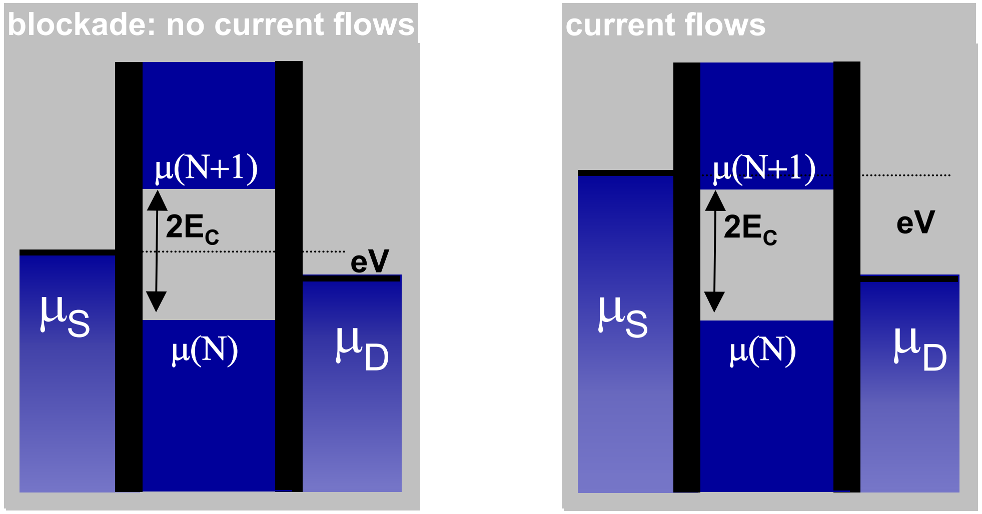
current through single-electron transistor

From non-equilibrium thermodynamics, we know that a current is driven by a chemical potential difference – hence we should compare the chemical potential on the device,

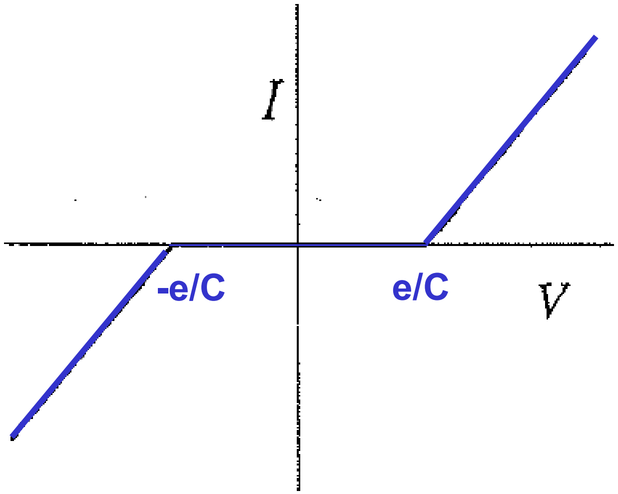
$$\mu(N) = U(N) - U(N-1) = (N - 1/2)\frac{e^2}{C} - eV_{ext} \,,$$

with that of the source and drain in order to see whether a current is flowing through the device. Current flows if –within the transport window eV- a DOS is present on the dot.

energy diagrams



schematic I-V characteristic

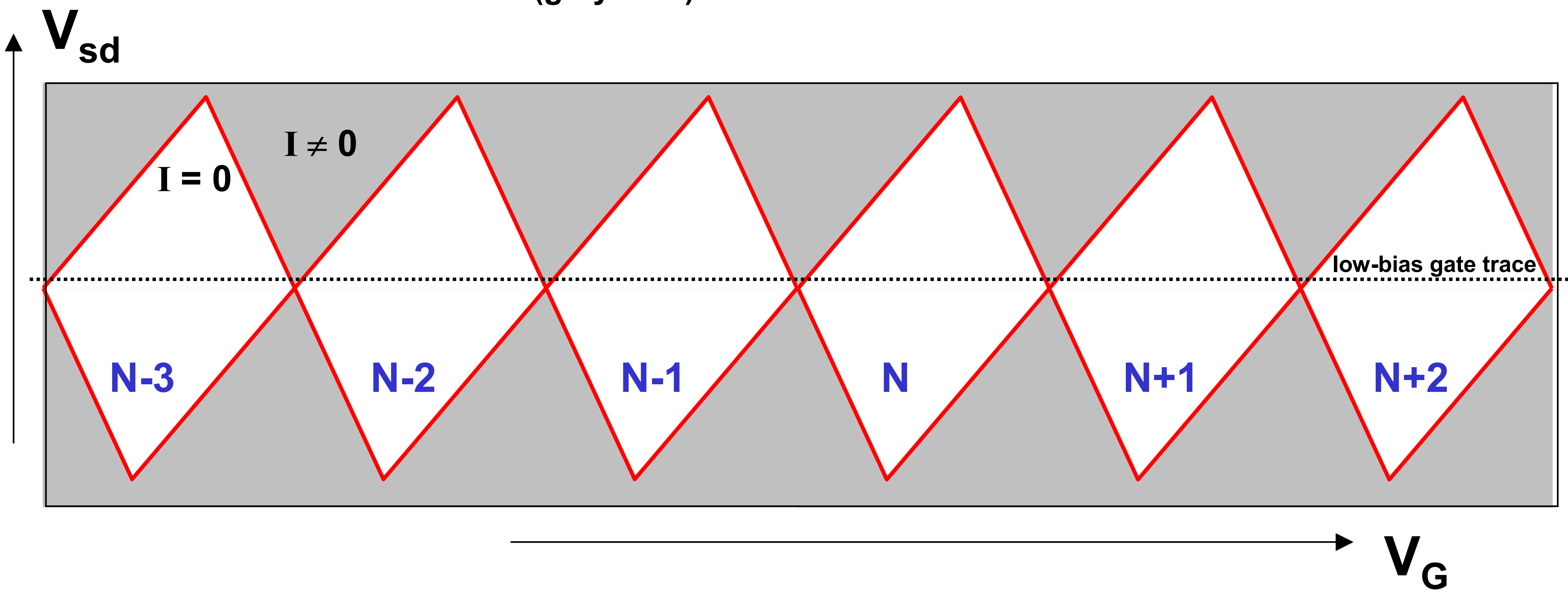


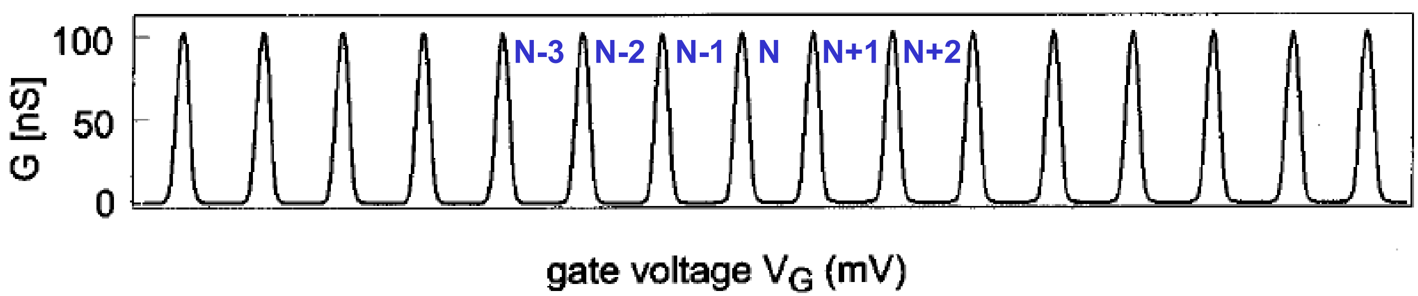
Why starts the current to flow at e/C and not at e/2C as before?

gate traces and stability diagram

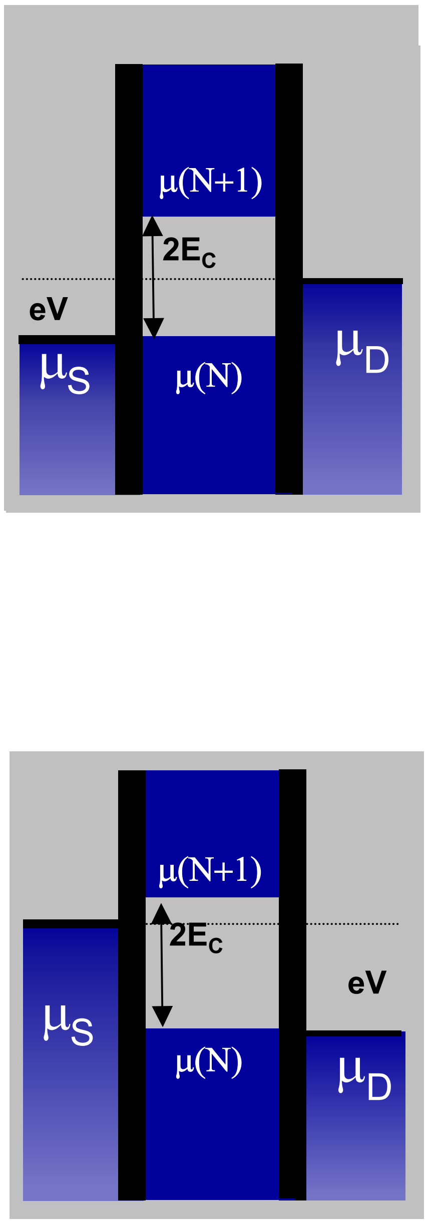
inside the Coulomb diamonds: the number of electrons on the island is fixed and no current flows

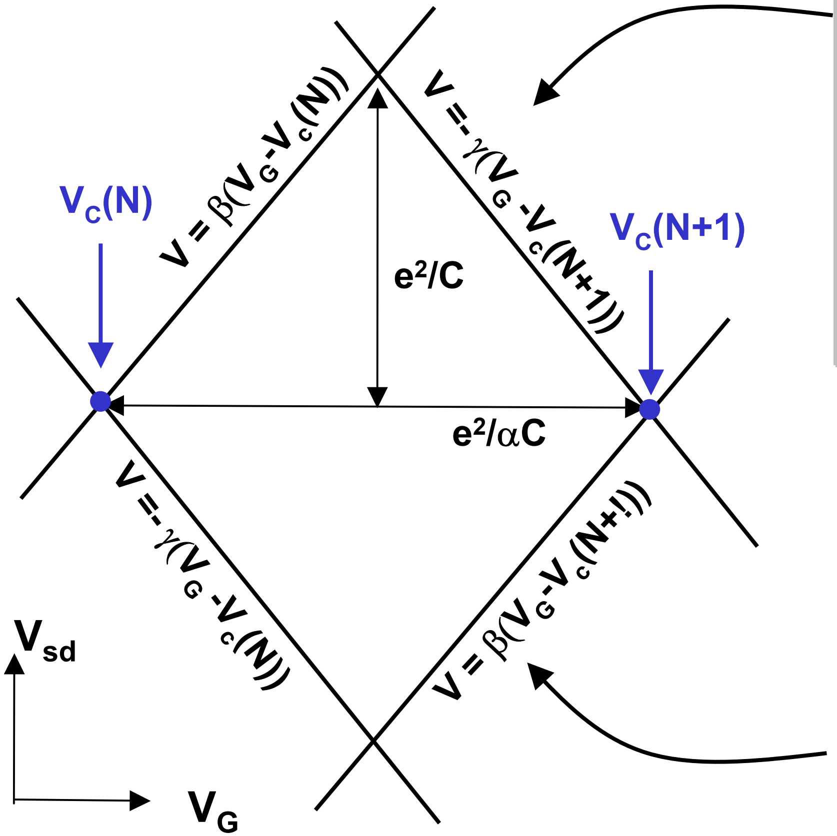
outside the Coulomb islands: the number of electrons fluctuates and current flows (gray area)

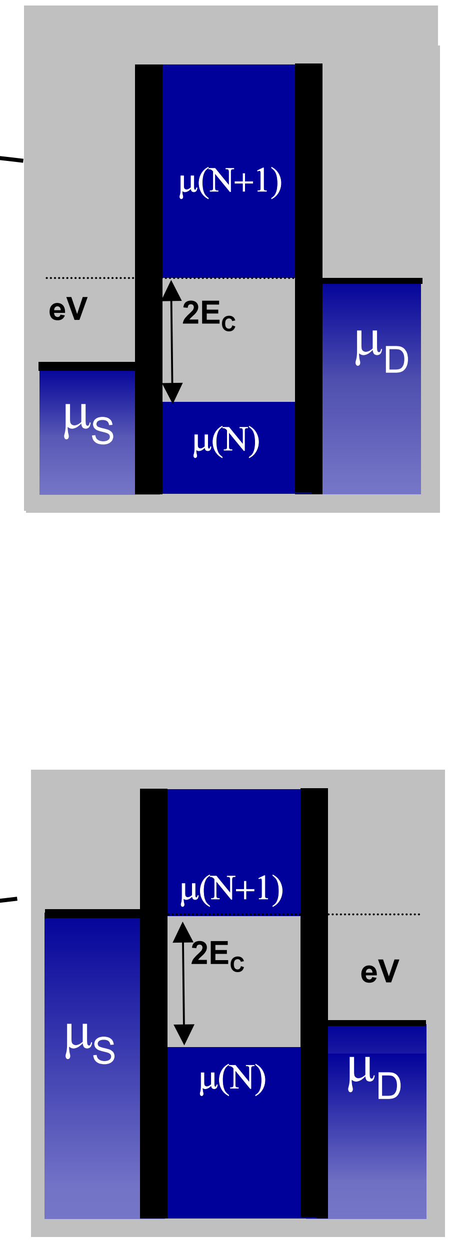




Coulomb diamonds







The lines define a region in which there is no current. This region is called the Coulomb diamond. At zero bias, current flows at the degeneracy points indicated in blue below.

Coulomb diamonds: the equations

From $\mu_S = \mu(N)$ we find $V = \beta(V_G - V_C)$ with $\beta = C_G/(C_G + C_D)$

From $\mu_D = \mu(N) = 0$ we find $V = -\gamma(V_G - V_C)$ with $\gamma = C_G / C_S$

$V_C = (N - 1/2)e / C$, i.e. the voltage corresponding to the chemical potential on the dot in the absence of an external potential.

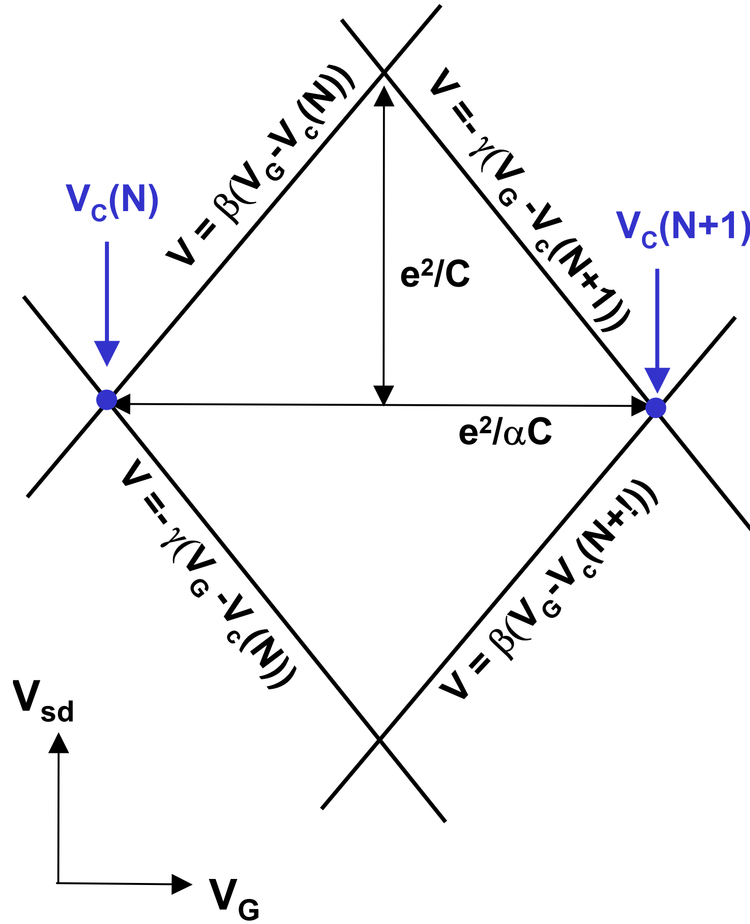
The energy required to put an extra electron on the island (having already N-electrons is called the addition energy:

$$E_{add} = \mu(N+1) - \mu(N) = \frac{e^2}{C} = 2E_C$$

In a measurement the addition energy can be read off from the height of the Coulomb diamonds or from the distance between adjacent crossing point ($V_C(N+1) - V_C(N) = e / C_G = 2E_C / \alpha$).

The latter term contains a factor α, which is the gate coupling parameter: the potential on the island varies linearly with the gate voltage, $\Delta V_I = \alpha \Delta V_G$ where

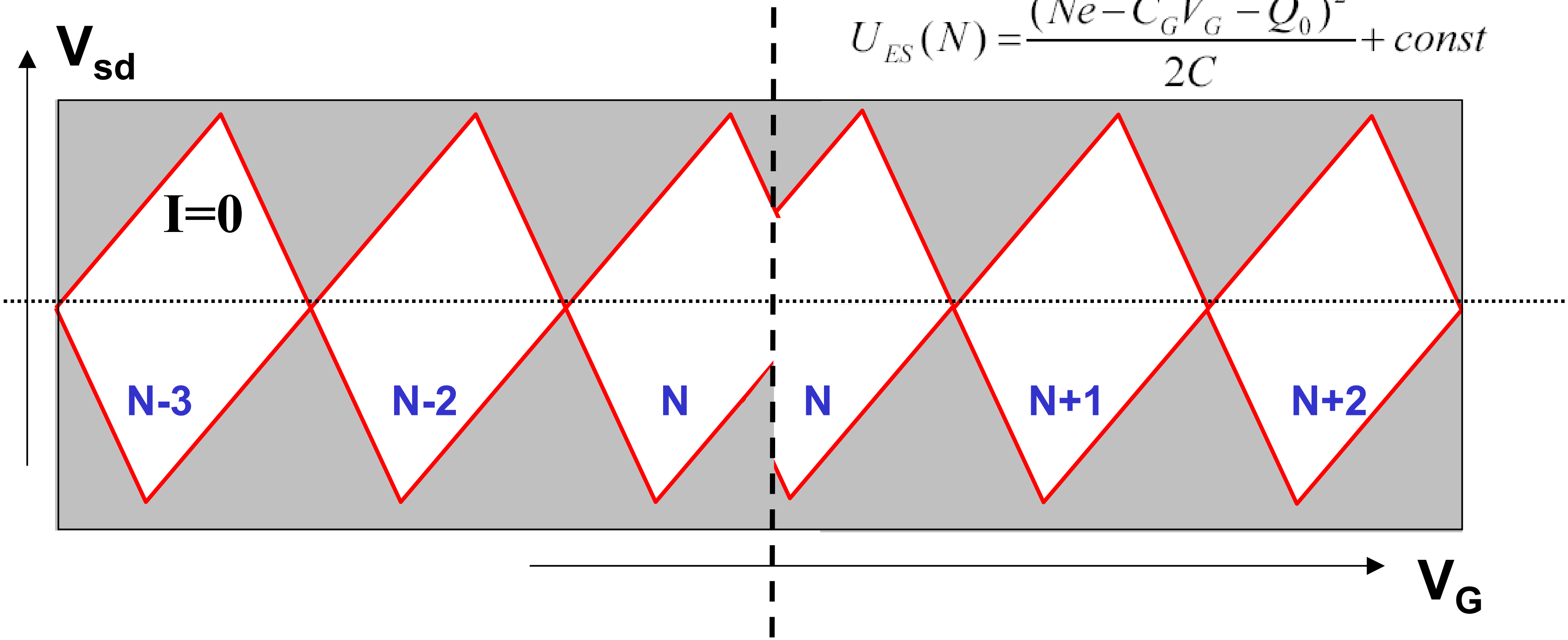
$$\frac{C}{C_G} = \frac{1}{\alpha} = \frac{1}{\beta} + \frac{1}{\gamma}$$



offset-charges

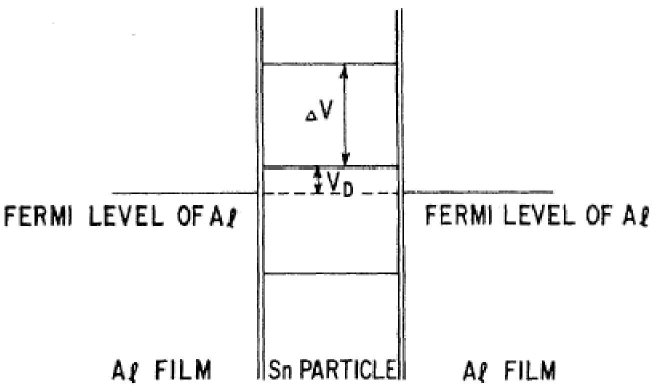
A sudden change in the electrostatic environment of the island (e.g. an electron in the gate oxide that moves from one trap to another) may cause a sudden change in the off-set charge Q_0 on the island: a switch in the stability diagram occurs. The offset charge can take any value is it is due to a polarization of the island.
These offset charges are a problem for more complicated SET circuits, because they are random. Each junction can have its own Q_0, making it difficult to predict device operation.

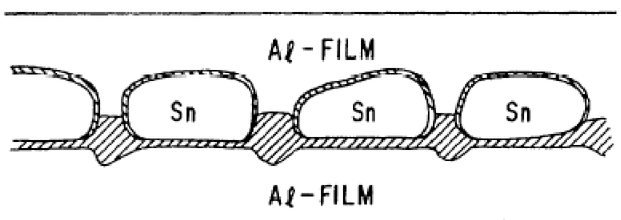
$$U_{ES}(N) = \frac{(Ne - C_G V_G - Q_0)^2}{2C} + const$$

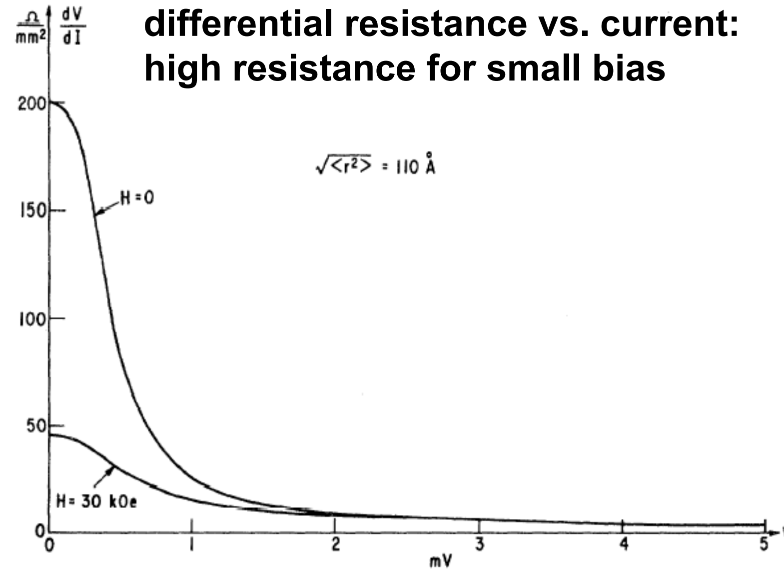


How does the low-bias gate trace looks like in the case of such a switch event?

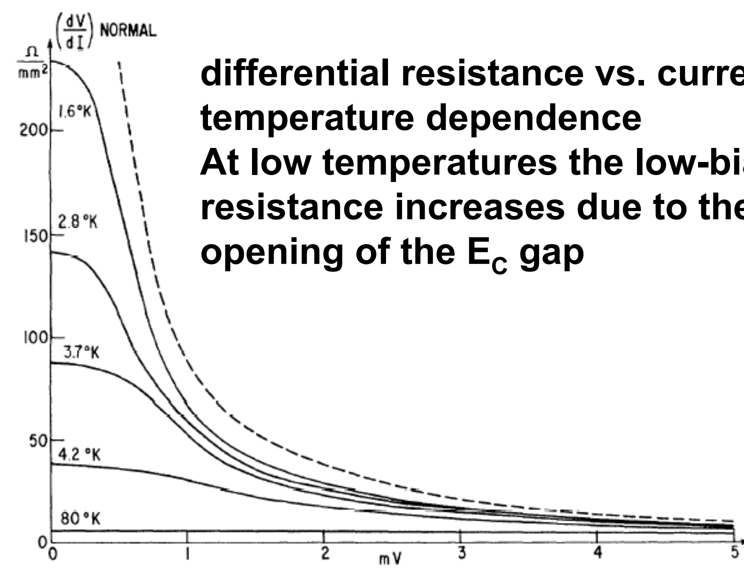
first measurements Coulomb effects







differential resistance vs. current:
high resistance for small bias



differential resistance vs. current:
temperature dependence
At low temperatures the low-bias (diff.) resistance increases due to the opening of the E_C gap

Sn disk –like metallic (superconducting) particles inbedded in aluminium-oxide matrix; diameter particles >2 nm

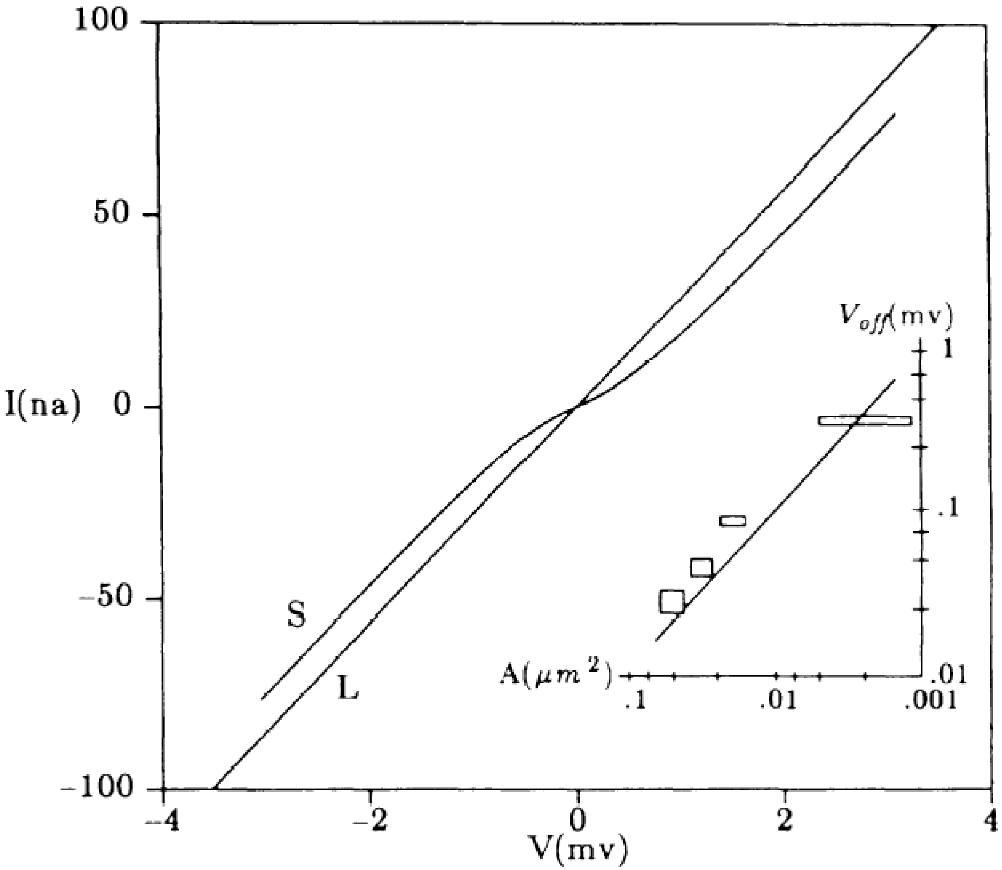
Figures:
V_C (V_{off}) = e/2C ≈ 1-2 meV
C ≈ 100 aF
E_C = 10 K

I. Giaever and H.R. Zeller, Phys. Rev. Lett. ***20*** *(1998) 1504*

VOLUME 59, NUMBER 1 PHYSICAL REVIEW LETTERS 6 JULY 1987

Observation of Single-Electron Charging Effects in Small Tunnel Junctions

T. A. Fulton and G. J. Dolan
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 6 Mar



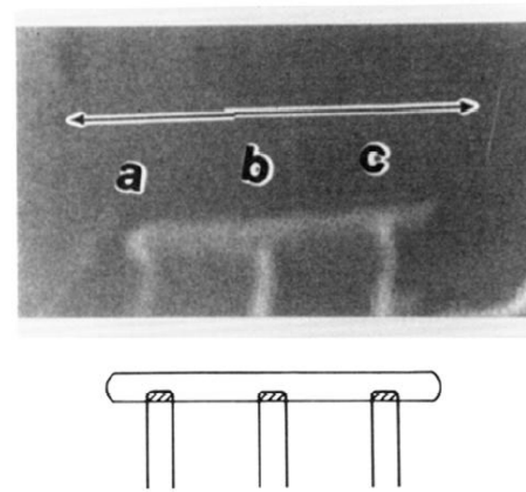


FIG. 2. A scanning-electron micrograph of a typical sample. Junctions labeled a, b, and c are formed where the vertical electrodes overlap and contact the longer horizontal central electrode. The bar is 1 μm long. The configuration is also shown in the accompanying drawing.

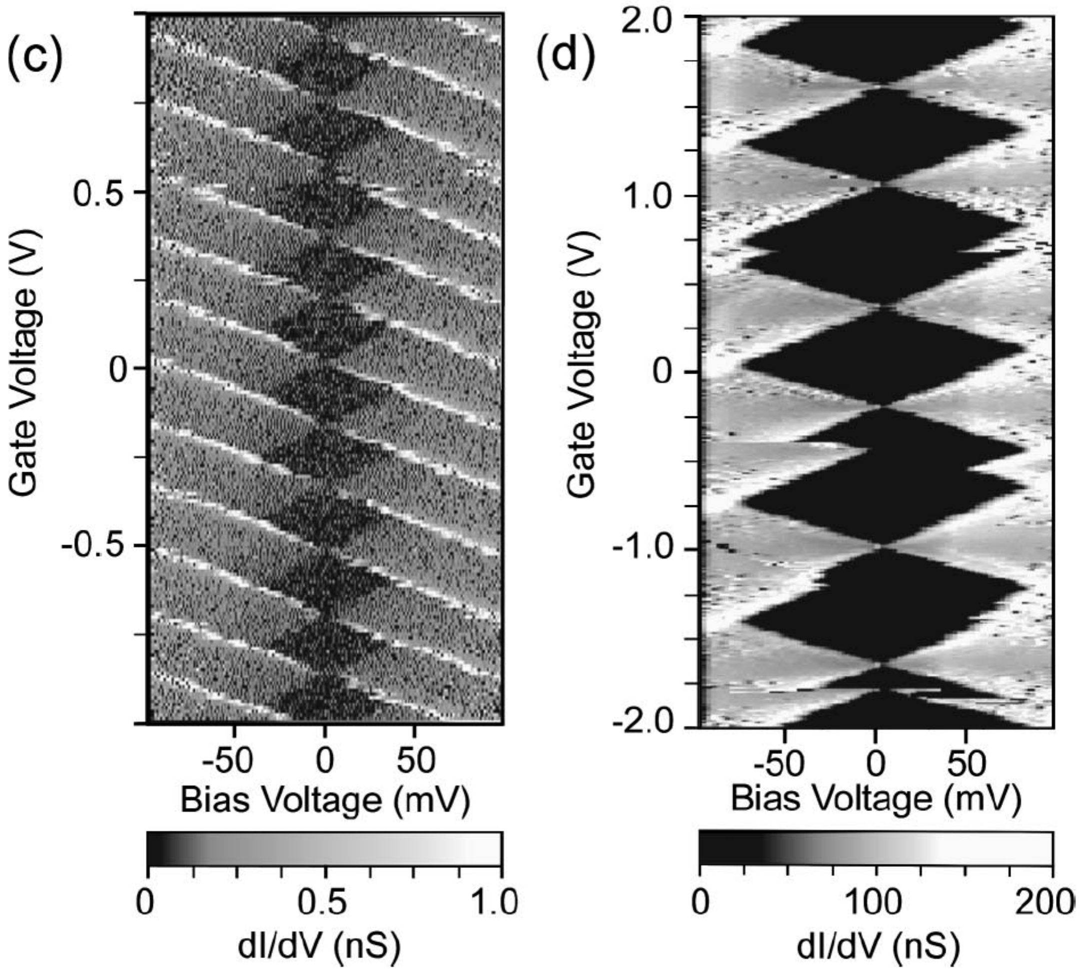
Aluminum tunnel junctions fabricated with a shadow evaporation method

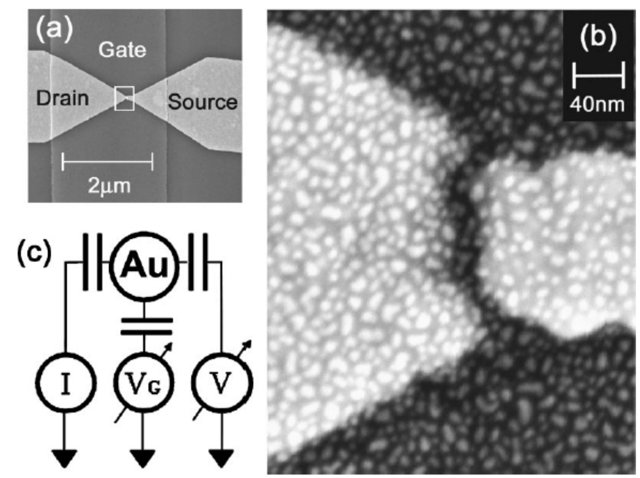
Main figure:
V_C (V_{off}) = e/2C = 0.25-0.3 meV
C = 0.20-0.25 fF

APPLIED PHYSICS LETTERS VOLUME 84, NUMBER 16 19 APRIL 2004

Metal-nanoparticle single-electron transistors fabricated using electromigration

K. I. Bolotin, F. Kuemmeth, A. N. Pasupathy, and D. C. Ralph[a)]
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New Yark 14853



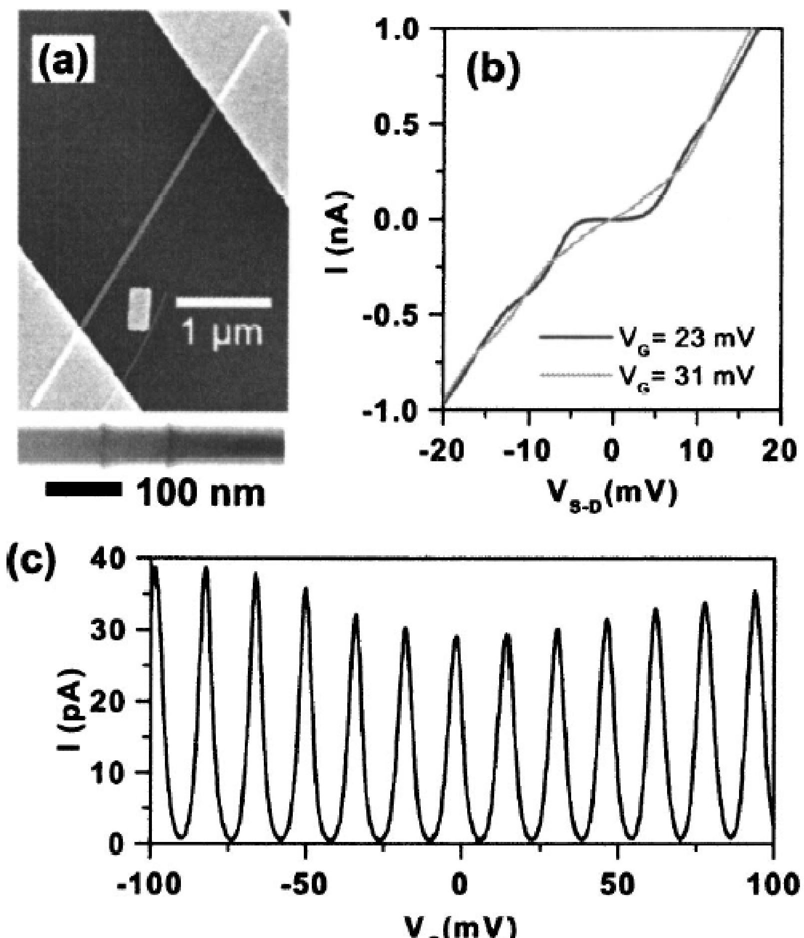


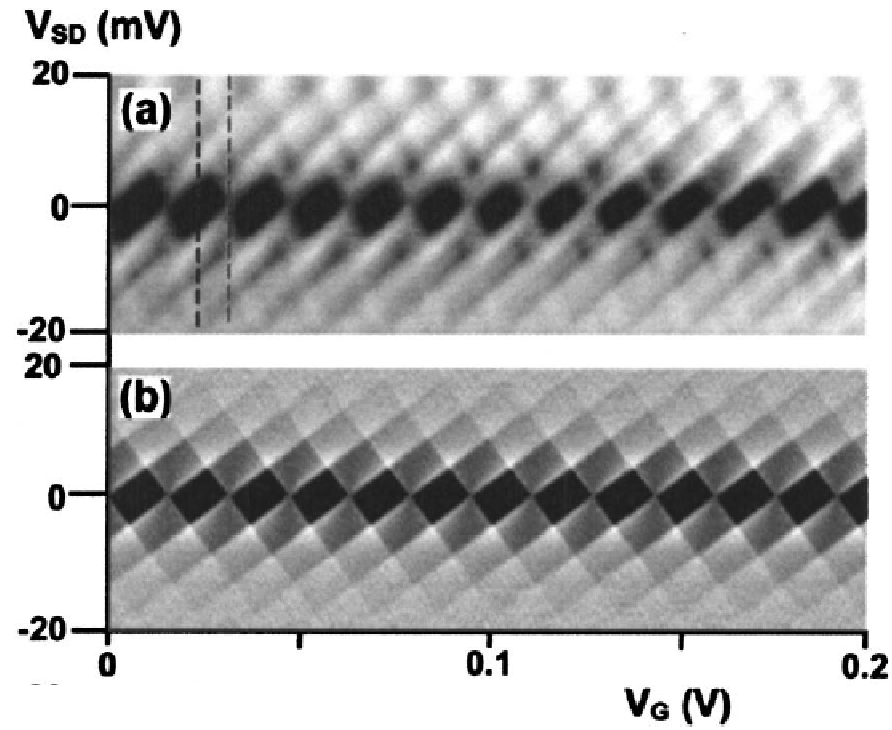
As we have seen before from the stability diagrams all capacitances can be obtained.

APPLIED PHYSICS LETTERS VOLUME 83, NUMBER 10 8 SEPTEMBER 2003

Single-electron transistors in heterostructure nanowires

C. Thelander,[a)] T. Mårtensson, M. T. Björk, B. J. Ohlsson, M. W. Larsson,[b)]
L. R. Wallenberg,[b)] and L. Samuelson
Solid State Physics/Nanometer Consortium, Lund University, P.O. Box 118, SE-221 00 Lund, Sweden





temperature effects

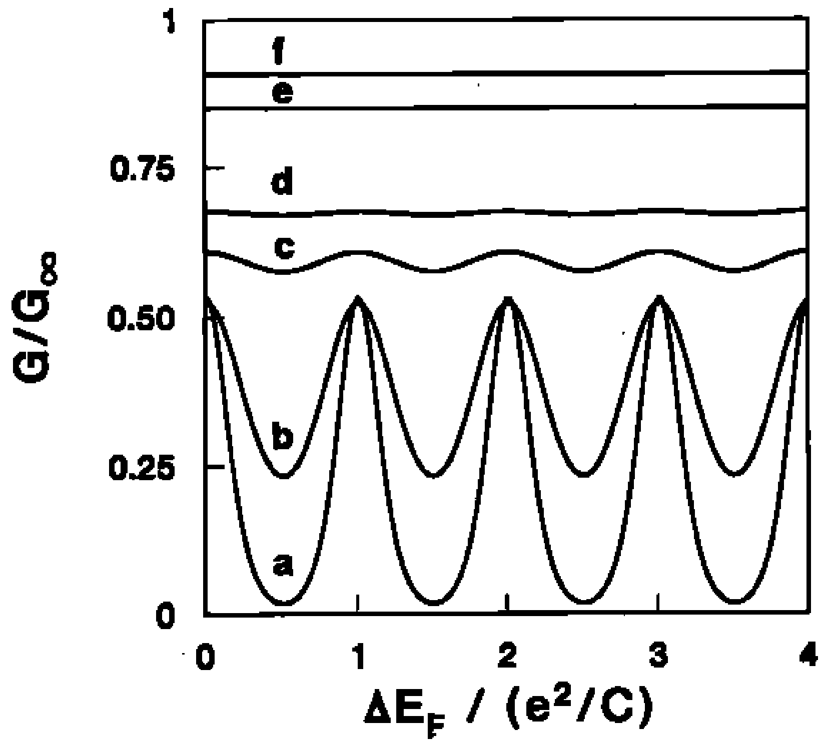


FIG. 6 Temperature dependence of the Coulomb-blockade oscillations as a function of Fermi energy in the classical regime $k_B T \gg \Delta E$. Curves are calculated from Eq. (2.18) with $\Delta E = 0.01\, e^2/C$, for $k_B T/(e^2/C) = 0.075$ (a), 0.15 (b), 0.3 (c), 0.4 (d), 1 (e), and 2 (f). Level-independent tunnel rates are assumed, as well as equidistant non-degenerate energy levels.

peak shape:

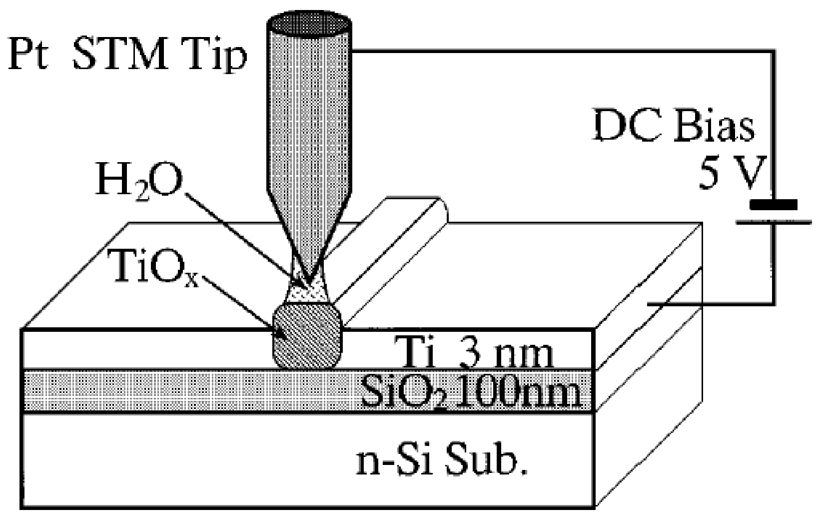
$$\frac{G}{G_\infty} \approx \frac{1}{2} \cosh^{-2}\left[\frac{\delta}{2.5 k_B T}\right]$$

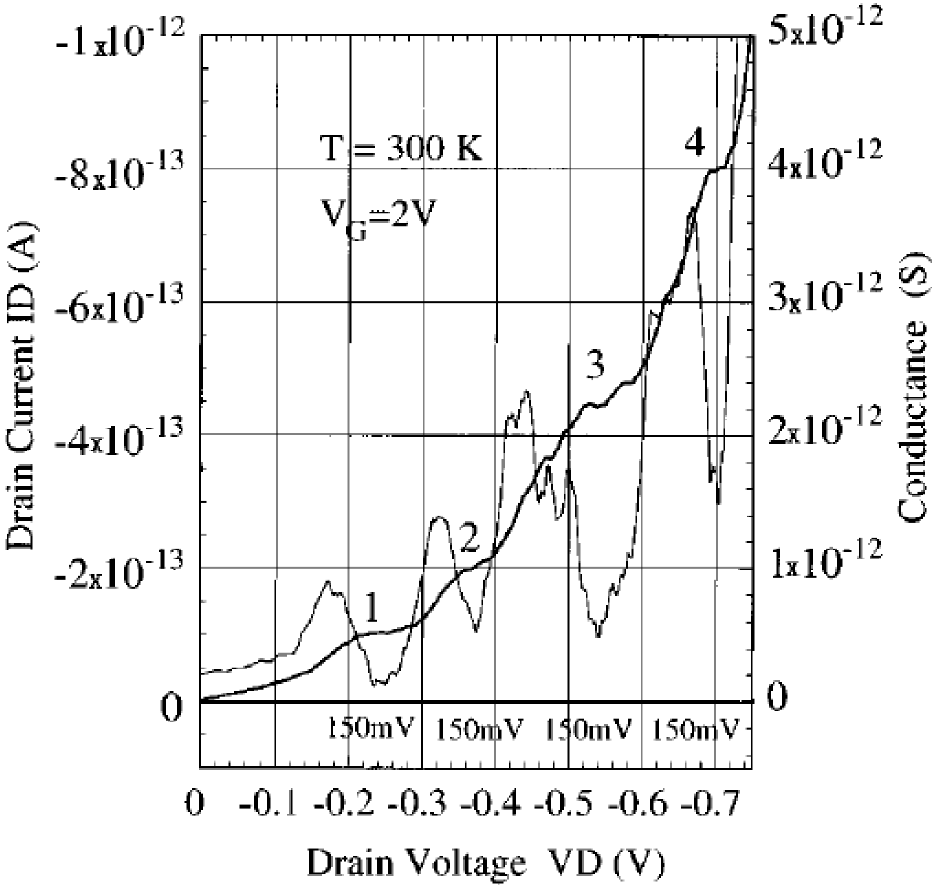
$$\delta = e\frac{C_G}{C}\left|V_{G,res} - V_G\right|$$

asymmetric coupling: Coulomb staircase

In case of asymmetric coupling, one barrier is much thicker than the other (tunneling through is more difficult). In this case, an electron on the island when tunneling to the thick barrier to the drain has to wait a long time before this to happen. If the bias is large enough to provide enough energy a second electron can hop on the dot, thereby suddenly increasing the probability for electrons to be transported. The result is a step-wise increase of the current (Coulomb staircase).

For symmetric barriers this is less likely to happen; the electron already tunnels out before the next one comes in.

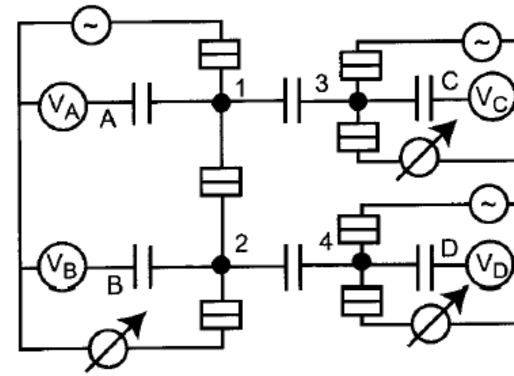




Matsumoto et al., Appl. Phys. Lett. ***68*** *(1996) 34*

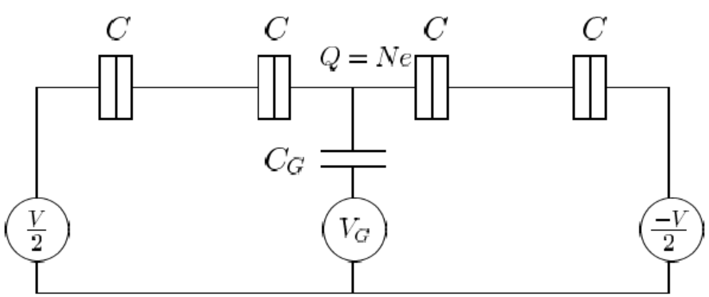
SET applications

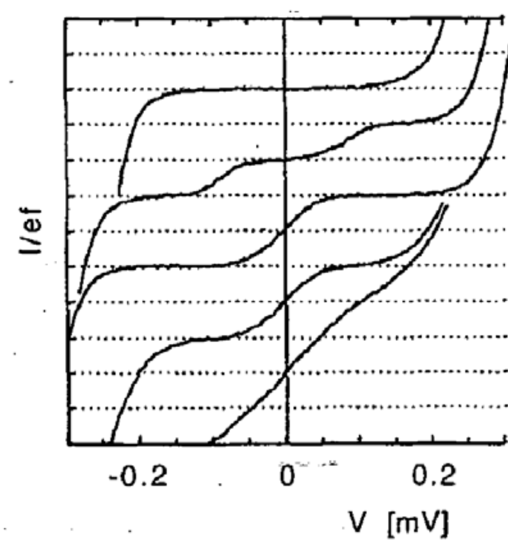
• sensitive charge measurements (10^{-5} e/√Hz)

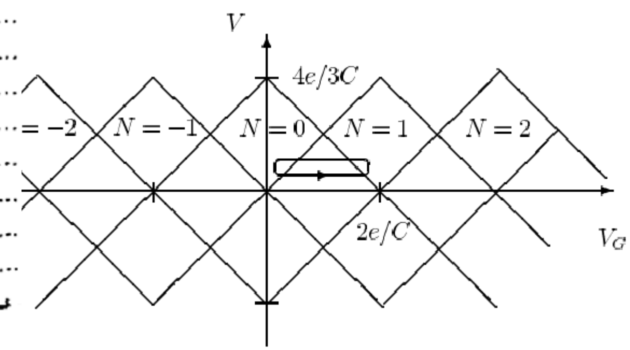


Experiment: SETs 3 and 4 work as electrometers to measure charges at the islands 1 and 2

• current standard (turnstile)







•Single-electron logic, memory

Single-electron logic and memory devices

ALEXANDER N. KOROTKOV†‡

INT. J. ELECTRONICS, 1999, VOL. 86, NO. 5, 511–547

issues:
random offset charges
room-temperature operation

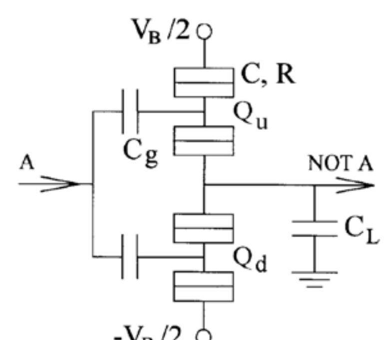
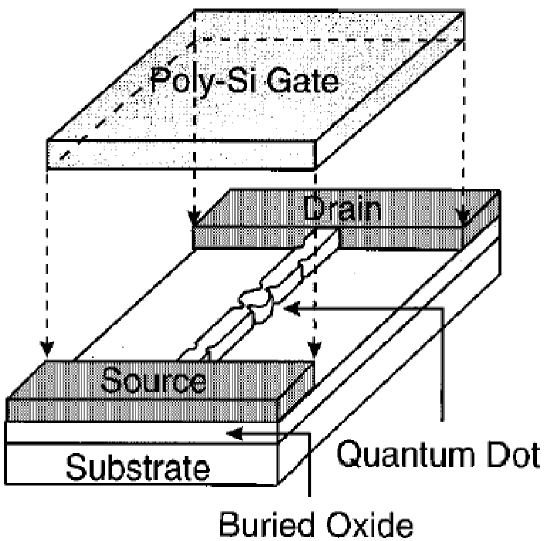


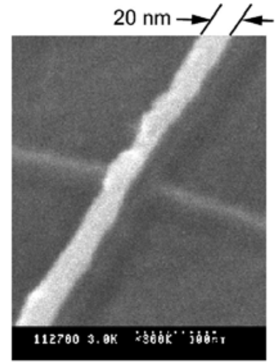
Figure 5. The complementary inverter made of two SET-transistors.

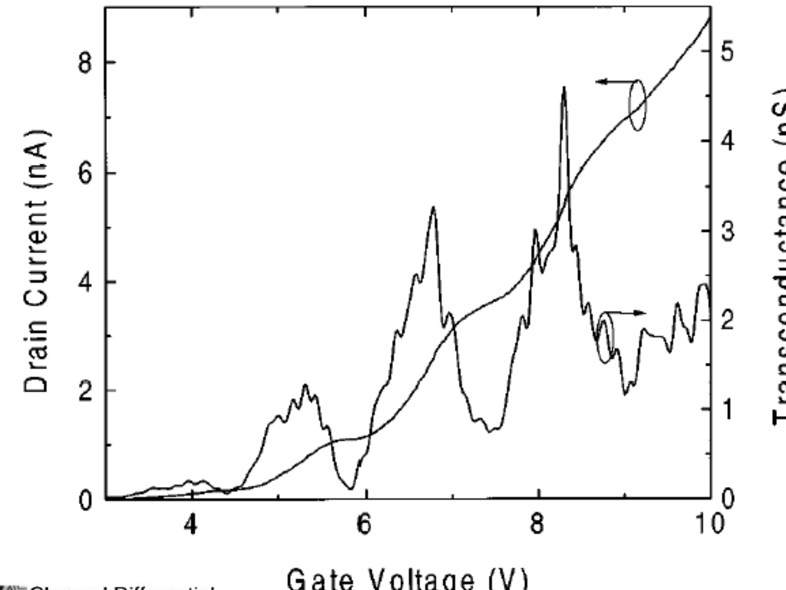
APPLIED PHYSICS LETTERS VOLUME 72, NUMBER 10 9 MARCH 1998

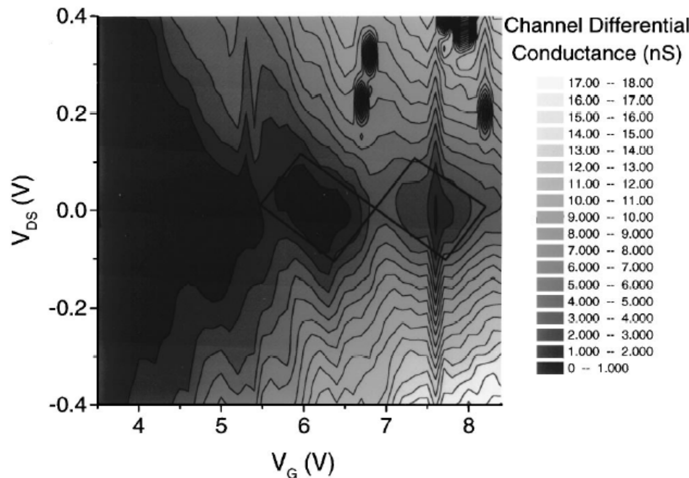
Silicon single-electron quantum-dot transistor switch operating at room temperature

Lei Zhuang,[a)] Lingjie Guo, and Stephen Y. Chou[b)]
Department of Electrical Engineering, NanoStructure Laboratory, University of Minnesota, Minnesota 55455 and Princeton University, New Jersey 08544









classical dots vs. quantum dots

$$U(N) = \frac{(Ne)^2}{2C} - NeV_{\text{ext}} + \sum_{n=1}^{N} E_n$$

- addition energy contains level spacing

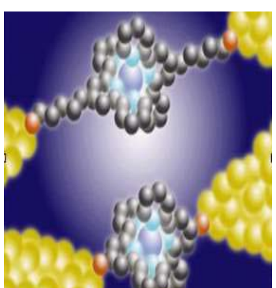
$$\mu(N+1) - \mu(N) = \frac{e^2}{C} + E_{N+1} - E_N$$

- current-voltage characteristic and T-dependence Coulomb peak are different

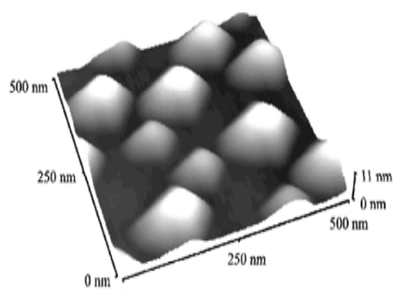
- level spectroscopy: discrete energy spectrum can be measured

- quantum dots: manipulation of a single electron in the dot

single molecule

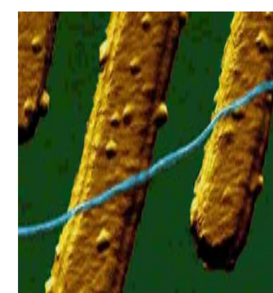


self-assembled QD



semiconducting QD

nanotube

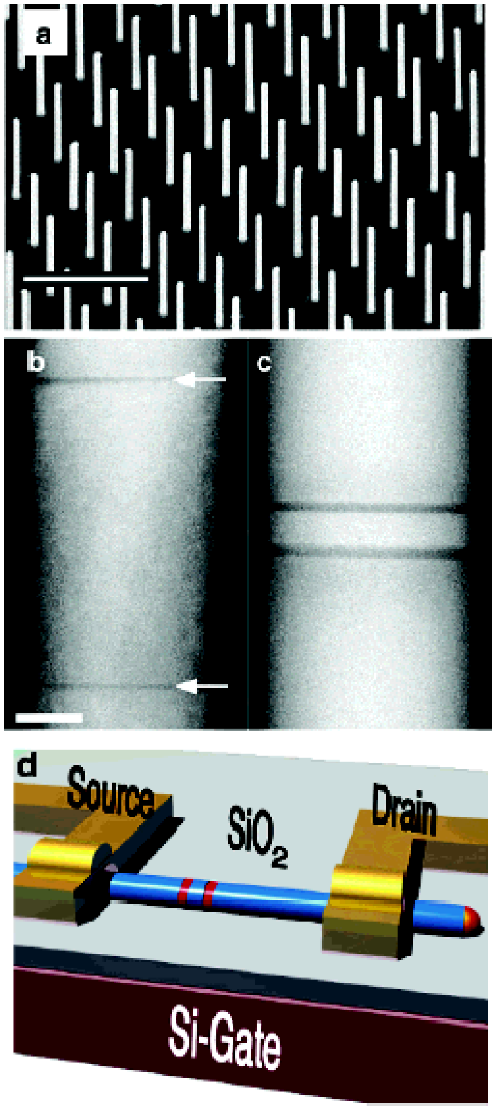


1 nm | 10 nm | 100 nm | 1µm

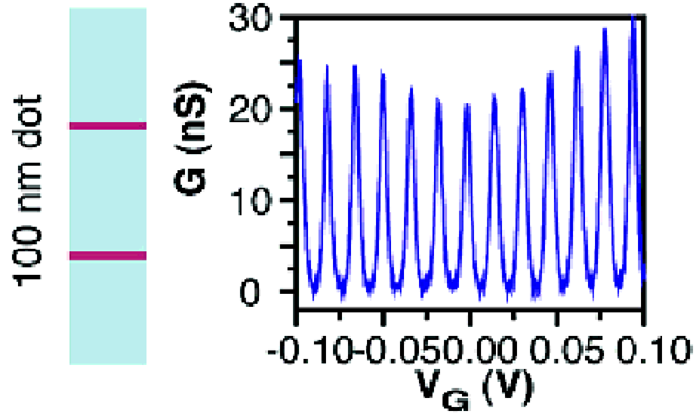
Few-Electron Quantum Dots in Nanowires

Mikael T. Björk,[†,§] Claes Thelander,[†,§] Adam E. Hansen,[†] Linus E. Jensen,[†] Magnus W. Larsson,[‡] L. Reine Wallenberg,[‡] and Lars Samuelson*[,†]

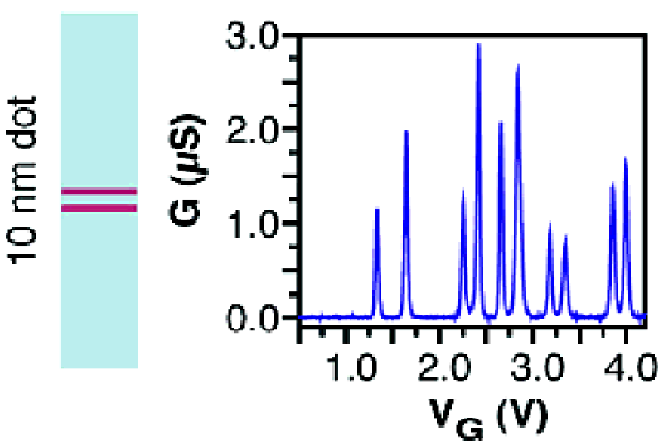
NANO LETTERS
2004
Vol. 4, No. 9
1621−1625



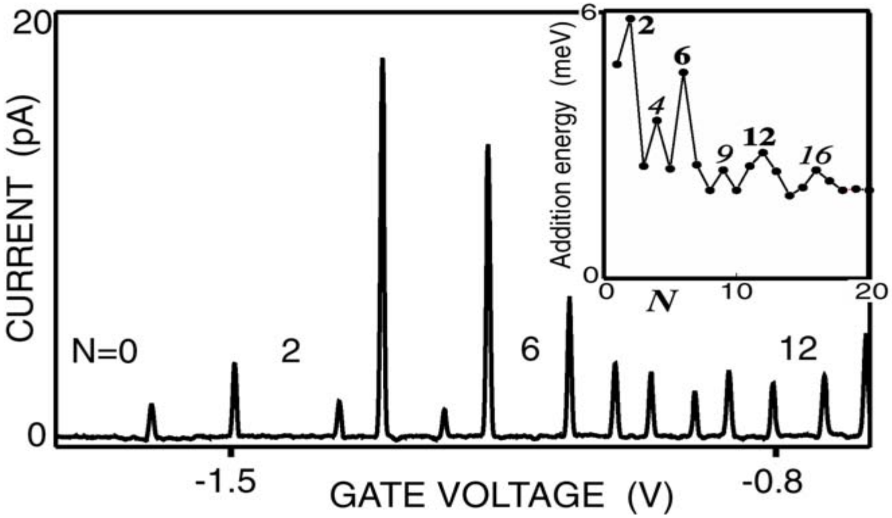
classical dot: regular spaced Coulomb peaks

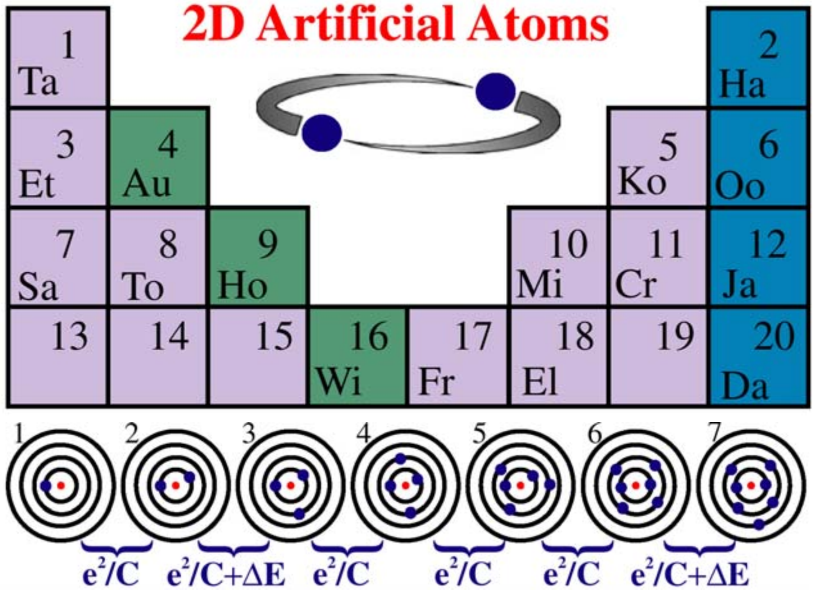


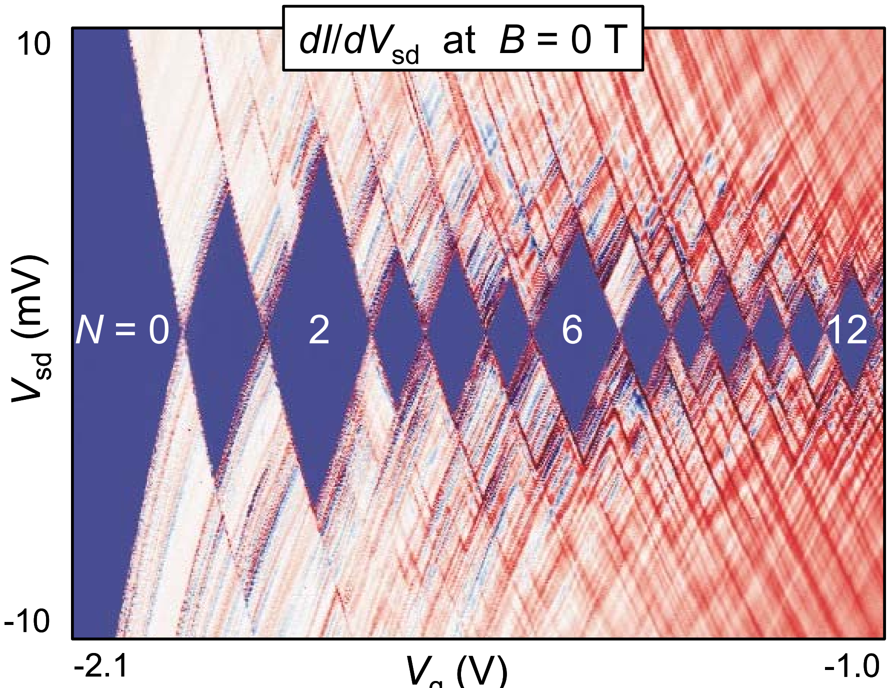
quantum dot: irregular spaced Coulomb peaks



few-electron quantum dots

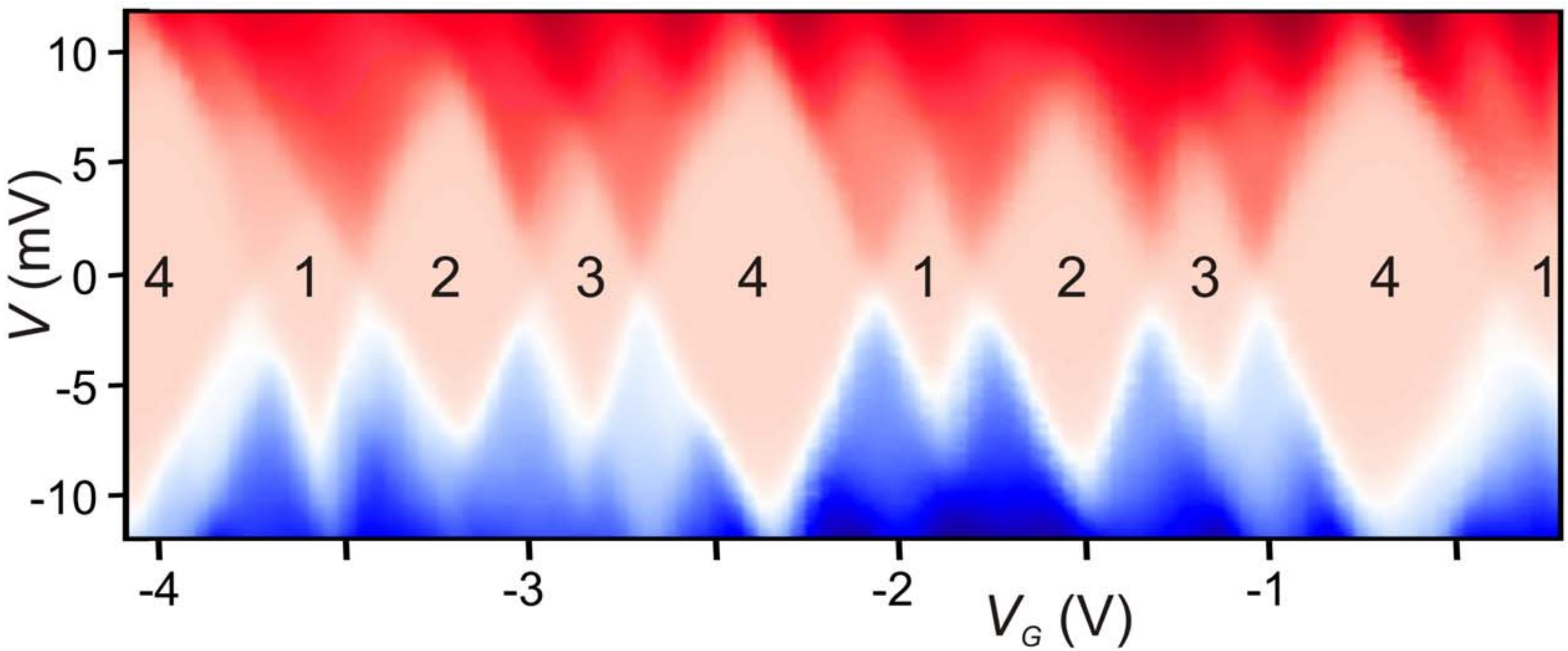






Kouwenhoven et al., Science ***278****, 1788 (`97)*

single-wall nanotube quantum dot



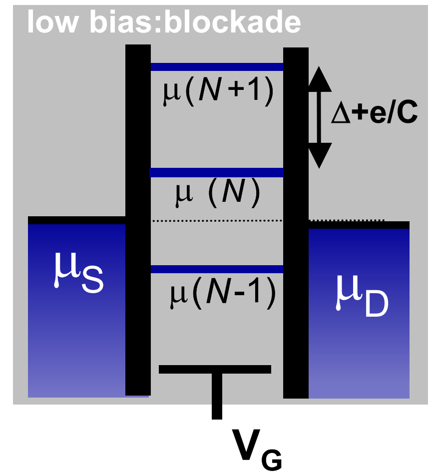
Give an estimate of the charging energy and the level spacing.

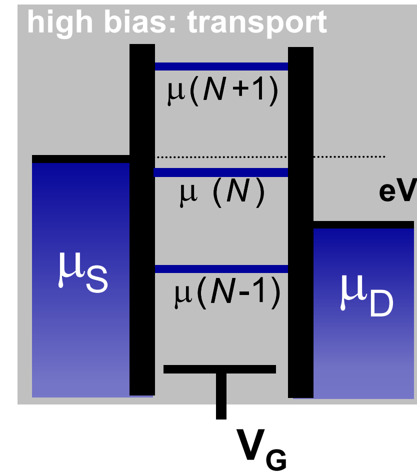
Compare the level spacing with the theoretical value.

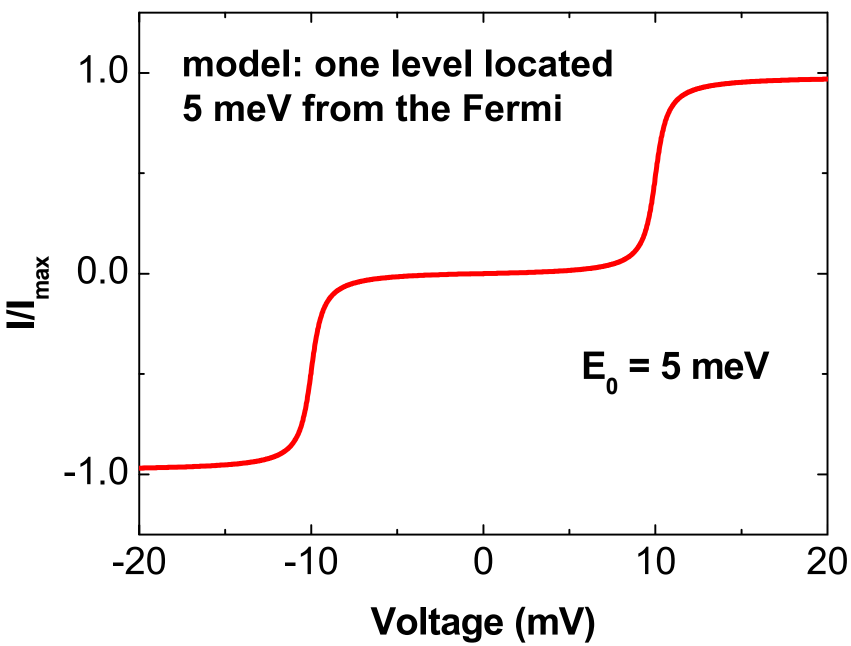
current-voltage characteristic quantum dot

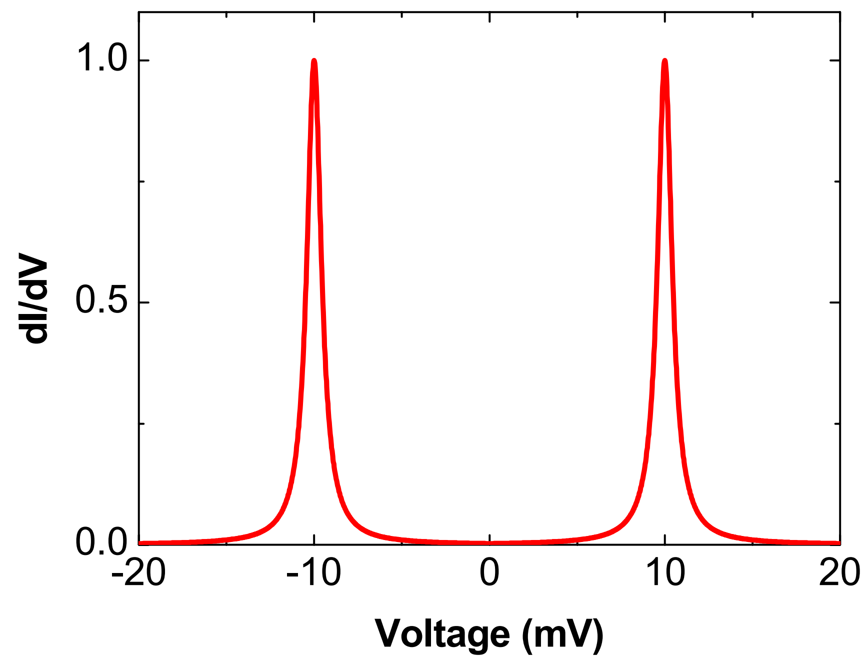
Current-voltage characteristic shows step-wise increase (discrete level structure on the dot; note the difference in the energy diagram with a classical dot)!

Conductance on resonance reaches the conductance quantum value indicating perfect transmission, despite the fact that the two individual barriers can have T<<1!!!!





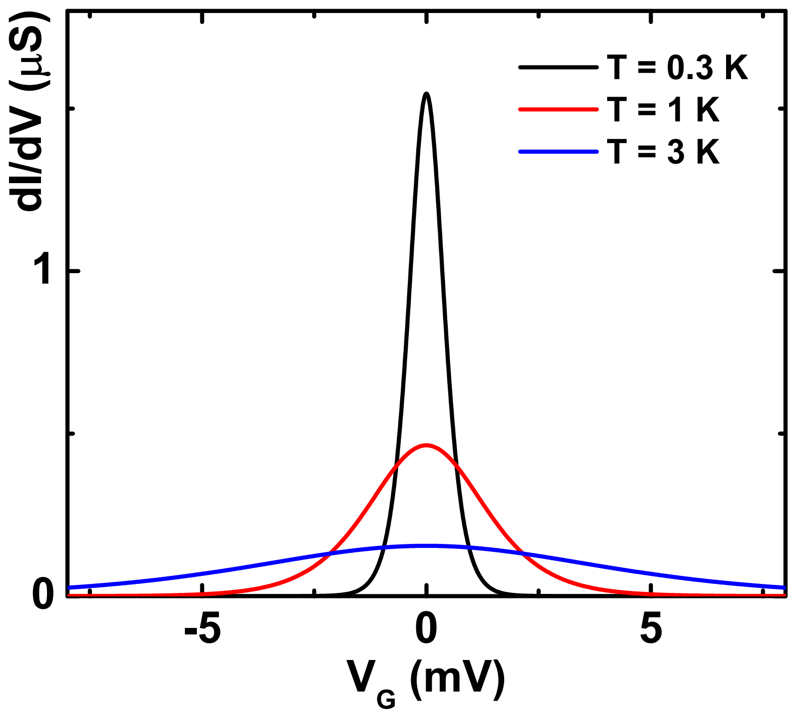


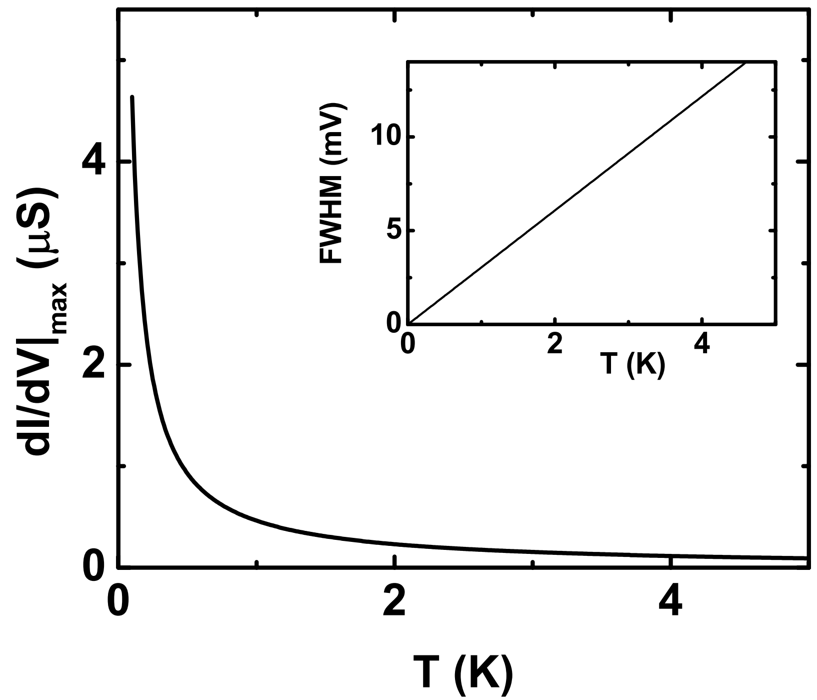


Why is the total gap 20 meV and not 5 or 10?

temperature effects: zero-bias conductance quantum dots

Quantum dots: the conductance increases as temperature is lowered and approaching perfect transmission for $T \rightarrow 0$ (resonant transmission)
(classical dots: peak height remains the same!!!!)





Left: Temperature dependence of the Coulomb peak height in the resonant transport model showing the characteristic increase as temperature is lowered. Right: peak height as a function of temperature. The inset shows the full width half maximum (FWHM) of the Coulomb peak as a function of temperature (see next slides). Calculations are performed with $\Gamma = 10^9$ 1/s and a gate coupling of 0.1 in the regime $\Gamma < k_B T$.

full-width half maximum conductance peaks

quantum $(\text{FWHM} = 3.5\, k_B T)$

$$\frac{G}{G_\infty} \approx \frac{\Delta E}{4k_B T} \cosh^{-2}\left[\frac{\delta}{2k_B T}\right]$$

classical $(\text{FWHM} = 4.35\, k_B T)$

$$\frac{G}{G_\infty} \approx \frac{1}{2} \cosh^{-2}\left[\frac{\delta}{2.5k_B T}\right]$$

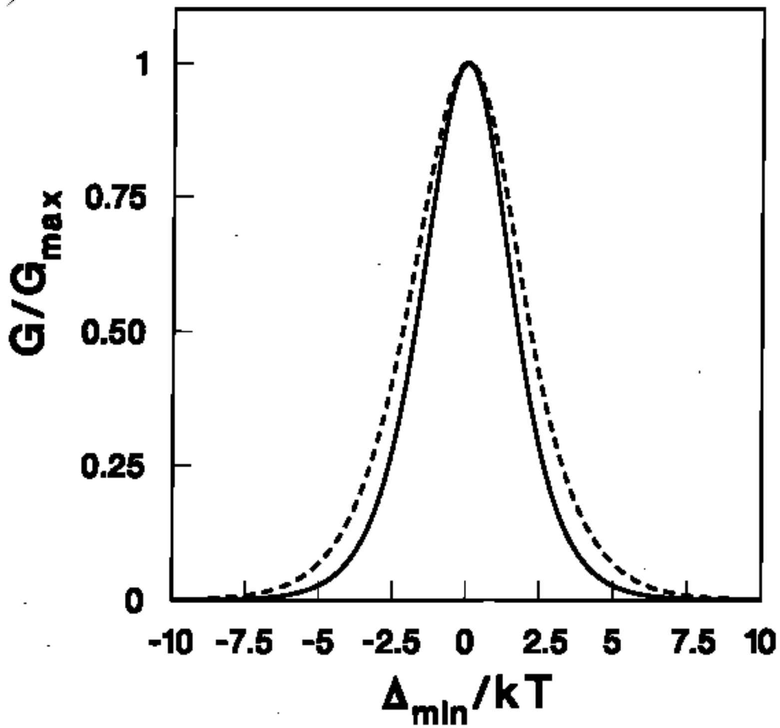
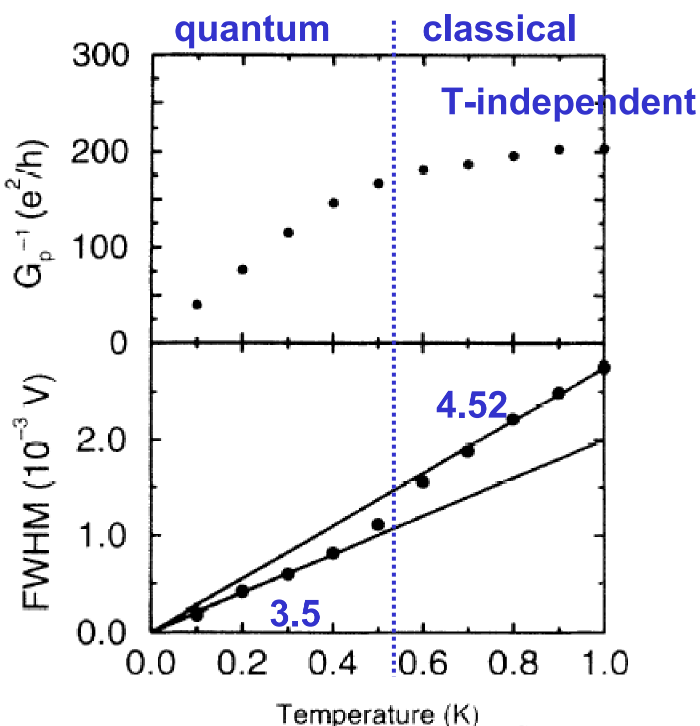
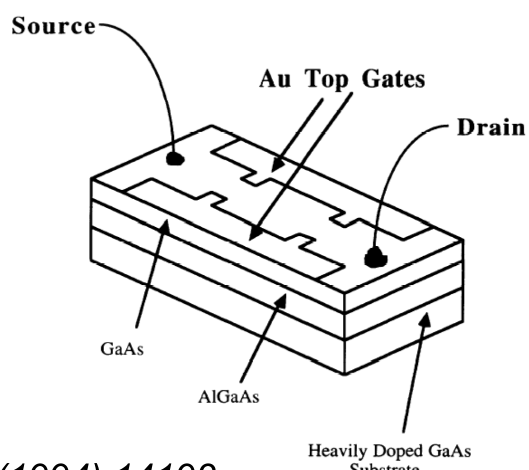


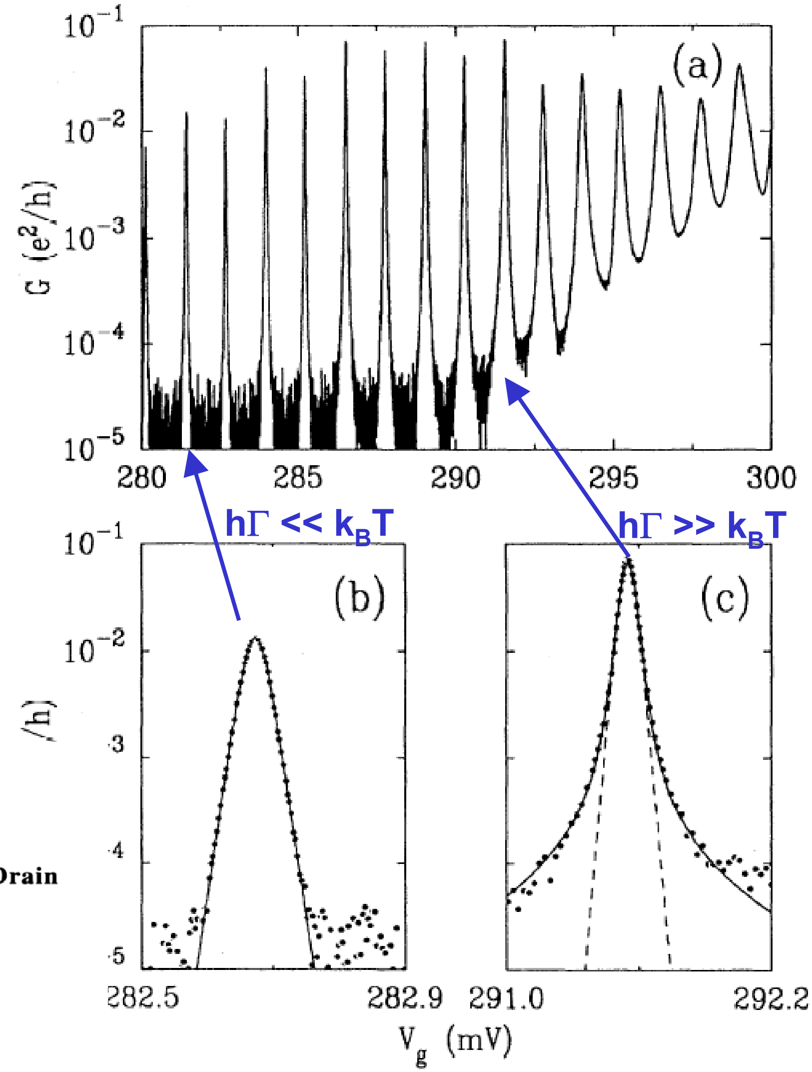
FIG. 7 Comparison of the lineshape of a thermally broadened conductance peak in the resonant tunneling regime $h\Gamma \ll k_B T \ll \Delta E$ (solid curve) and in the classical regime $\Delta E \ll k_B T \ll e^2/C$ (dashed curve). The conductance is normalized by the peak height G_{max}, given by Eqs. (2.25) and (2.28) in the two regimes. The energy Δ_{min} is proportional to the Fermi energy in the reservoirs, cf. Eq. (2.26). (From Beenakker.[19])

experimental low-bias peak shapes



transition to quantum regime at low T

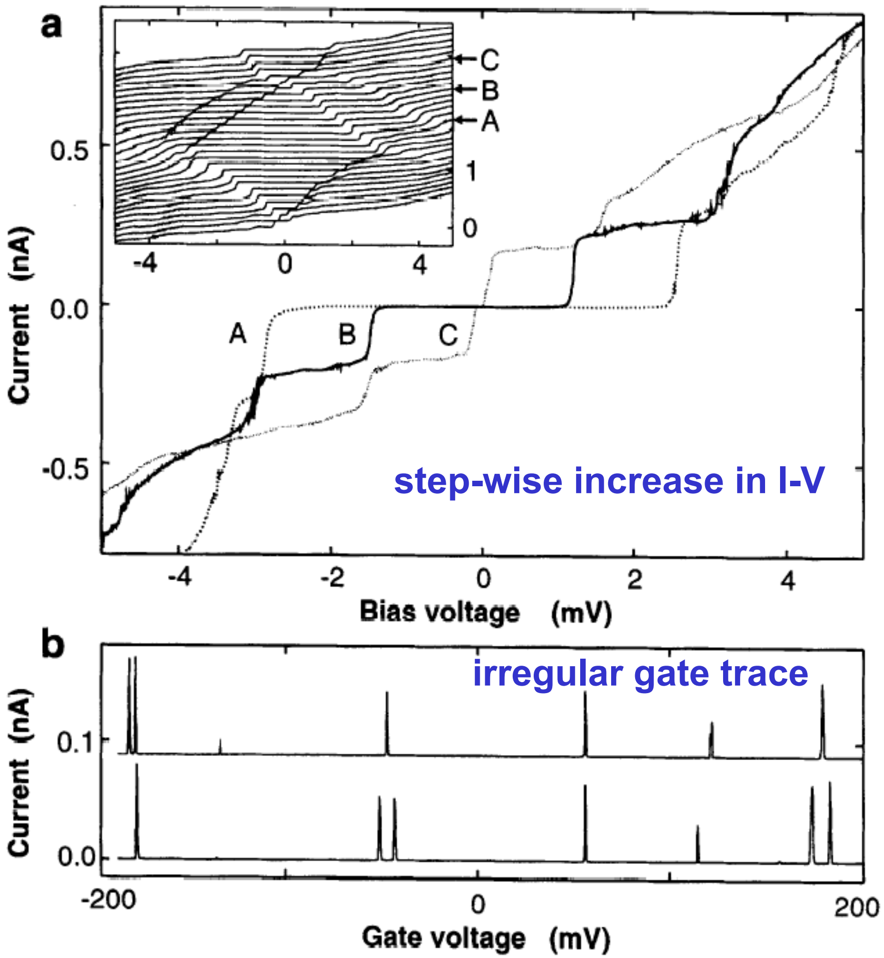


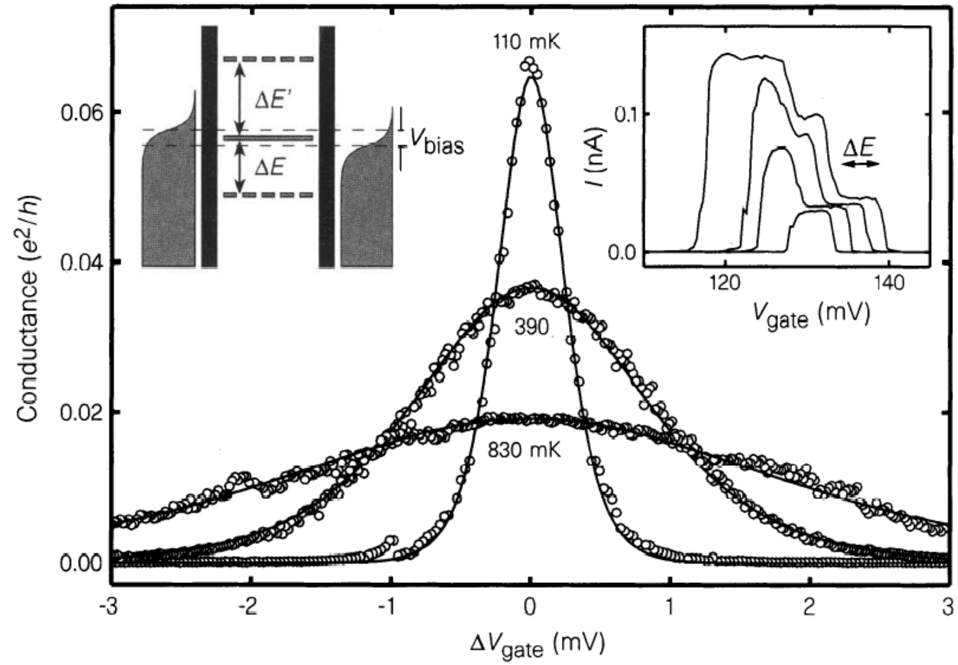


Foxman et al., Phys. Rev. B 50 (1994) 14193

Foxman et al., Phys. Rev. B 47 (1993) 10020

the first nanotube quantum dot



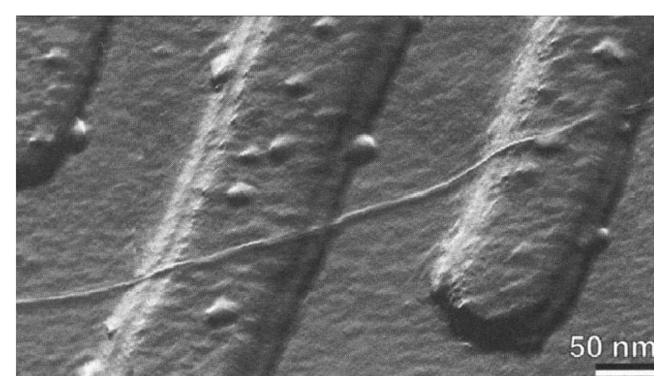


Coulomb peak height increases when temperature is lowered

Individual single-wall carbon nanotubes as quantum wires

Sander J. Tans*, Michel H. Devoret*†, Hongjie Dai‡, Andreas Thess‡, Richard E. Smalley‡, L. J. Geerligs* & Cees Dekker*

NATURE | VOL 386 | 3 APRIL 1997



summary different regimes zero-bias conductance

classical (FWHM $= 4.35\, k_B T$)

$$\frac{G}{G_\infty} \approx \frac{1}{2}\cosh^{-2}\left[\frac{\delta}{2.5 k_B T}\right] \qquad h\Gamma, \Delta E << k_B T << e^2/C$$

quantum (FWHM $= 3.5\, k_B T$)

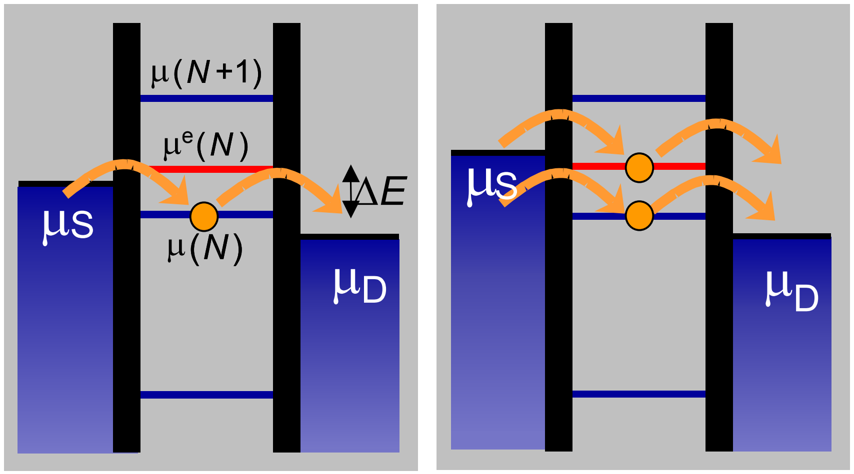
$$\frac{G}{G_\infty} \approx \frac{\Delta E}{4 k_B T}\cosh^{-2}\left[\frac{\delta}{2 k_B T}\right] \qquad h\Gamma < k_B T << \Delta E, e^2/C$$

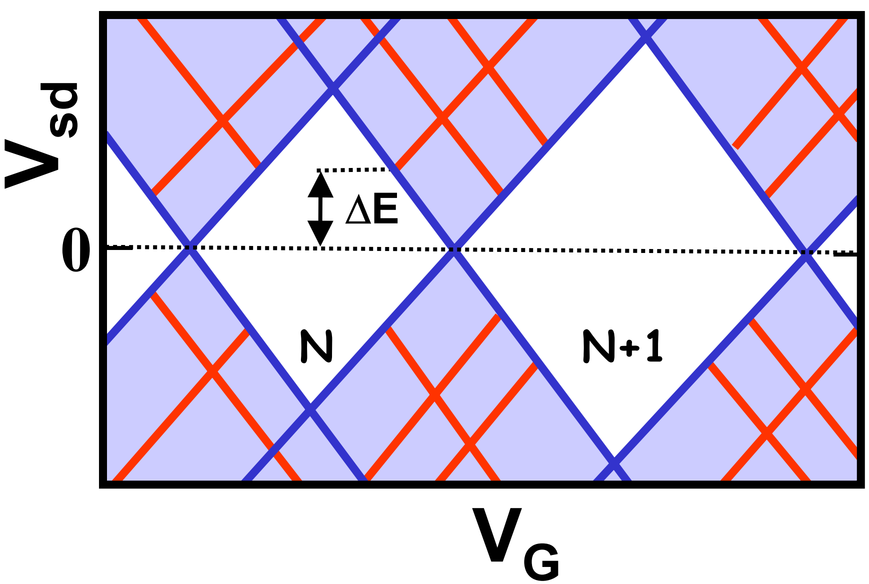
life – time limited (Lorentzian shaped Breit - Wigner formula)

$$\mathrm{G} = \frac{2e_2}{4}\,\frac{(h\Gamma)^2}{(h\Gamma)^2 + \delta^2} \qquad T = 0, e^2/C << h\Gamma,\ \Delta E$$

$$\delta = e\frac{C_G}{C}\left|V_{G,res} - V_G\right|$$

level spectroscopy: excited states

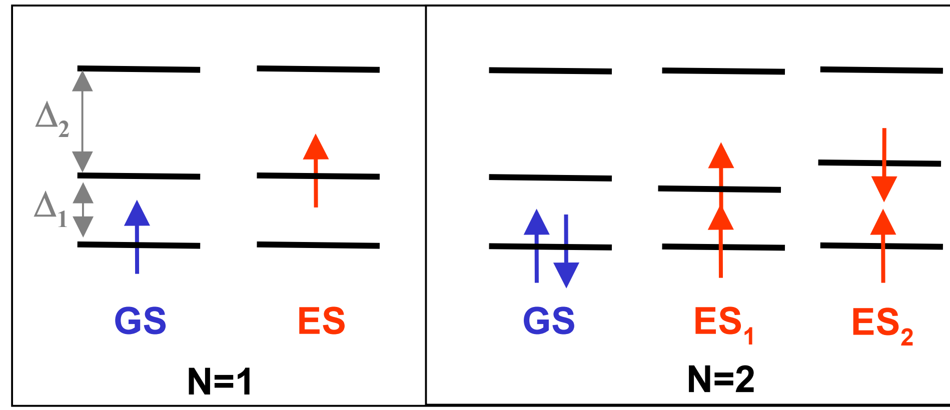


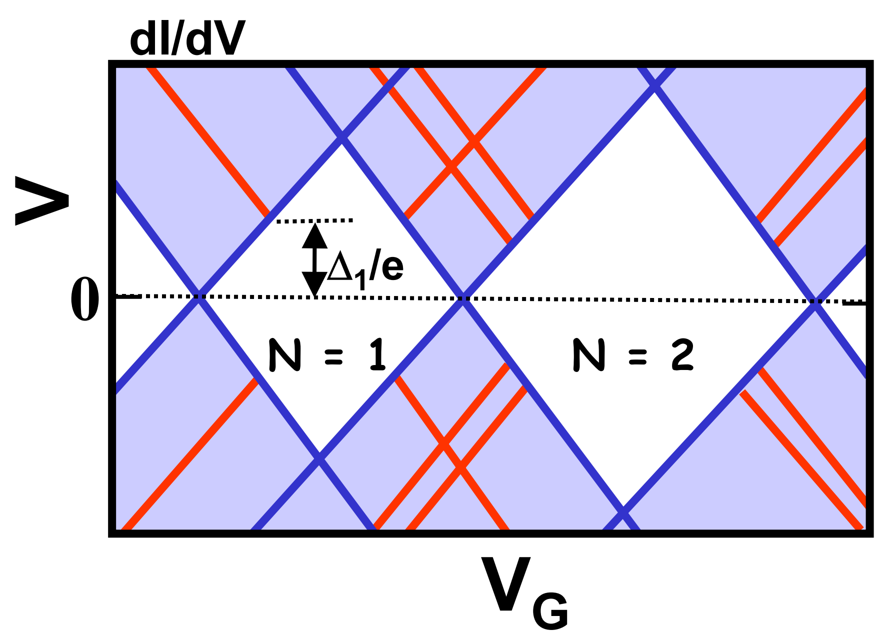


- when an excited level enters the transport window, an additional transport channel opens up leading to a **step-wise increase of the current**. In the differential resistance (which is often plotted in the stability diagram), these steps appear as lines running parallel to the diamond edges (red lines)
- the energy of the excited state can directly be read off from the diagram as indicated in the figure
- excitations can probe electronic spectrum, spin or vibrational states

manipulating single charges

Top: Schematic drawing of the ground state (GS) filling and the excited states (ES). Left: the island contains one electron and the first excited state involves a transition to the nearest unoccupied level. (In zero magnetic field there is an equal probability to find a down spin on the dot.) Right: two electrons with opposite spin occupy the lowest level. The first excited state involves the promotion of one of the spins to the nearest unoccupied level. A ferromagnetic coupling favors a spin flip. The antiparallel configuration (ES_2) has a higher energy.
Bottom: corresponding stability diagram.

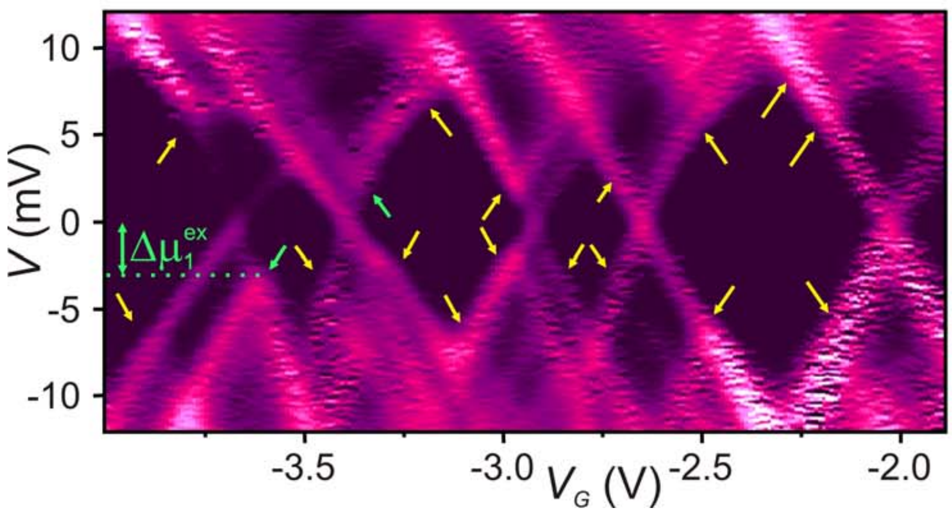




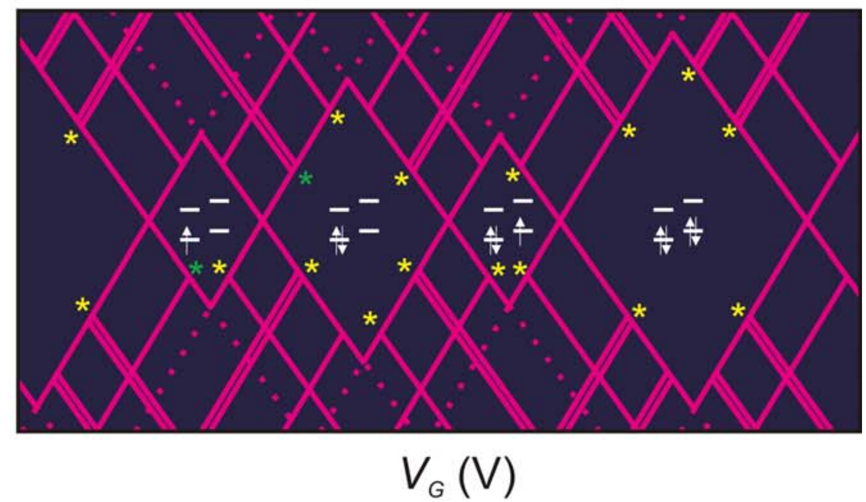
How large are the excitation energies for the N=2 case?

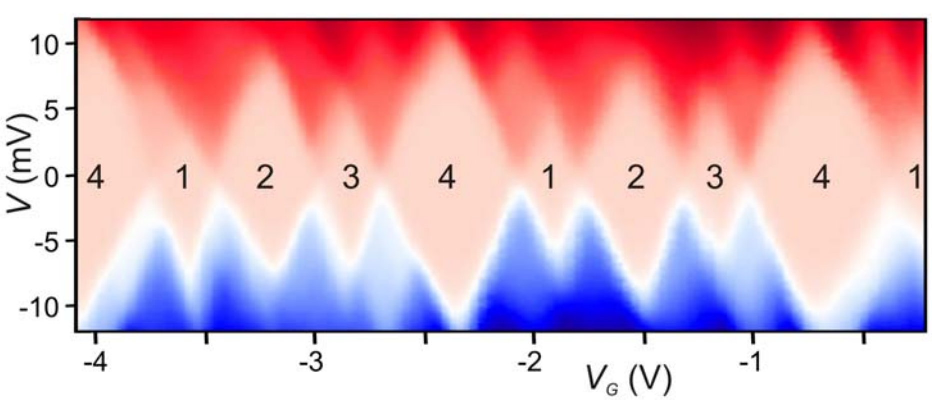
metallic SWNTs: full understanding of level structure including excited states

measurements L = 180 nm



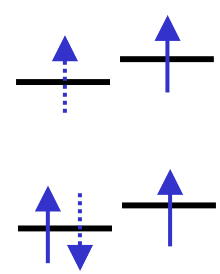
calculation





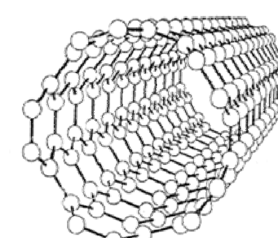
Nanotubes: two levels with two electrons

Δ = 9.0 meV (= $hv_F/2L$); E_c = 4.3 meV
δ = 3.2 meV; J = 0.4 meV



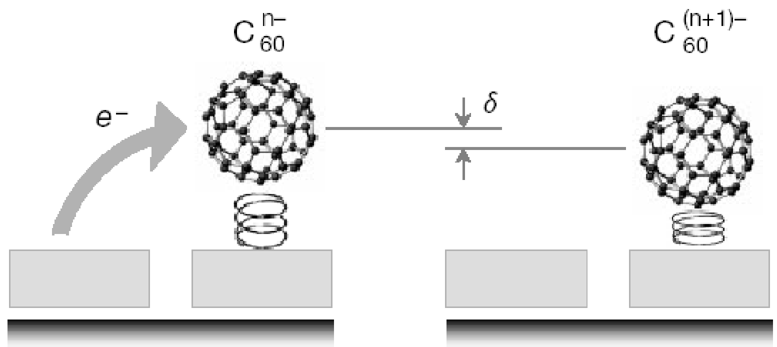
$\Delta\mu_1^{ex} = \delta$

$\Delta\mu_2^{ex} = \delta - J$



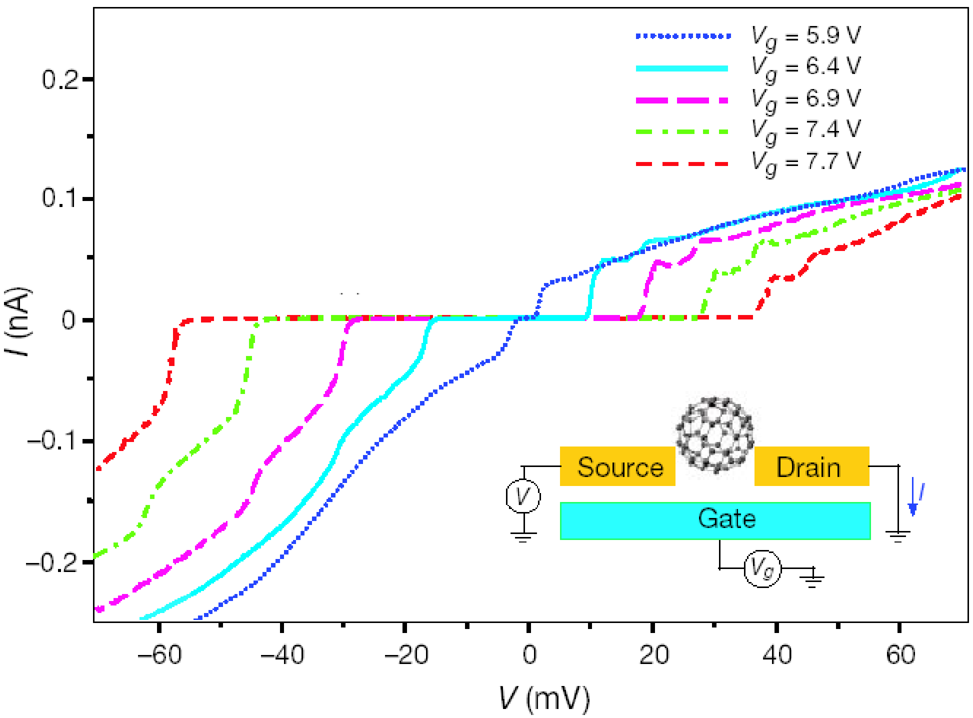
S. Sapmaz et al. PRB ***71*** *(2005) 153402*

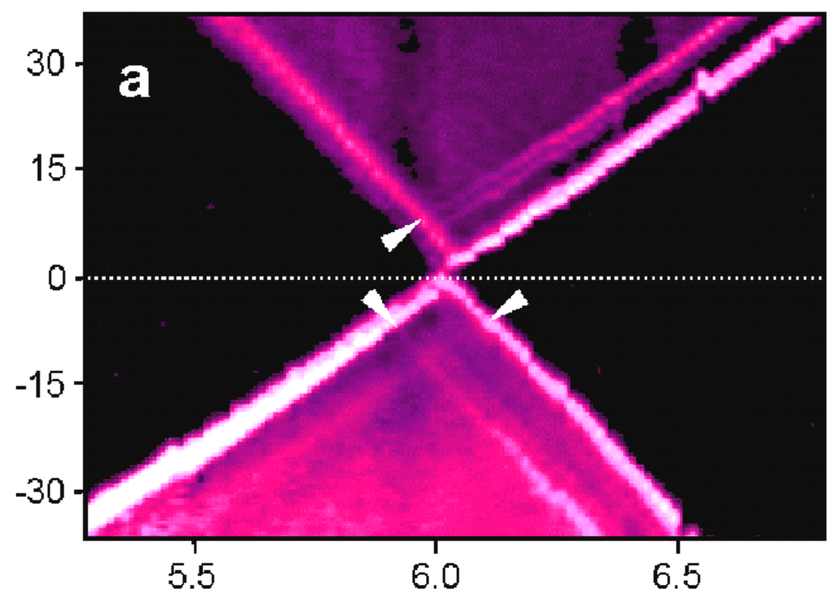
vibration assisted tunnelling in a C_{60} transistor



single electron tunneling events excite and probe the mechanical motion of the C60 bucky ball

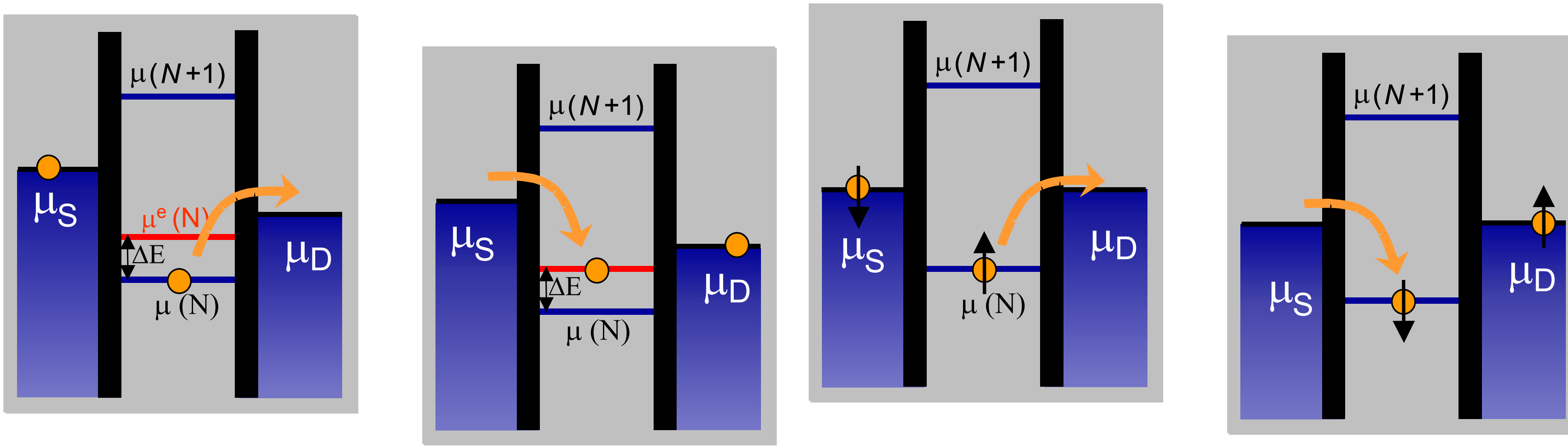
vibrational mode adds another transport channel: step in current-voltage characteristic





*H. Park et al. Nature **407**, 57 (2000)*

not so weak coupling to the leads: higher-order processes (Γ~U)



inelastic co-tunneling: excitation spectrum

Kondo-effect (elastic): spin filling