nanomechanics and mesoscopic phonons

Integrated Mechanical Circuits

Courtesy CT-Cnyugen (Berkeley)
Why NEMS? Device applications

- Smaller, cheaper, faster, lower power consumption
- “Phones of the future”: NEM-devices are in the right frequency range (1-5 GHz) to replace elements in cell phones
- Better frequency selectivity (higher Q)
- Bulk passive components replaced by smaller size MEMS/NEMS components
- New sensor applications

needed: high Q; high frequency

Nanotube RAM (including movie):
http://www.nantero.com/index.html
mass sensing on the level of single molecules

\[ \frac{\partial \omega_0}{\partial \omega_{0,\text{min}}} \approx \frac{1}{2} \frac{\omega_0}{Q} \]

Doubly-clamped beam

\[ \partial M_{\text{min}} \approx \frac{m}{Q} \]

<table>
<thead>
<tr>
<th>( f_0 )</th>
<th>( L \times w \times t ) (( \mu )m)</th>
<th>( M_{\text{eff}} )</th>
<th>( Q )</th>
<th>( \partial m_{\text{min}} ) (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.7 MHz</td>
<td>10 x 0.2 x 0.1</td>
<td>230 fg</td>
<td>10,000</td>
<td>1.4 x 10^8</td>
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<td></td>
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<td>1.4 x 10^7</td>
</tr>
<tr>
<td>380 MHz</td>
<td>1 x 0.05 x 0.05</td>
<td>3 fg</td>
<td>10,000</td>
<td>1.8 x 10^6</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.8 x 10^5</td>
</tr>
<tr>
<td>7.7 GHz</td>
<td>0.1 x 0.01 x 0.01</td>
<td>23 ag</td>
<td>10,000</td>
<td>1400</td>
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<td>140</td>
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</tbody>
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low \( M \) (high \( f \)); high \( Q \)
carbon-nanotube oscillators towards zeptogram detection

FIG. 2. SEM image of a carbon nanotube oscillator. The inset shows the deposit induced by an electron beam deposition process.

M. Nishio et al. APL 86 (2005) 133111
nanorelays: **low-power mechanical switches**

*S.N. Cha et al. APL 86 (2005) 083105*

*S.W. Lee et al. Nanoletters (2004)*
nanotube–nanomechanics: rotator

rotating mirror

low-friction internal motion of the shells of a multi-wall nanotube makes motion possible

bio-nanomechanics II

**bio-motors:** ATP fuelled biomotor with a fluorescent filament of a few micron length attached (time between pictures 133 ms)

nanomechanics of breaking an atomic gold wire

estimate force:

\[ x_0 = 0.2 \text{ nm} \]
\[ E_b = 3 \text{ eV} \]
\[ k \approx \frac{\partial^2 V(x)}{\partial^2 x} \approx 10 \text{ N/m} \]
\[ F = kx_0 = 2 \text{ nN} \]
Measurement of the conductance of a hydrogen molecule

R. H. M. Smit*, Y. Noat†, C. Untiedt*, N. D. Lang‡, M. C. van Hemert§ & J. M. van Ruitenbeek*

* Kamerlingh Onnes Laboratorium, Universiteit Leiden, PO Box 9504, 2300 RA Leiden, The Netherlands
† IBM Research Division, Thomas J. Watson Research Center, Yorktown Heights, New York 10598, USA
‡ Leids Instituut voor Chemisch Onderzoek, Gorlaeus Laboratorium, Universiteit Leiden, PO Box 9502, 2300 RA Leiden, The Netherlands
vibrational modes in Pt-H$_2$-Pt junctions

Djukic et al. cond/mat 0409640
stretching the contact: longitudinal or transversal modes?

\[ \omega = \sqrt{\frac{k}{m}} \]
Casimir force: a force of nothing

inside the cavity: \[ E = \sum_{k} \hbar \omega = \frac{\pi \hbar c}{L} \sum_{n=1}^{\infty} n \]

outside the cavity: \[ E = \frac{\pi \hbar c}{L} \int_{0}^{\infty} n \, dn \]

\[ \Delta E = \frac{\pi \hbar c}{L} \left[ \sum_{n=1}^{\infty} n - \int_{0}^{\infty} n \, dn \right] \]

Euler - Maclauran:

\[ \sum_{n=0}^{\infty} f(n) - \int_{0}^{\infty} f(n) \, dn = \frac{1}{2} f(0) - \frac{1}{12} f'(0) + \frac{1}{720} f''(0) - \ldots \]

\[ f(n) = n \implies f(0) = 0, \ f'(0) = -\frac{1}{12}, \ f''(0) = 0 \]

\[ \Delta E = -\frac{\pi \hbar c}{12L} \implies \text{force} = \frac{\pi \hbar c}{12L^2} \]

can be generalized to other geometries (3D, plate-sphere), temperature effects and non-ideal conductors
vacuum fluctuations: Casimir effect

In absence of electrostatic forces, attractive force between plate and sphere: $F \propto z^{-3}$

MEMS technology
(MicroElectroMechanical Systems)

By nanofabrication techniques, we can enter domain where particle wavelength is comparable to channel dimensions.

\[
\lambda_{\text{max}} = \frac{c}{\nu_{\text{max}}} \approx \frac{hc}{3k_B T} = \frac{112 \text{ nm}}{T} 
\]

\[
\frac{h\nu_{\text{max}}}{k_B T} \approx 3
\]

Planck Distribution

\[
\lambda_{\text{max}} = 11 \text{ nm} \\
\nu_{\text{max}} = 625 \text{ GHz}
\]

\[
\lambda_{\text{max}} = 11 \mu\text{m} \\
\nu_{\text{max}} = 625 \text{ MHz}
\]

at 10K,

at 10 mK,

Courtesy Keith Schwab
the thermal conductance “quantum”

derivation analogous to that of the electrical conductance quantum but look at the energy flux (instead of electron flux) and use Bose-Einstein statistics (consider two reservoirs with a T difference $\Delta T$ coupled by a 1D channel)

$$J = \hbar \omega \eta(n)$$

$$J = \int \frac{dk}{2\pi} \hbar \omega(k) \eta(\omega) \nu(k)$$

$$\eta(\omega) = \frac{1}{e^{\hbar \omega / k_B T} - 1}; \quad \nu(k) = \frac{d\omega}{dk}$$

$$J_{TOT} = \int_{0}^{\infty} \hbar \omega \left[ \eta^+ - \eta^- \right] d\omega$$

$$\eta^+ - \eta^- = \frac{\partial \eta}{\partial T} \Delta T$$

$$G_{th} = \frac{J_{TOT}}{\Delta T} = \frac{\pi^2 k_B^2 T}{3\hbar}$$

the existence of a thermal conductance quantum also indicates that it is difficult to cool nano-objects to their ground state!!

Wiedemann-Franz law:

$G_{th} = LGT$, $L =$ Lorentz constant

With $G_0 = e^2/h$ one finds $G_{th} = \pi^2 k_B^2 T / 3h$
quantum phonon transport

\[ g_0 = \pi^2 k_B^2 T/3h \]

eigenfrequencies cantilevers and beams

classical elasticity, Euler-Bernoulli theory:

\[ \rho A \frac{\partial^2 u_y}{\partial t^2} + EI \frac{\partial^4 u_y}{\partial z^4} = 0, \quad 0 < z < L \]

solve and use appropriate boundary conditions
top-down fabrication: surface nanomachining

1. starting material: heterostructure with a single structural layer

2. mask definition: by optical and e-beam lithography and thin film deposition

3. pattern transfer: anisotropic etch defines structure vertically

4. selective etch: removes the sacrificial layer; final structures are suspended

- high resolution e-beam lithography (x,y)
- mono-crystalline epitaxial layers (z)
- GaAs-based systems
- Si-based systems (SOI)
- Silicon Carbide
eigenfrequencies cantilever

\[ \xi(0) = \xi'(0) = 0 \]
\[ \xi''(L) = \xi'''(L) = 0 \]

\[ I = \frac{WH^3}{12} \implies f_n = \frac{\alpha_n^2 H}{2\pi L^2} \sqrt{\frac{E}{12 \rho}} \]

boundary conditions:

\[ 1 + \cosh(kL) \cos(kL) = 0 \]
\[ \alpha_n = k_n L ; \quad \alpha_1 = 1.8751 , \quad \alpha_2 = 4.6941 , \quad \ldots , \quad \alpha_n = \frac{\pi}{2} (2n - 1) \]

sound velocity: \( v = (E/\rho)^{1/2} \)

for the first modes: not harmonic
eigenmodes of cantilever nanotube


P. Poncheral et al., Science 283 (1999) 1513
eigenfrequencies double clamped beams

\[ \xi(0) = \xi'(0) = 0 \]
\[ \xi(L) = \xi'(L) = 0 \]

boundary conditions:

\[ 1 - \cosh(k_n L) \cos(k_n L) = 0 \]

\[ \beta_n = k_n L; \quad \beta_1 = 4.7300, \quad \beta_2 = 7.8532, \ldots, \quad \beta_n = \frac{\pi}{2} (2n + 1) \]

silicon:

L=1 \, \mu m, H=W=0.1 \, \mu m, f_0 = 1 GHz

nanotube:

L= 100 nm, d = 1.4 nm, f_0 = 5 GHz

\[ I = \frac{WH^3}{12} \quad \Rightarrow \quad f_n = \frac{\beta_n^2 H}{2\pi L^2} \sqrt{\frac{E}{12 \rho}} \]
double clamped strings/beams

\[
EI \frac{\partial^4 u}{\partial z^4} - T \frac{\partial^2 u}{\partial z^2} - K = 0
\]

APL 78 (2001) 162

frequency \propto L^{-1}

frequency \propto L^{-2}
magnetomotive method: conducting beam (50 Ω)

\[ V(t) = \xi B L \frac{dy}{dt} \]

\[ \vec{F} = LBI(t) \]

\[ F(t) = m\dot{y} + \alpha y + ky = BLI(t) \quad \text{assume } \alpha^2 \ll mk \]

amplitude at resonance \( y_0 = QBLI_0/k \)

\[ V_0 = \xi BL y_0 \omega_0 = \xi QB^2L^2I_0\omega_0/k = \frac{\xi QB^2L^2}{m\omega_0} I_0 \equiv R_m I_0 \]

\( \xi = 0.83 \), depends on bending profile

Cleland and Roukes, Sensors and Actuators 72, 256 (1999)
For $Q > 10$: response of the damped, driven the same as that of a harmonic oscillator. Lorentzian peak shape; the width defines the Q-factor.
increasing drive: nonlinear response
(Duffing oscillator)

\[ F_{\text{eff}} = kx + k'x^3 \]
\[ x_c = \frac{2d}{\sqrt{Q/2(1-\nu^2)}} \]

“Jimmy Hendrix regime”

Husain et al, APL 83 (2003) 1240

interesting for applications (parametric amplification) and for new quantum effects (Thorwart et al. (2005))
The MiniGRAIL detector is a cryogenic 68 cm diameter spherical gravitational wave antenna made of CuAl(6%) alloy with a mass of 1400 Kg, a resonance frequency of 2.9 kHz and a bandwidth around 230 Hz, possibly higher. The quantum-limited strain sensitivity $dL/L$ would be $\sim 4 \times 10^{-21}$. The antenna will operate at a temperature of 20 mK. An other similar detector is being built in São Paulo, which will strongly increase the chances of detection by looking at coincidences. The sources we are aiming at are for instance, non-axisymmetric instabilities in rotating single and binary neutron stars, small black-hole or neutron-star mergers etc.

MiniGRAIL: Frossati (Leiden University)
quantum effects in a harmonic oscillator

\[ H = \frac{1}{2} k x^2 + \frac{1}{2} \frac{p^2}{m} \]

\[ \Rightarrow E_n = \hbar \omega \left( n + \frac{1}{2} \right) \]

zero-point motion:

\[ x_0 = \sqrt{\frac{\hbar}{m \omega_0}} \]

energy stored in the oscillator:

\[ \langle E \rangle = \frac{\hbar \omega}{2} + \frac{\hbar \omega}{e^{\hbar \omega / k_B T} - 1} \]

quantum regime if occupation number \((N_{th})\) is of order one: \(1\ GHz \Rightarrow T_{sample} < 50\ mK\)
zero point motion

expected rms displacement noise of a 4.4 GHz nanotube resonator as a function of temperature

needed: high Q; high frequency, low mass
MEMS devices as electrometers

- Measured sensitivity (300 K): $0.1\, eHz^{-1/2}$
- Ultimate sensitivity (300 K): $2 \times 10^{-5}\, eHz^{-1/2}$

towards zero-point motion detection using a SET and a mixing technique

\[ x_0 = \frac{\hbar}{m \omega_0} \]

**Figure 1** The device used in the experiment.  
**a.** Scanning electron micrograph of the device, showing the doubly clamped GaAs beam, and the aluminum electrodes (coloured) forming the single electron transistor and beam electrode. Scale bar, 1 μm. The AlAlO₃/Al tunnel junctions have approximately 50 × 50 nm² overlap.  
**b.** A schematic of the mechanical and electrical operation of the device.

\[ x_0 \text{ a factor 100 above the quantum limit; } N_{th} = 10 \]

*Knobel and Cleland, Nature 424 (2003) 291*
almost quantum detection limit with RF-SET

$x_0$ a factor 4.3 above the quantum limit; $N_{th} = 58$

Fig. 3. Charge noise power around the mechanical resonance with $V_{NR} = 15$ V. Right peak is taken at 100 mK and is fit with a Lorentzian, shown as a red line. This noise power is used to scale the left peak taken with the refrigerator at 35 mK and corresponds to a resonator noise temperature of $T_N^{NR} = 73$ mK. This then scales the white-noise floor, which corresponds to a system-noise temperature of $T_N^{SSET} = 16$ mK = 18 $T_{cl}$. Using the equipartition relation, the displacement resolution is $3.8$ fm/\(\sqrt{\text{Hz}}\). The inset shows the driven response, approximately 800 pm on resonance, with the data as circles and a Lorentzian fit as the solid lines. All SSET measurements are taken with the SSET biased near the double Josephson quasiparticle resonance peak.

LaHaye et al., Science 304 (2004) 74
Coulomb blockade in SET that can move

- usual CB theory does not apply since the gate capacitance is distance dependent
- mechanical degrees of freedom via classical theory of elasticity
- nanotube modeled as an elastic rod
- applicable to other suspended structures
- nanotube: hope for high Q-factors (smooth on a nanometer level)

S. Sapmaz et al., PRB 67 (2003) 235414
bottom-up fabrication: suspended nanotubes

electrodes

ISWNTs

catalyst

AFM markers

catalyst + nanotubes

electrodes

ISWNT

lateral Gate
bottom-up fabrication: suspended nanotubes

before etching

after etching in acid (BHF)

Nygård and Cobden, APL 79 (2001) 4216
electrostatic term in Coulomb Blockade is displacement dependent

\[ W_{\text{el-st}}(z[x]) = \frac{(ne)^2}{2C_\Sigma} - neV_G \]

analytical case: \( C_L = C_R = 0 \)

\[ C_G(z[x]) = \int \frac{dx}{2 \ln \left[ \frac{2(R - z)}{r} \right]} \]

\[ W_{\text{el-st}}(z[x]) = \frac{(ne)^2}{2C_0} - neV_G - \frac{(ne)^2}{L^2R} \int z[x] \, dx \]

Electrostatic force \((z << R)\): mechanical correction proportional to \((ne)^2\)

\[ W \neq \frac{(ne + Q)^2}{2C_\Sigma} \]
the nanotube moves in discrete steps every time an additional electron tunnels onto it

\[
\begin{align*}
\text{weak bending:} & \quad z_{\text{max}} < r, \quad z_{\text{max}} \propto V_G^2 \\
\text{strong bending:} & \quad z_{\text{max}} > r, \quad z_{\text{max}} \propto V_G^{2/3}
\end{align*}
\]

As soon as \( z_{\text{max}} > d \), the strain due to deformation has to be taken into account.

S. Sapmaz et al., PRB 67 (2003) 235414
**electron-vibron coupling in a movable dot**

![Diagram](image)

\[ \hat{H} = \frac{p^2}{2m} + \frac{1}{2} m \omega_0^2 x^2 + \lambda x \]

\( \lambda: \) e-ph coupling

\[ g = \frac{1}{2} \lambda^2 = \frac{1}{2} \left( \frac{\ell}{\ell_0} \right)^2 \quad \ell_0 = \sqrt{\frac{\hbar}{m \omega}} \]

\[ \Gamma_{L,R} \Rightarrow \Gamma_{L,R} \left| \langle \Psi_{\text{after}} | \Psi_{\text{before}} \rangle \right|^2 = \Gamma_{L,R} P_{nm} \]

Frank - Condon factors determine step heights in IV

\[ P_{n0} = \left| \langle \Psi(x - \ell) | \Psi(x) \rangle \right|^2 = \frac{e^{-g} g^n}{n!} \]

in practice: steps visible if \( g > 0.1 \)

*Braig and Flensberg, PRB 68 (2003) 205323*
Frank-Condon factors reproduce step heights in the experiment reasonably well.

The e-ph coupling constant is about 1 and approximately length and gate independent: displacement ~1 pm!

A nanotube of 1 micron length acts as a single quantum harmonic oscillator ($\hbar\omega >> k_B T$) !!!

A better fit may be obtained if the influence of a gate voltage and asymmetric coupling is considered (Braig and Flensberg, 2003) or if non-equilibrium phonons are present (Mitra, Aleiner and Milles, 2004); Negative Differential Resistance not understood.
some vibrational modes nanotubes

**bending mode**
- Frequency: $f \approx 2.5 \text{ GHz}-25 \text{ MHz}$
- Length dependence: $(100 \text{ nm}-1 \text{ µm})$
- Energy: $E \approx 10-0.1 \text{ µeV}$

**squashing mode**
- Frequency: $f \approx 0.54-1.0 \text{ THz}$
- Energy: $E \approx 2.2-4.2 \text{ meV}$

**stretching (longitudinal) mode**
- Frequency: $f \approx 1.3-0.13 \text{ THz}$
- Length dependence: $(100 \text{ nm}-1 \text{ µm})$
- Energy: $E \approx 600-60 \text{ µeV}$

**radial breathing mode**
- Frequency: $f \approx 4.5-6.2 \text{ THz}$
- Energy: $E \approx 18-25 \text{ meV}$
data fit longitudinal (stretching) modes the best

\[ E = 18.7 \text{ meV} \]
\[ E = 1.67 \text{ meV} \mu m^{-1} \]
\[ E = 110 \mu V \mu m^{-1} \]
\[ E = 0.1 \mu V \mu m^{-2} \]

Sapmaz et al. PRL 96 (2006) 26801