Offshore Hydromechanics

Module 1 : Hydrostatics
Constant Flows
Surface Waves

OE4620 Offshore Hydromechanics

Ir. W.E. de Vries

November 2007

Offshore Engineering

TU Delft

Delft University of Technology

Today

First hour:

- Schedule for remainder of hydromechanics module 1
- Assignment stability
- Basic flow properties
- Potential flow concepts

Second hour:

- Potential flow elements
- Superposition of flow elements
- Cylinder in uniform flow

Schedule OE4620 Module 1

- 13 November – Constant potential flow phenomena
- 20 November – Constant real flow phenomena 1
- 27 November – Constant real flow phenomena 2
-   4 December – Ocean surface waves 1 (!)
- 11 December – Ocean surface waves 2
- 18 December – Extra/Examples

Assignment: Stability

- Make the assignment for next lecture
- Next lecture the assignment will be worked out briefly
- Questions regarding the assignment or the subjects of module 1? Contact:

    - Room 2.78
    - Tel. 015 278 7568
    - E-mail w.e.devries@tudelft.nl

CONSTANT POTENTIAL FLOW PHENOMENA

Flows in this chapter:

- Obey simplified laws of fluid mechanics
- Are subject to limitations
- Give qualitative impressions of flow phenomena

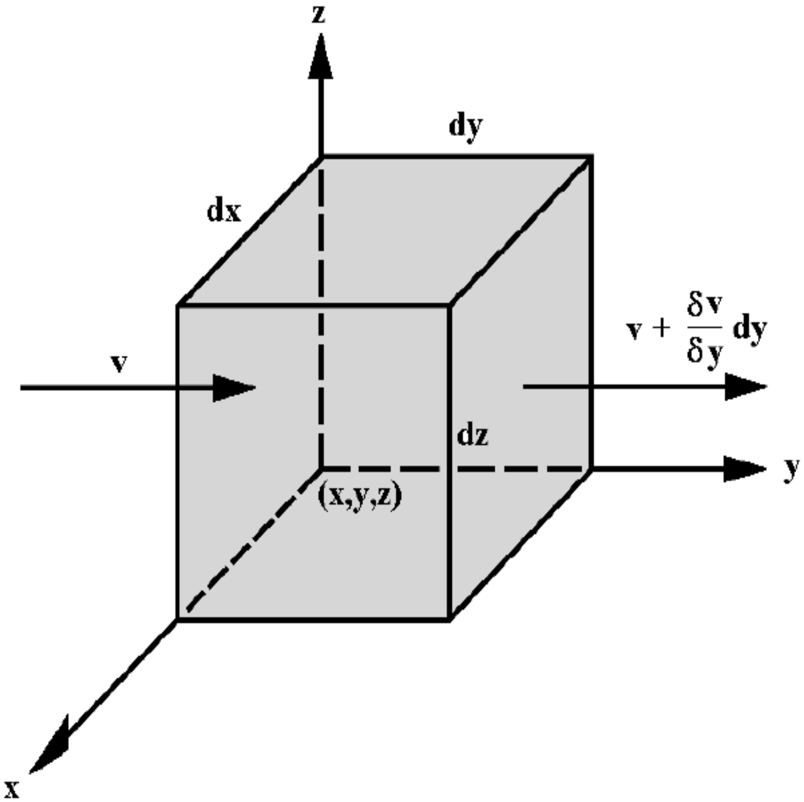
Basic Flow Properties

- Ideal fluid :
  - Non – viscous
  - Incompressible
  - Continuous
  - Homogeneous

Continuity Condition

•Increase of mass per unit time:

$$\frac{\partial m}{\partial t} = -\frac{\partial}{\partial t}\left(\rho \cdot dxdydz\right)$$



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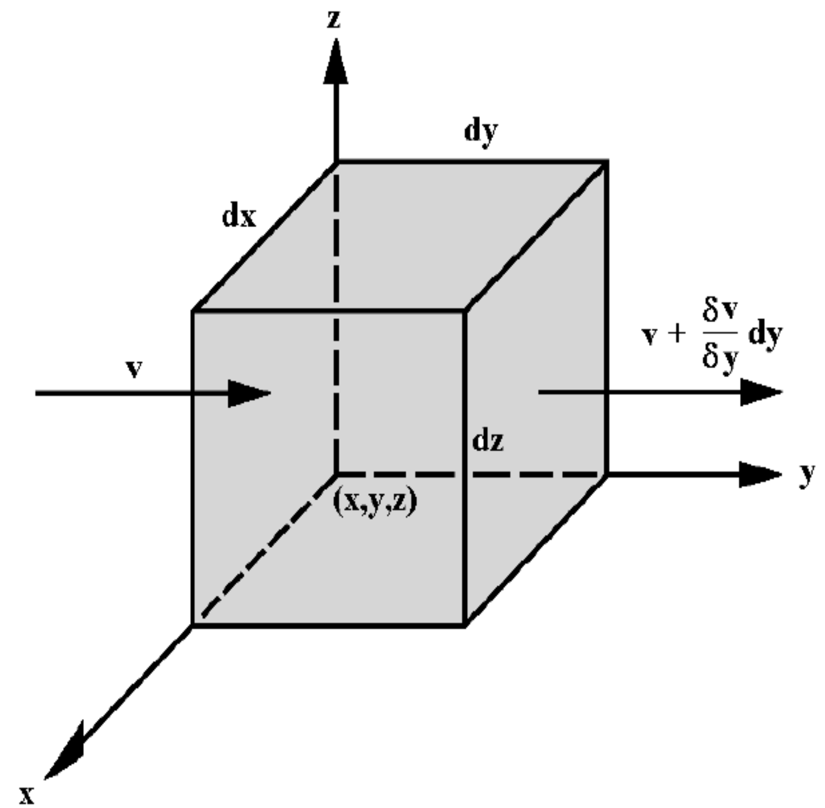
Continuity Condition

•Mass through a plane *dxdz* during a unit of time:

$$m_{in} = \rho \cdot v \cdot dxdzdt$$

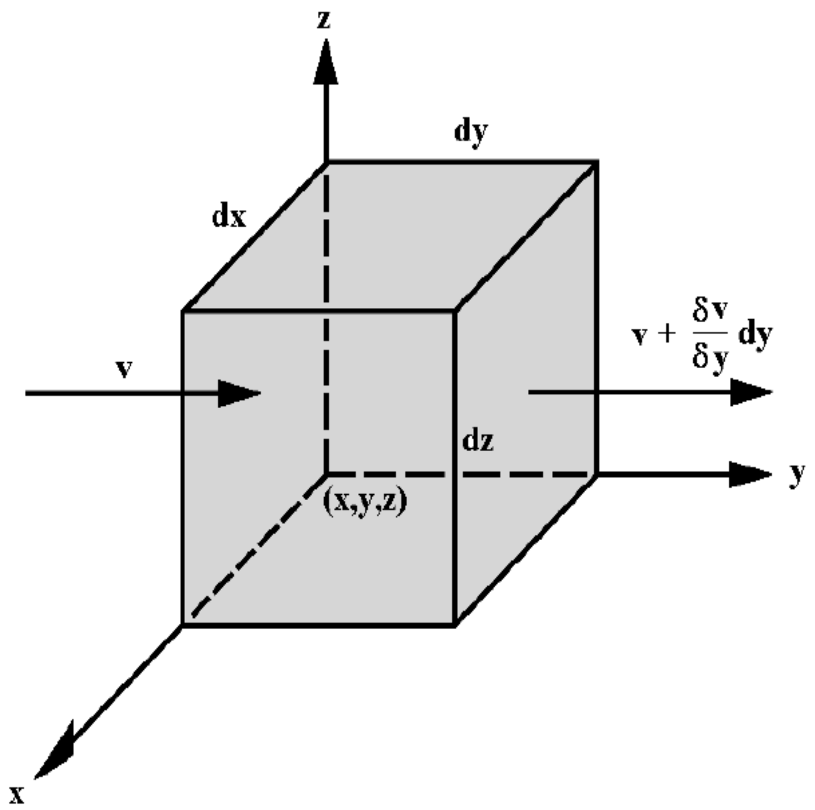


Continuity Condition

•Mass through a plane *dxdz* during a unit of time:

$$m_{in} = \rho \cdot v \cdot dxdzdt$$

$$m_{out} = \left[\rho v + \frac{\partial(\rho v)}{\partial y} dy \right] \cdot dxdzdt$$



Continuity Condition

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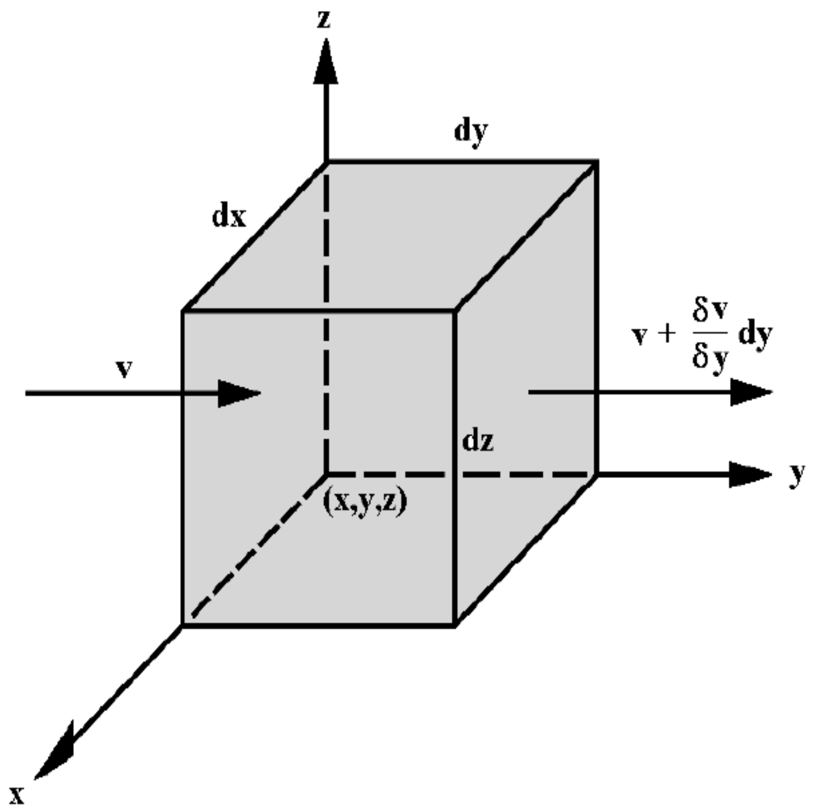
$$m_{out} = \left[\rho v + \frac{\partial(\rho v)}{\partial y} dy \right] \cdot dxdzdt$$

$$\left(\frac{\partial m}{\partial t} \right) = \left[\rho v + \frac{\partial(\rho v)}{\partial y} dy \right] \cdot dxdz - \rho u dxdz$$



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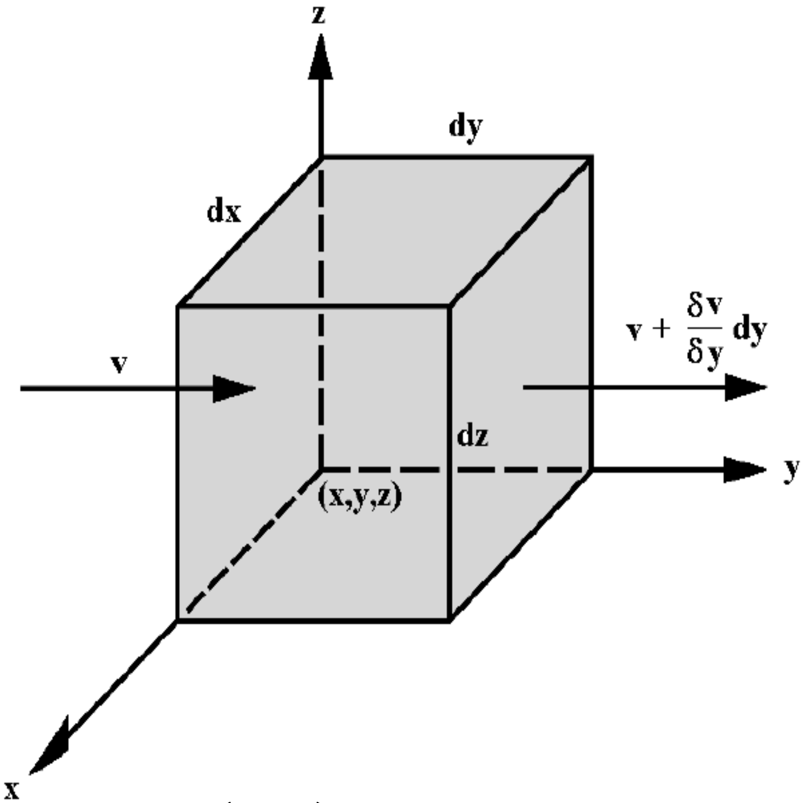
Continuity Condition



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$$\left(\frac{\partial m}{\partial t} \right) = \left[\rho v + \frac{\partial(\rho v)}{\partial y} dy \right] \cdot dxdz - \rho u dxdz = \frac{\partial(\rho u)}{\partial y} dxdydz$$

Continuity Condition

Combining for *x*, *y* and *z* direction yields the **Continuity Equation**

$$\frac{\partial m}{\partial t}$$

Continuity Condition

Combining for *x*, *y* and *z* direction yields the **Continuity Equation**

$$\frac{\partial m}{\partial t} = -\frac{\partial \rho}{\partial t} dxdydz$$

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Continuity Condition

Combining for *x*, *y* and *z* direction yields the **Continuity Equation**

$$\frac{\partial m}{\partial t} = -\frac{\partial \rho}{\partial t} dxdydz = \left\{ \frac{\partial \left(\rho u \right)}{\partial x} + \frac{\partial \left(\rho v \right)}{\partial y} + \frac{\partial \left(\rho w \right)}{\partial z} \right\} dxdydz$$

Continuity Condition

Combining for *x*, *y* and *z* direction yields the **Continuity Equation**

$$\frac{\partial \rho}{\partial t}+\frac{\partial\left(\rho u\right)}{\partial x}+\frac{\partial\left(\rho v\right)}{\partial y}+\frac{\partial\left(\rho w\right)}{\partial z}=0$$

Continuity Condition

For an ideal, *incompressible* fluid:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Continuity Condition

For an ideal, *incompressible* fluid:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Or, with $\nabla=\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}$: $\nabla\vec{V}=0$

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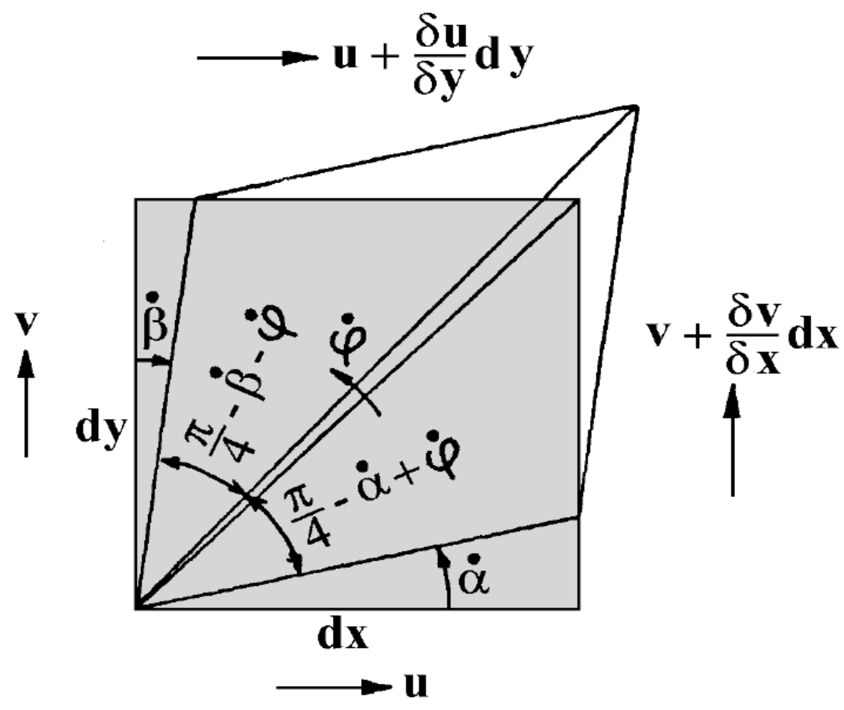
Deformation of flow particles

Deformation

$$\frac{\partial v}{\partial x} = \tan \dot{\alpha} \approx \dot{\alpha} \qquad \frac{\partial u}{\partial y} = \tan \dot{\beta} \approx \dot{\beta}$$



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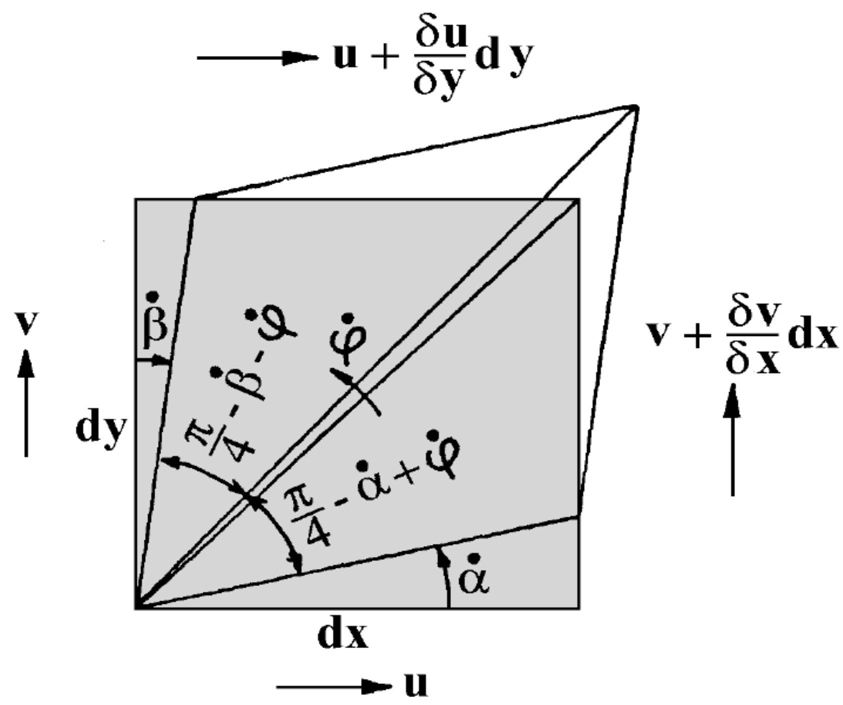
Deformation of flow particles

Deformation

$$\frac{\partial v}{\partial x} = \tan \dot{\alpha} \approx \dot{\alpha} \qquad \frac{\partial u}{\partial y} = \tan \dot{\beta} \approx \dot{\beta}$$

Deformation velocity (dilatation):

$$\frac{\dot{\alpha} + \dot{\beta}}{2} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$



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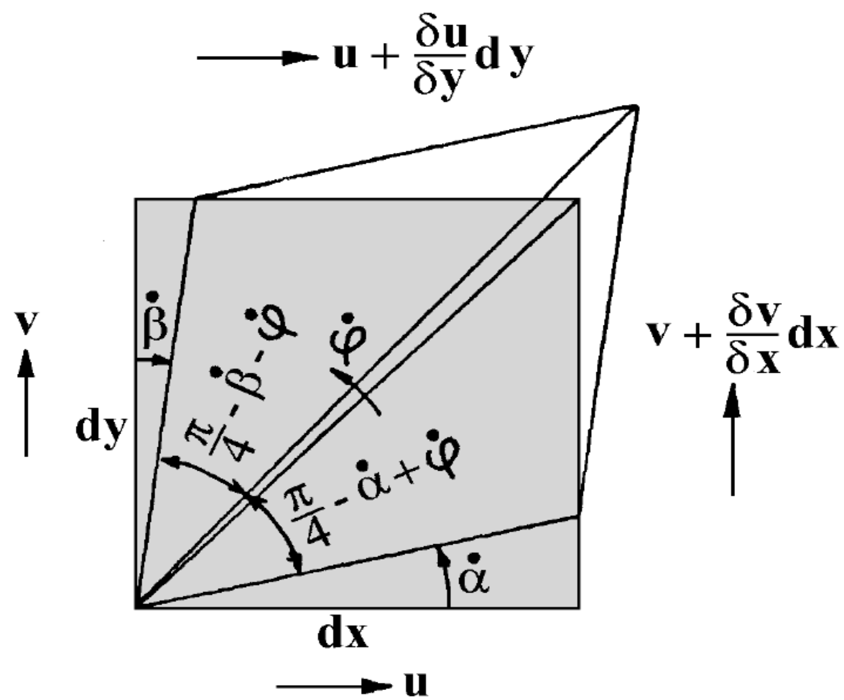
Rotation (in 2 D) :

$$\dot{\varphi} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$



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Velocity Potential

A scalar function, Φ

The velocity in any point of the fluid and in any direction is the derivative to that direction of the potential

$$u = \frac{\partial \Phi}{\partial x} \qquad v = \frac{\partial \Phi}{\partial y} \qquad w = \frac{\partial \Phi}{\partial z}$$

Properties of Velocity Potentials

- Combining the continuity condition with the velocity potentials results in the *Laplace* equation :

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad \text{or} \quad \nabla^2 \Phi = 0$$

- Potential flow is rotation-free by definition :

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0 \qquad \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

Properties of Velocity Potentials

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Euler and Bernoulli

- Euler: Apply Newton's 2nd law to non viscous and incompressible fluid

$$dm\frac{Du}{Dt}$$

Euler and Bernoulli

- Euler: Apply Newton's 2nd law to non viscous and incompressible fluid

$$dm\frac{Du}{Dt} = \rho \cdot dxdydz \cdot \frac{Du}{Dt} =$$

Euler and Bernoulli

- Euler: Apply Newton's 2nd law to non viscous and incompressible fluid

$$dm\frac{Du}{Dt} = \rho \cdot dxdydz \cdot \frac{Du}{Dt} = -\frac{\partial p}{\partial x}dx \cdot dydz$$

Mass* acceleration = pressure*area

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Euler and Bernoulli

- Euler Equations:

$$\frac{\partial u}{\partial t} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

With

$$u\frac{\partial u}{\partial x} = \frac{\partial \Phi}{\partial x}\cdot\frac{\partial^2 \Phi}{\partial x^2} = \tfrac{1}{2}\cdot\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial x}\right)^2$$

$$v\frac{\partial u}{\partial y} = \frac{\partial \Phi}{\partial y}\cdot\frac{\partial^2 \Phi}{\partial x \partial y} = \tfrac{1}{2}\cdot\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial y}\right)^2$$

$$w\frac{\partial u}{\partial z} = \frac{\partial \Phi}{\partial z}\cdot\frac{\partial^2 \Phi}{\partial x \partial z} = \tfrac{1}{2}\cdot\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial z}\right)^2$$

Euler and Bernoulli

- Differentiate w.r.t. *x*, *y* or *z* gives 0:

$$\frac{\partial}{\partial x}\left\{\frac{\partial \Phi}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial \Phi}{\partial x}\right)^2+\left(\frac{\partial \Phi}{\partial y}\right)^2+\left(\frac{\partial \Phi}{\partial z}\right)^2\right]+\frac{p}{\rho}\right\}=0$$

(similar for *y*, *z*)

Euler and Bernoulli

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$$\frac{\partial}{\partial x}\left\{\frac{\partial \Phi}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial \Phi}{\partial x}\right)^2+\left(\frac{\partial \Phi}{\partial y}\right)^2+\left(\frac{\partial \Phi}{\partial z}\right)^2\right]+\frac{p}{\rho}\right\}=0$$

(similar for *y*, *z*)

- This expression is a function of time only, providing the **Bernoulli Equation:**

$$\frac{\partial \Phi}{\partial t}+\frac{1}{2}V^2+\frac{p}{\rho}+gz=C(t)$$

Euler and Bernoulli

- For a *stationary* flow along a streamline :

$$\frac{1}{2}V^2 + \frac{p}{\rho} + gz = C$$

- This can be written in two alternative ways :

$$\frac{V^2}{2g} + \frac{p}{\rho g} + z = C \qquad\qquad \tfrac{1}{2}\rho V^2 + p + \rho gz = C$$

Stream Function Ψ

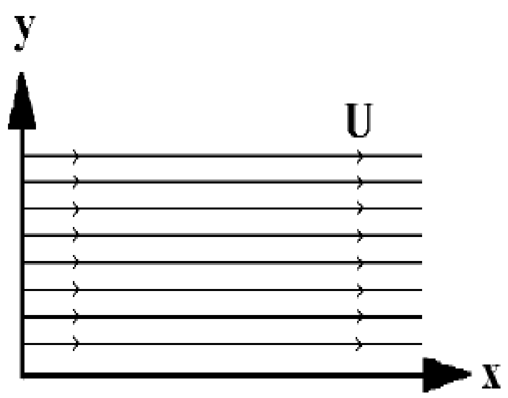
- Ψ= constant along a streamline
- Definition : $u = \dfrac{\partial \Psi}{\partial y}$ and $v = -\dfrac{\partial \Psi}{\partial x}$
- Stream lines and equipotential lines are orthogonal
- No cross flow, impervious boundary of stream tube

Potential Flow Elements

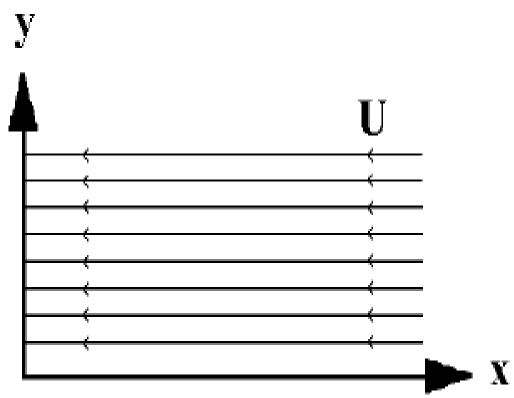
- Uniform flow
- Source
- Sinks
- Circulation

Superposition is allowed

Uniform Flow



a. Uniform Flow
(in positive x-direction)



b. Uniform Flow
(in negative x-direction)

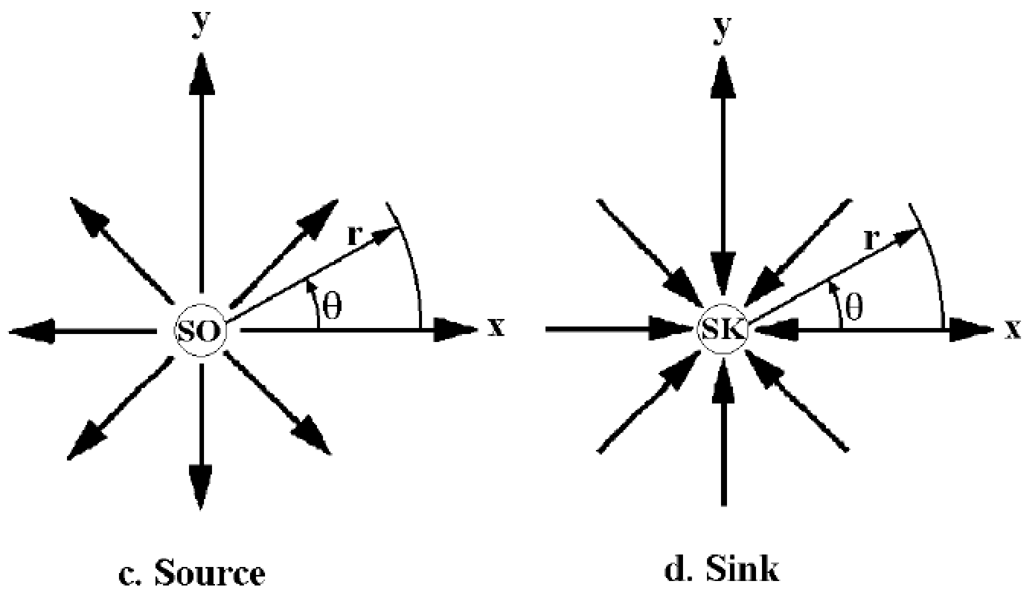
$$\Phi = +U \cdot x$$

$$\Psi = +U \cdot y$$

$$\Phi = -U \cdot x$$

$$\Psi = -U \cdot y$$

Source & Sink



$$\Phi = -\frac{Q}{2\pi}\cdot \ln r \qquad \Phi = +\frac{Q}{2\pi}\cdot \ln r$$

$$\Psi = +\frac{Q}{2\pi}\cdot \theta \qquad \Psi = -\frac{Q}{2\pi}\cdot \theta$$

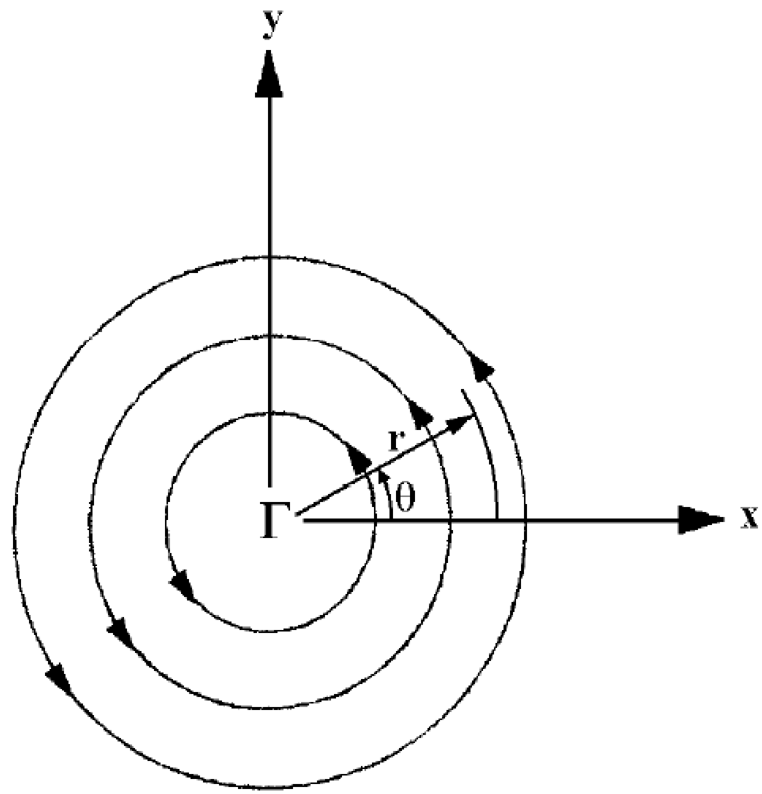
Circulation (Vortex)

- Counter clockwise flow:

$$\Phi = +\frac{\Gamma}{2\pi}\cdot\theta \qquad \Psi = -\frac{\Gamma}{2\pi}\cdot\ln r$$

- Clockwise flow

$$\Phi = -\frac{\Gamma}{2\pi}\cdot\theta \qquad \Psi = +\frac{\Gamma}{2\pi}\cdot\ln r$$



e. Circulation

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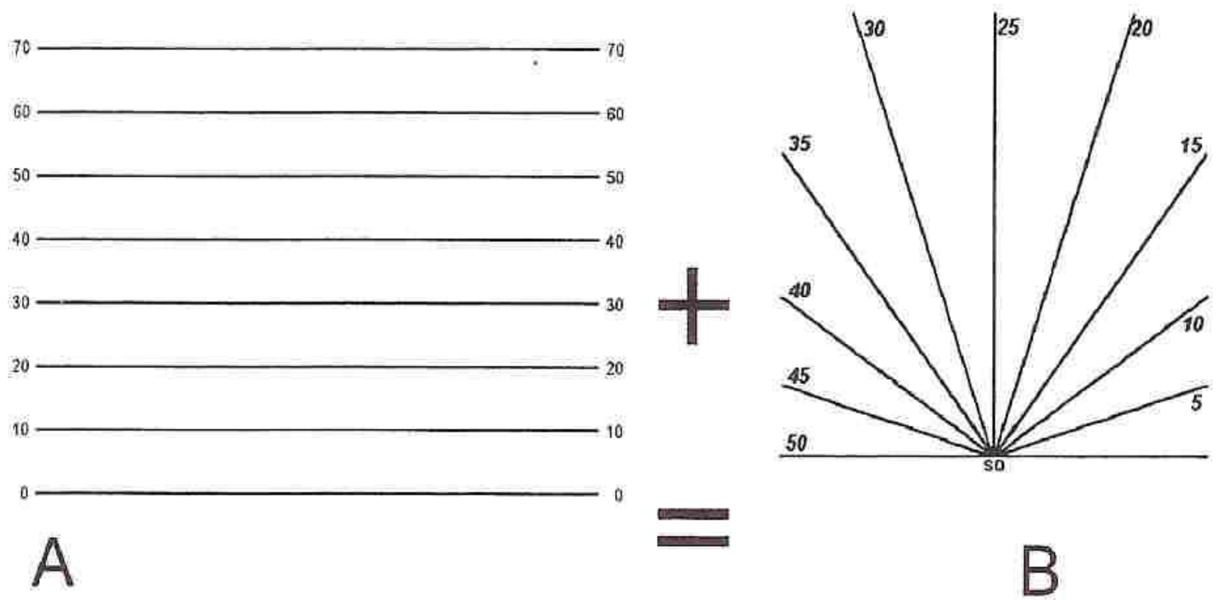
Superposition - principle



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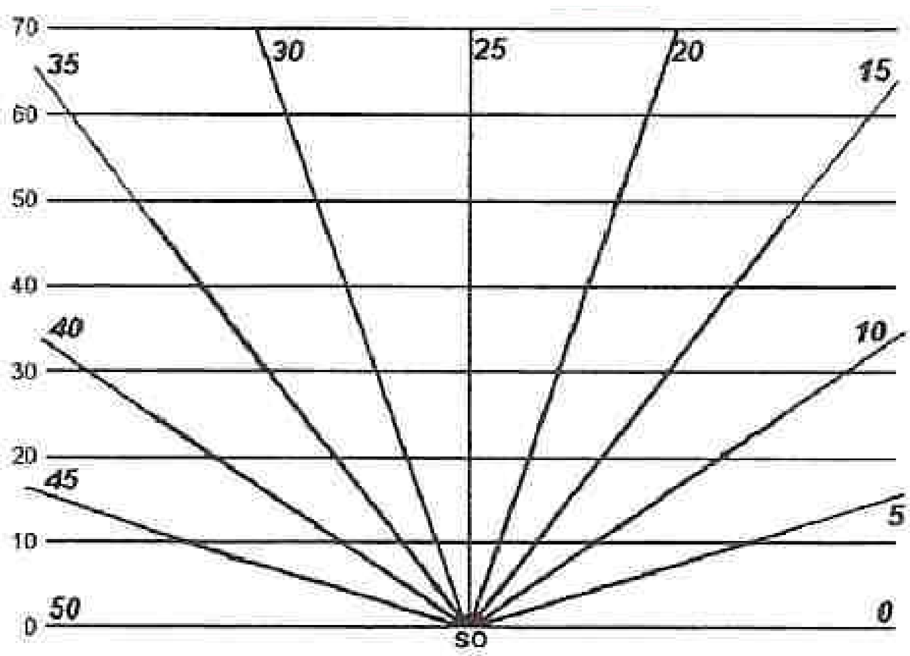
Superposition - principle



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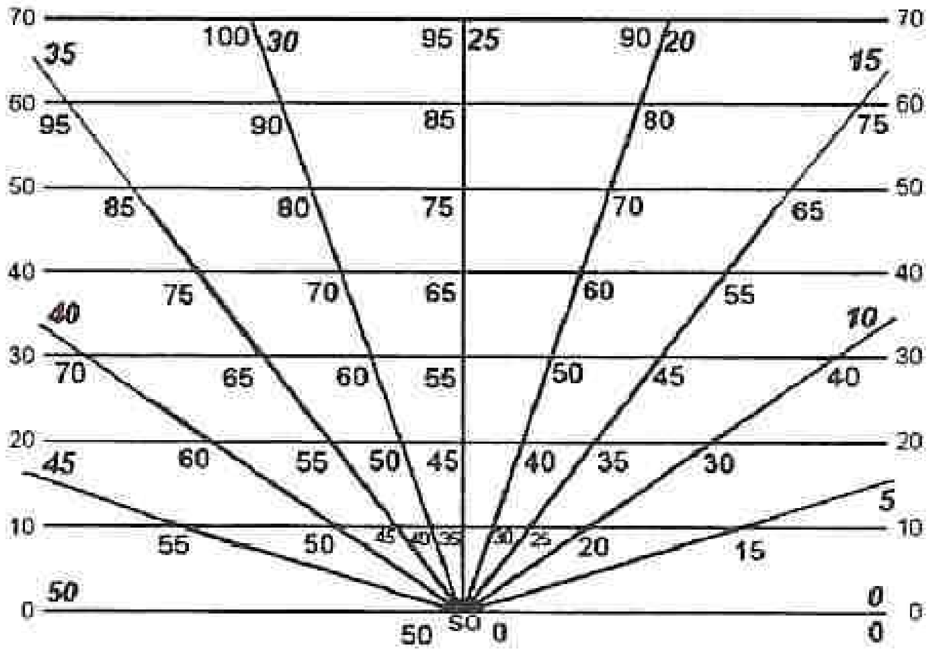
Superposition - principle



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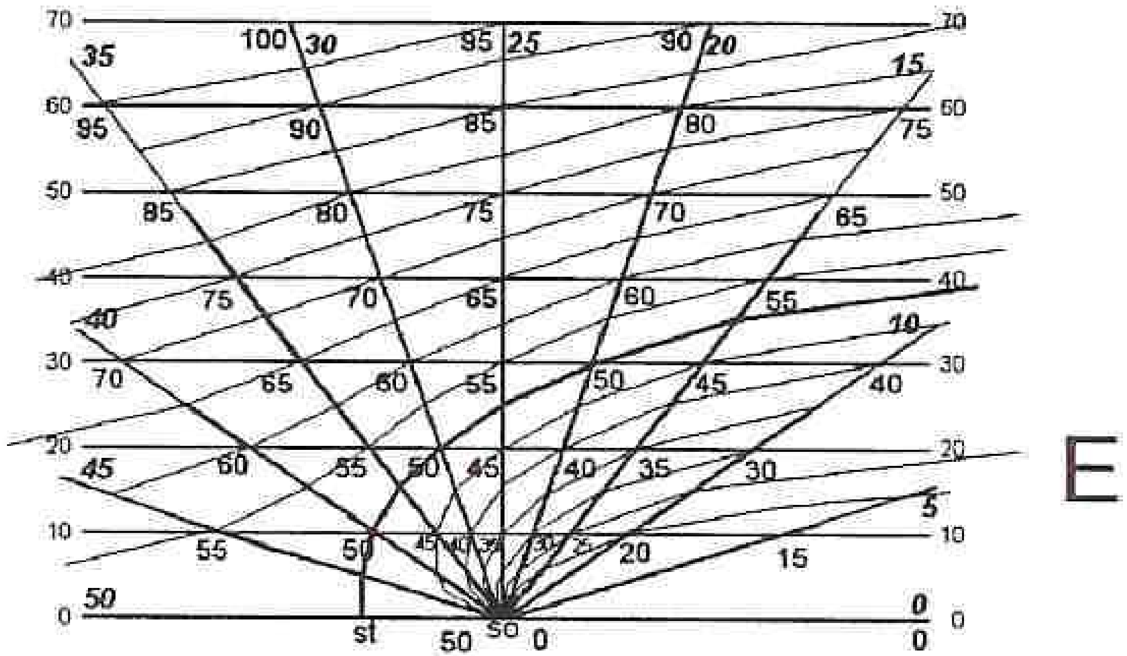
Superposition - principle



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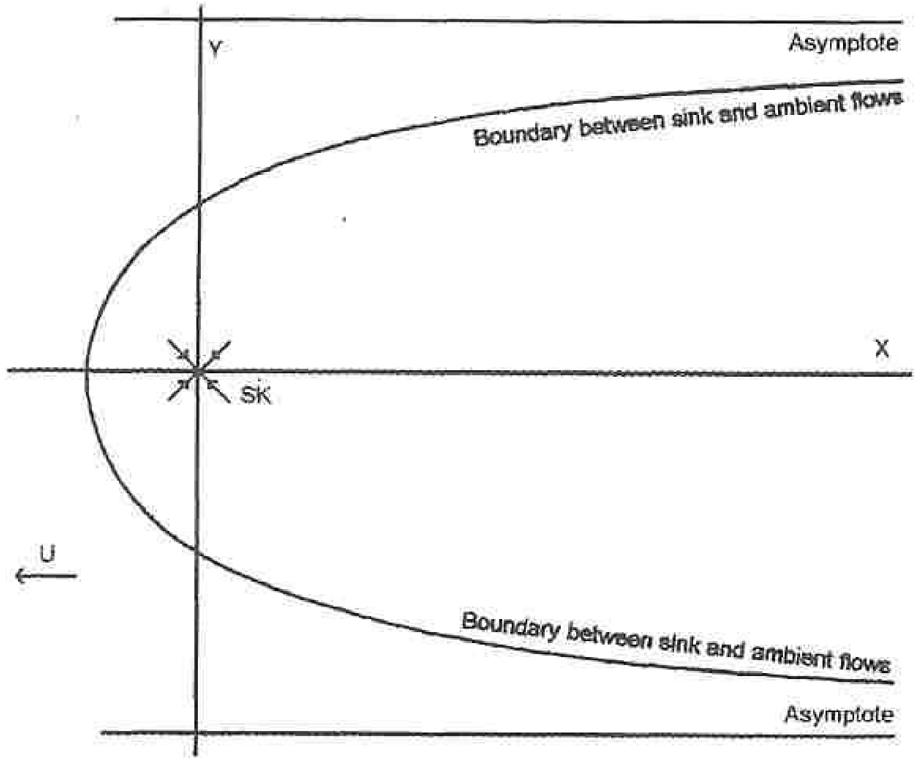
Uniform flow + sink



$$\Psi = -\frac{Q}{2\pi} \cdot \arctan\left(\frac{y}{x}\right) - U_\infty \cdot y$$

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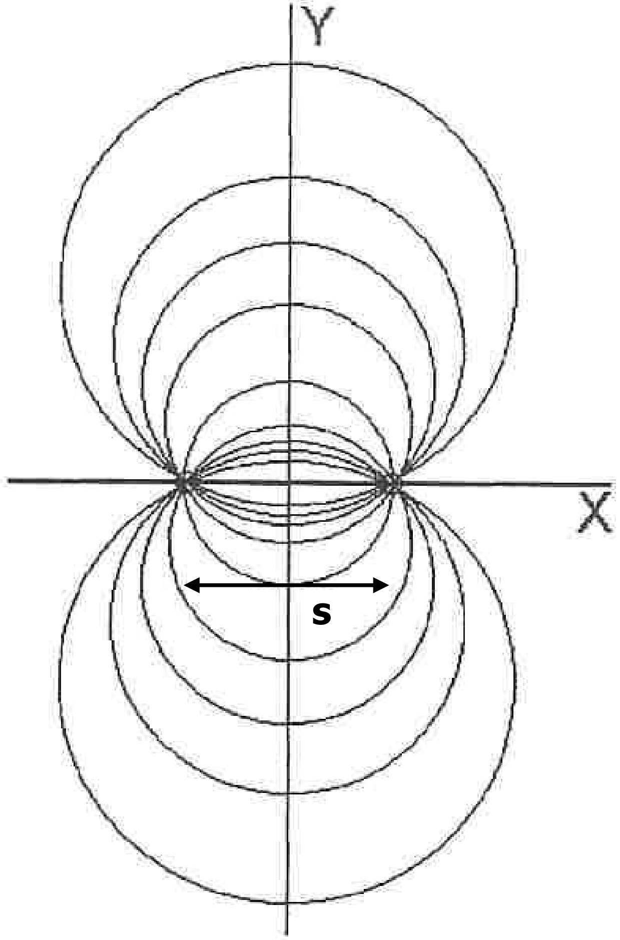
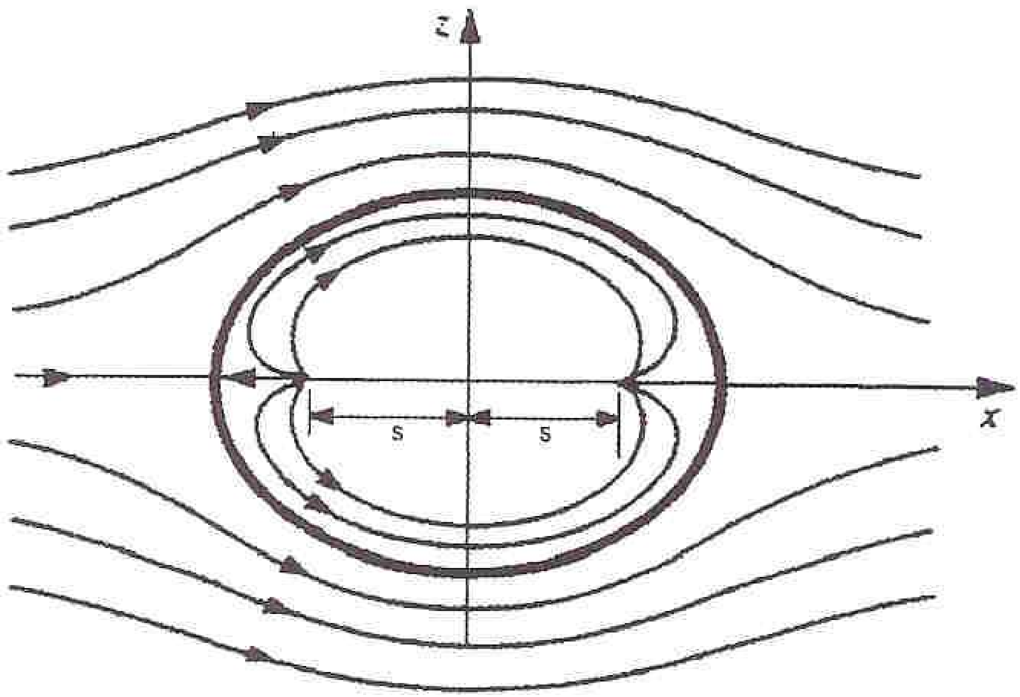
Separated Source + sink



$$\Psi = \frac{Q}{2\pi} \cdot \arctan\left(\frac{2ys}{x^2 + y^2 + s^2} \right)$$

Uniform flow + source + sink

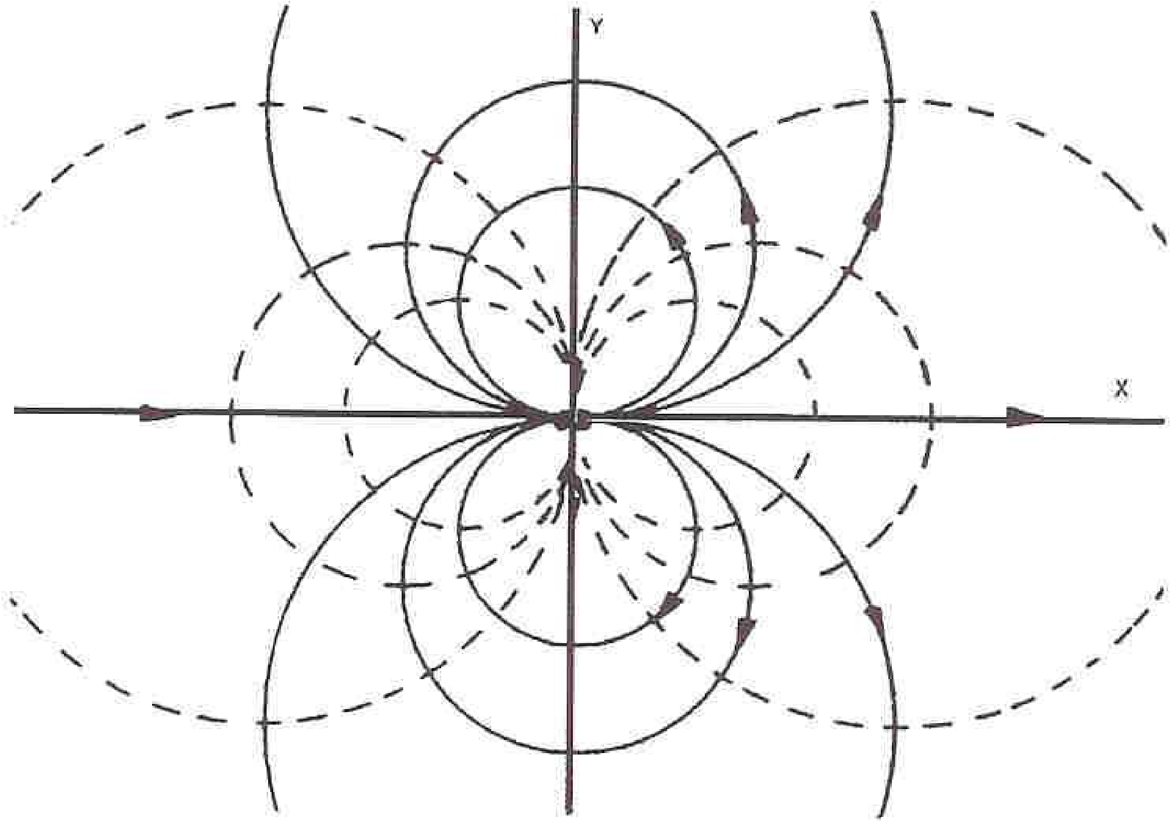
- No flow through boundary
  (= streamline)
- Rankine ship forms



$$\Psi = \frac{Q}{2\pi} \cdot \arctan\left(\frac{2ys}{x^2 + y^2 - s^2} \right) + U_\infty \cdot y$$

Doublet or Dipole

- A source and sink pair placed very close together
- (s -> 0)

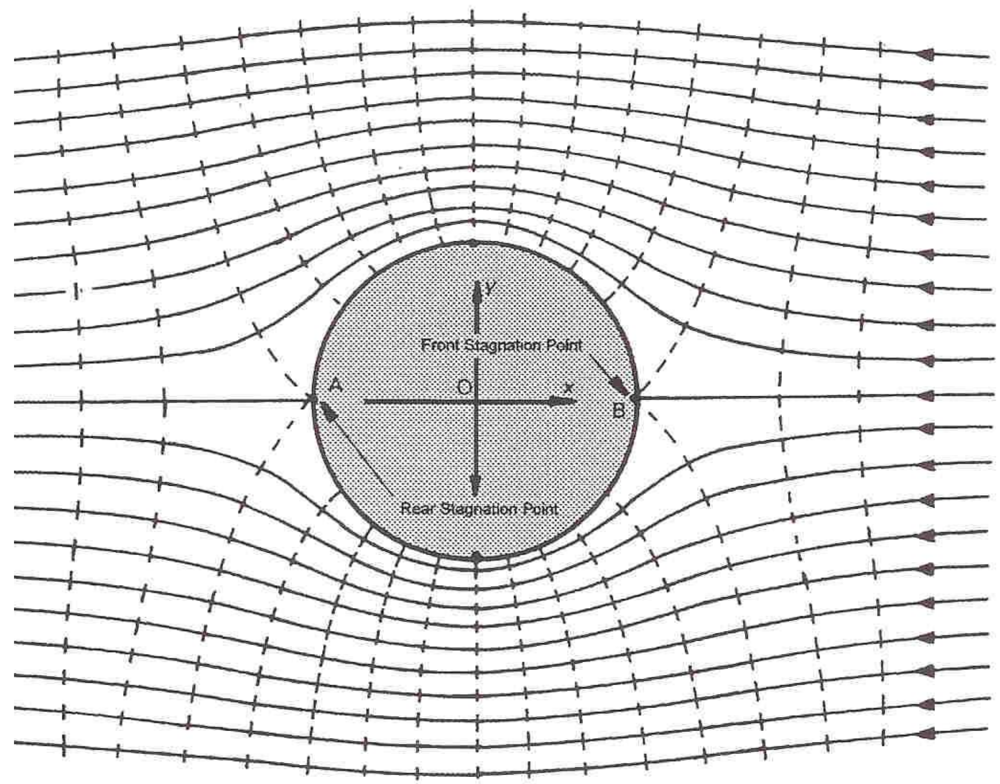


$$\Psi = \mu \cdot \frac{y}{x^2 + y^2} \qquad \Phi = \mu \cdot \frac{x}{x^2 + y^2}$$

Dipole + uniform flow

- Model for cylinder
- *X*-axis is a streamline
- Boundary at $R = \sqrt{\dfrac{\mu}{U_\infty}}$

  is also a streamline



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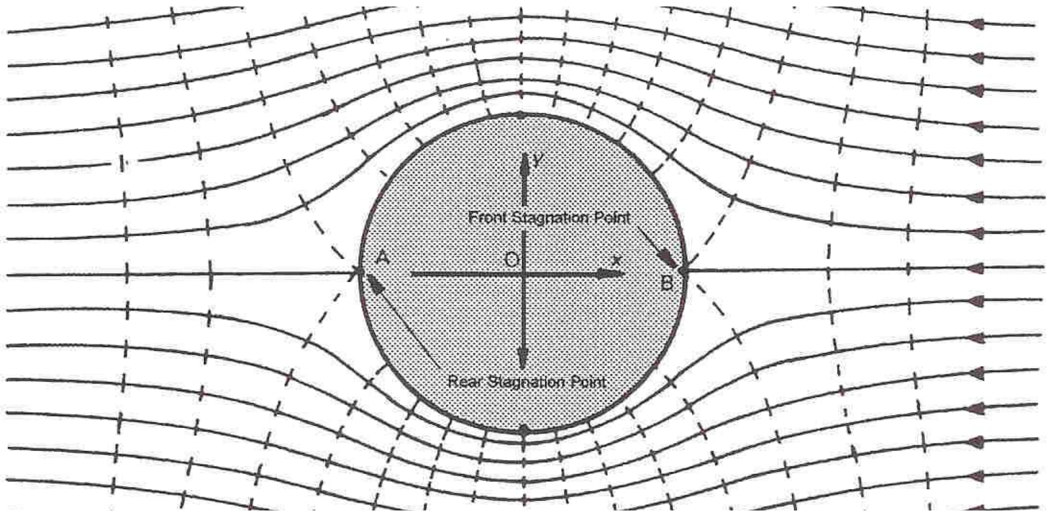
Uniform flow around a cylinder

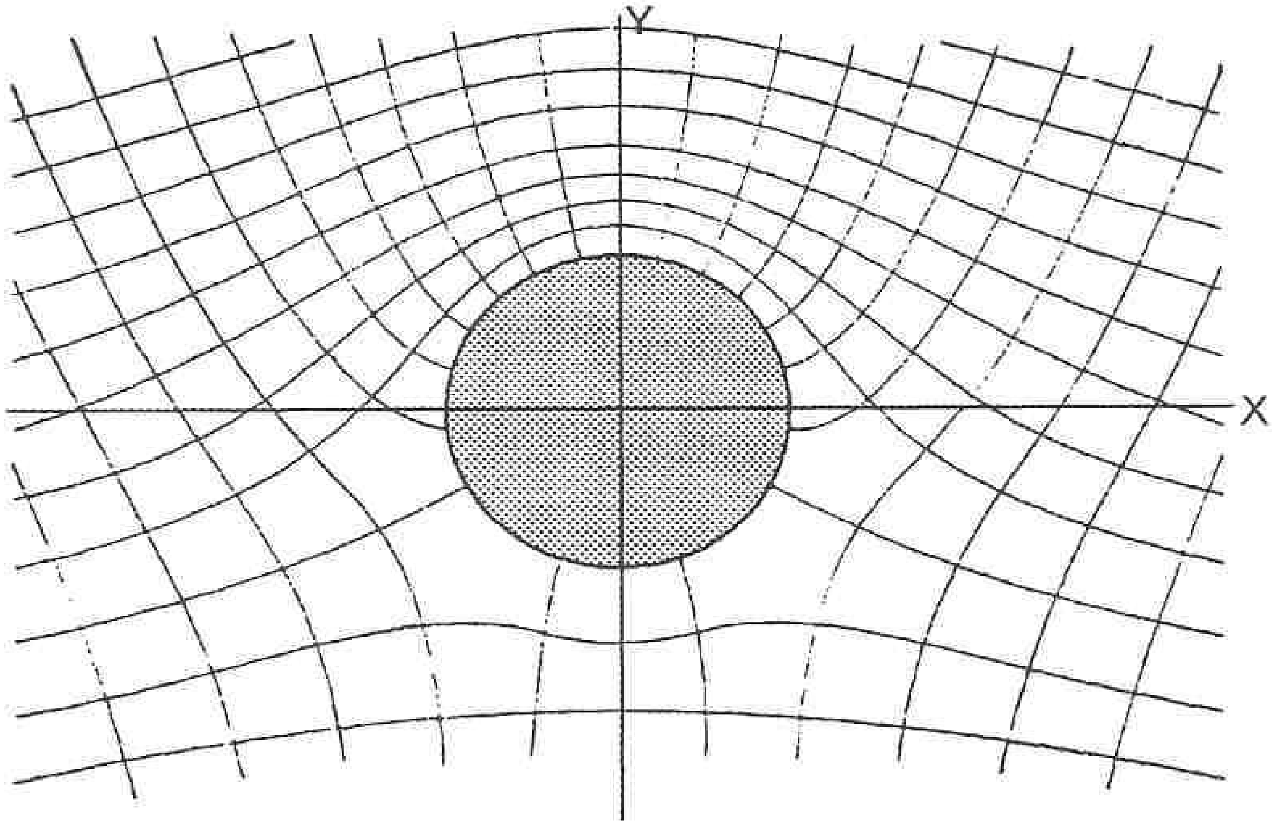


$$v_\theta = -\left[\frac{\partial \Psi}{\partial r}\right]_{r=R} = -\frac{\partial}{\partial r}\left\{\frac{\mu \sin\theta}{r} - U_\infty r \sin\theta\right\}_{r=R} \qquad v_\theta = -2U_\infty \sin\theta$$

- $V_\theta = 0$ at stagnation points
- $V_\theta = 2^*U_\infty$ at sides of cylinder

Uniform flow + circulation

Pressure asymmetry creates a lift force



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Continuity Condition

Continuity Equation, general case

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

Ideal fluid, incompressible

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Deformation of flow particles

Deformation velocity or dilatation:

$$\frac{\dot{\alpha}+\dot{\beta}}{2}=\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)$$

Rotation (in 2 D) :

$$\dot{\varphi}=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

Velocity Potential

A scalar function, Φ

The velocity in any point of the fluid and in any direction is the derivative to that direction of the potential

$$u = \partial\Phi/\partial x \quad \text{etc}$$

Properties of Velocity Potentials

- The continuity condition results in the *Laplace* equation :

$$\boxed{\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0} \qquad \text{or:} \qquad \boxed{\nabla^2 \Phi = 0}$$

- Potential flow is rotation-free by definition :

$$\boxed{\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0}$$

Euler and Bernoulli

Force = mass * acceleration : Euler

non viscous and incompressible fluid

$$\boxed{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial x}}$$

Energy conservation along a flowline : Bernoulli

non viscous and incompressible fluid

$$\frac{\partial}{\partial x}\left\{\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2\right] + \frac{p}{\rho}\right\} = 0 \qquad \boxed{\frac{\partial \Phi}{\partial t} + \frac{1}{2}V^2 + \frac{p}{\rho} + gz = C(t)}$$

stationary flow : $\boxed{\frac{1}{2}\rho V^2 + p + \rho g z = Constant}$

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Stream Function Ψ

- Companion of the Velocity Potential
- Definition : $\boxed{u = \frac{\partial \Psi}{\partial y}}$ (a) and $\boxed{v = -\frac{\partial \Psi}{\partial x}}$ (b)
- Stream lines and equipotential lines are orthogonal
- No cross flow, impervious boundary of stream tube

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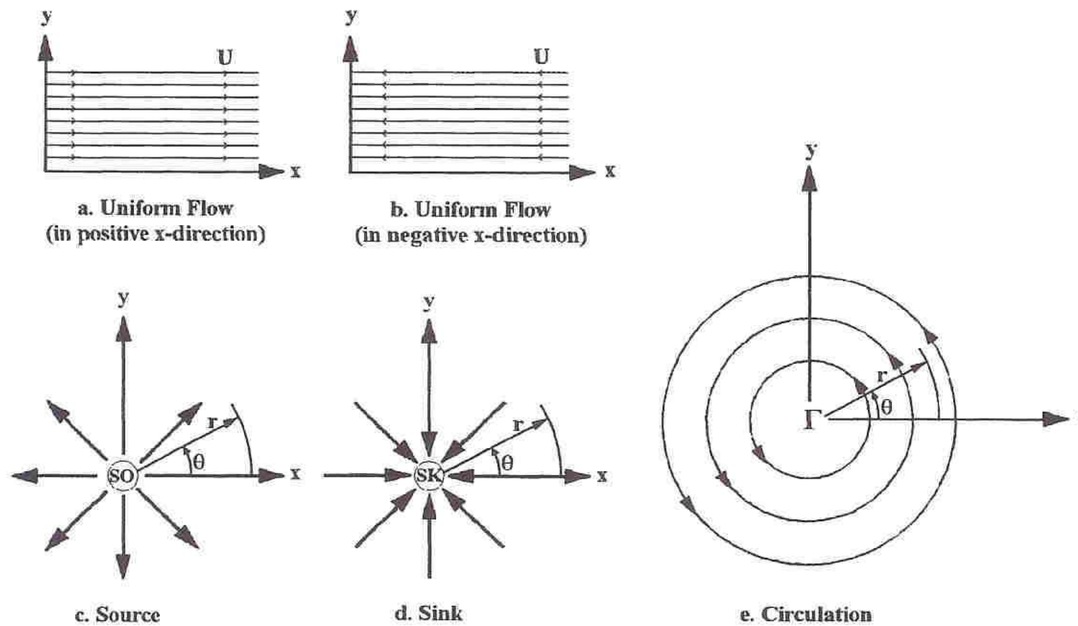
Potential Flow Elements

- Uniform flow
- Sources and sinks
- Circulating flows

Superposition is allowed



Polar Coordinate Description

Basic definitions : $\boxed{v_r = +\frac{\partial \Phi}{\partial r} = \frac{1}{r} \cdot \frac{\partial \Psi}{\partial \theta}}$ (a) and $\boxed{v_\theta = \frac{1}{r} \cdot \frac{\partial \Phi}{\partial \theta} = -\frac{\partial \Psi}{\partial r}}$ (b)

Source :

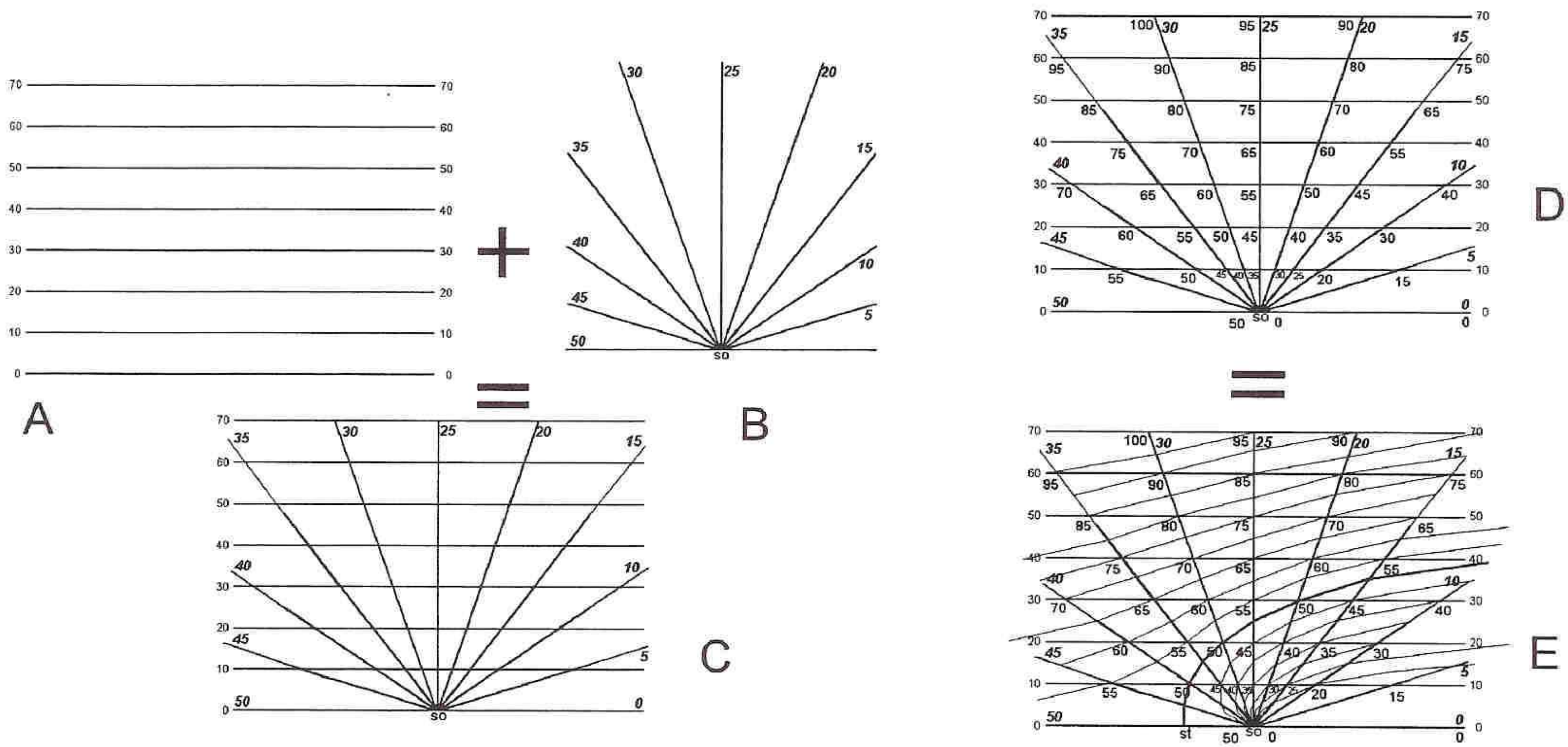
(Sink = - source)

$\boxed{\Phi = +\frac{Q}{2\pi} \cdot \ln r}$ (a) and $\boxed{\Psi = +\frac{Q}{2\pi} \cdot \theta}$ (b)

Vortex : $\boxed{\Phi = +\frac{\Gamma}{2\pi} \cdot \theta}$ (a) and $\boxed{\Psi = -\frac{\Gamma}{2\pi} \cdot \ln r}$ (b)

$$\Gamma = \oint v_\theta \cdot ds = 2\pi r \cdot v_\theta = constant$$

Superposition - principle



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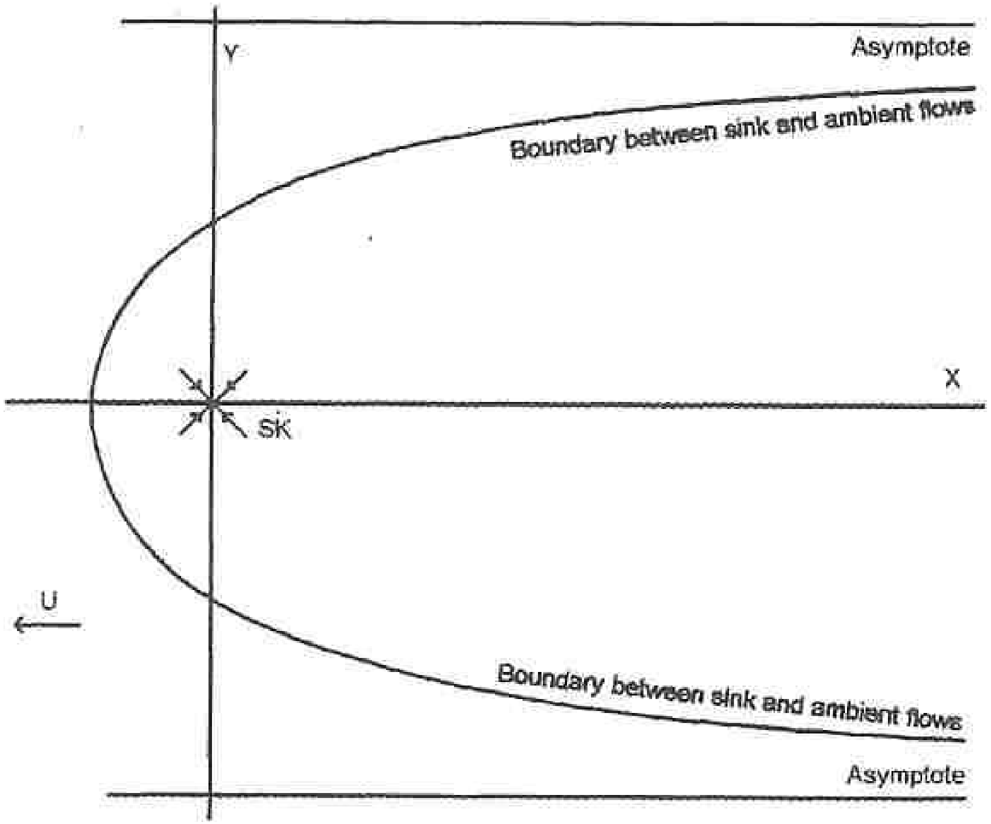
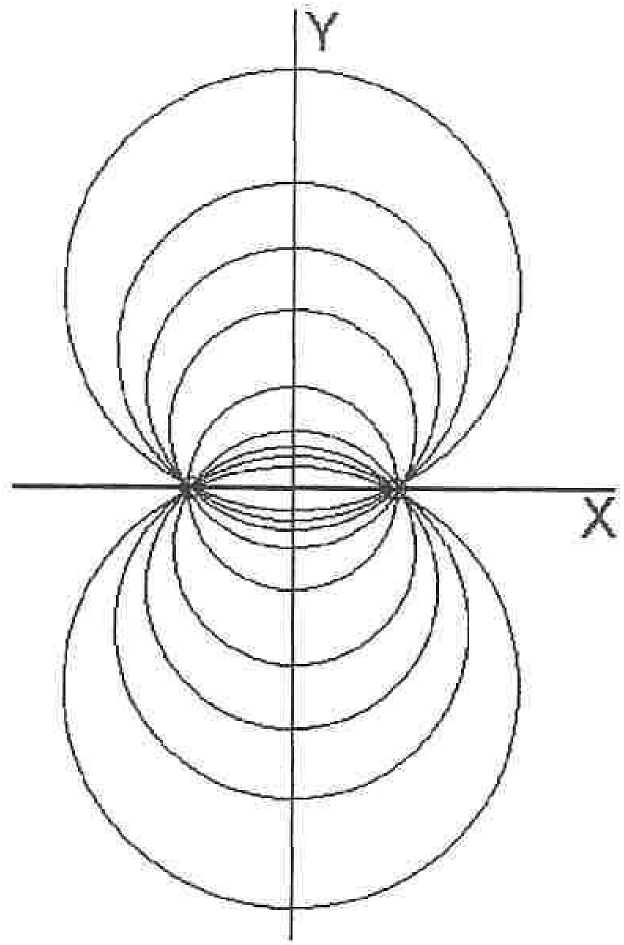
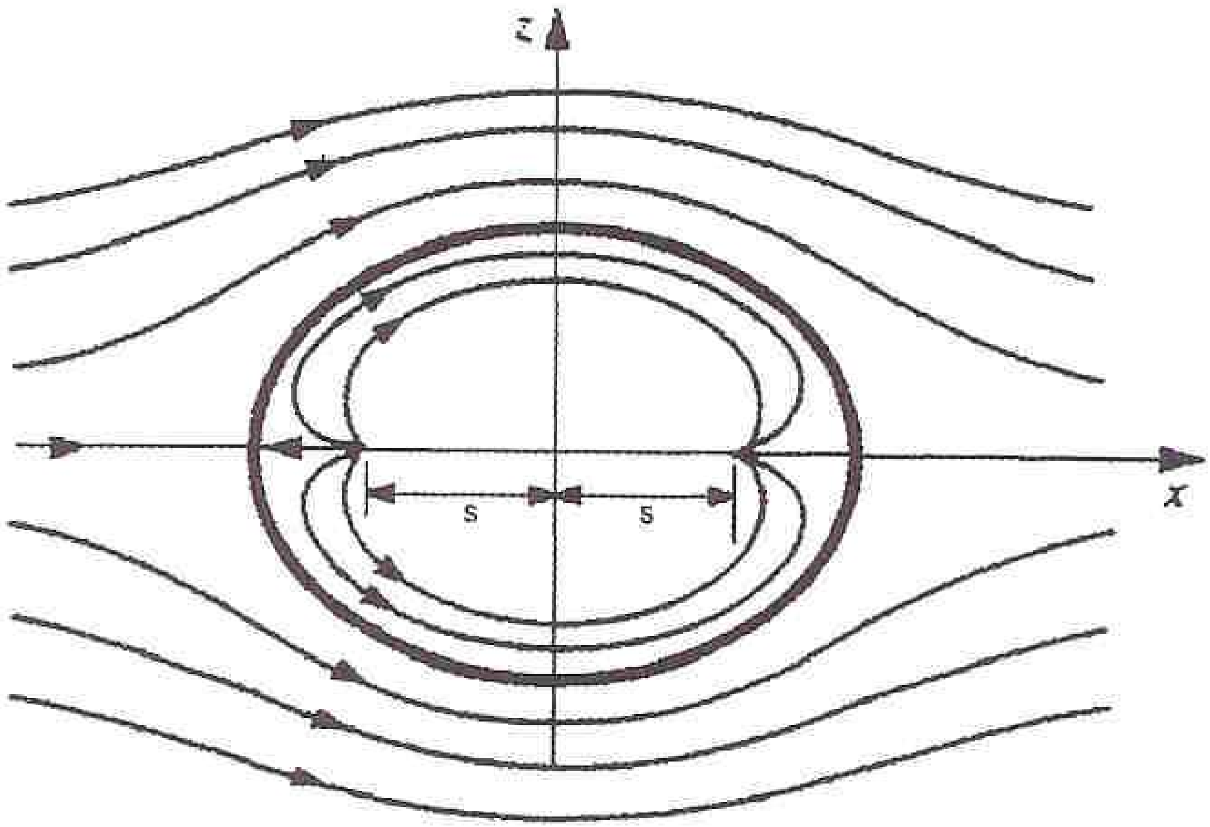
Uniform flow + sink



Source + sink



Uniform flow + source + sink



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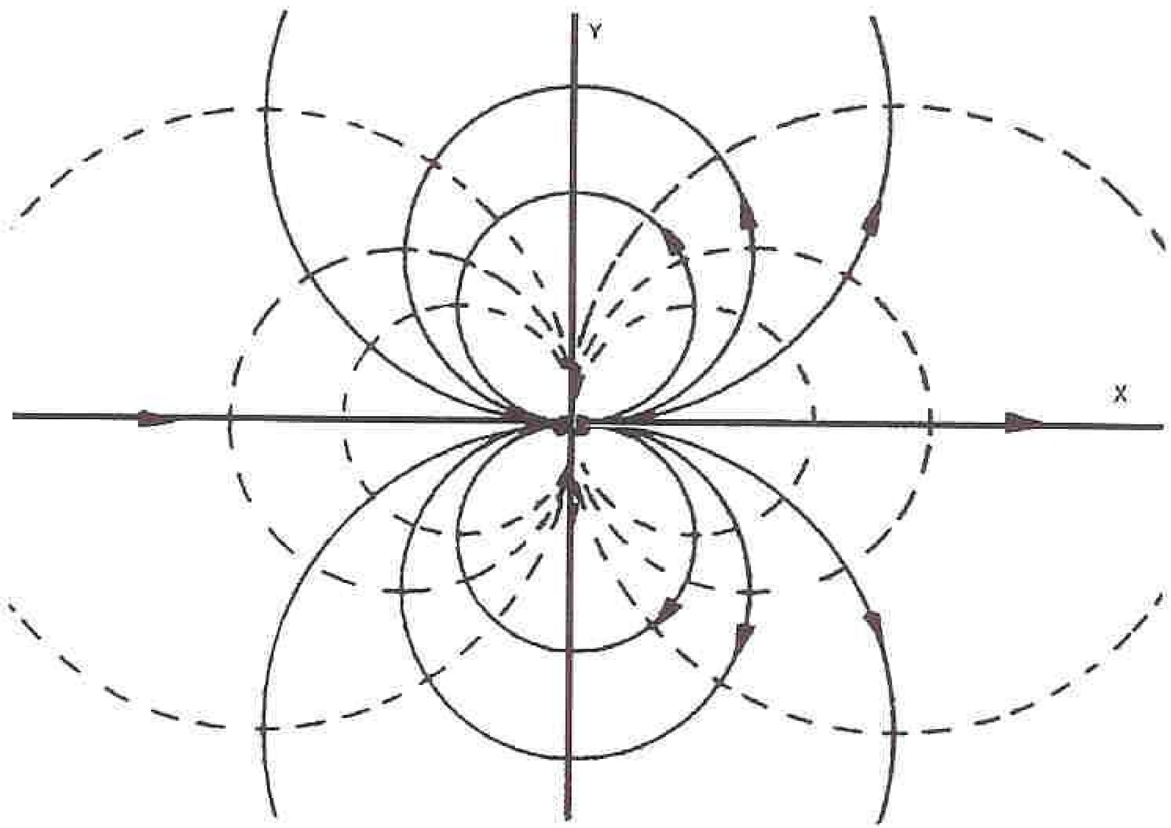
Dipole

Strength $\mu = Qs/\pi$



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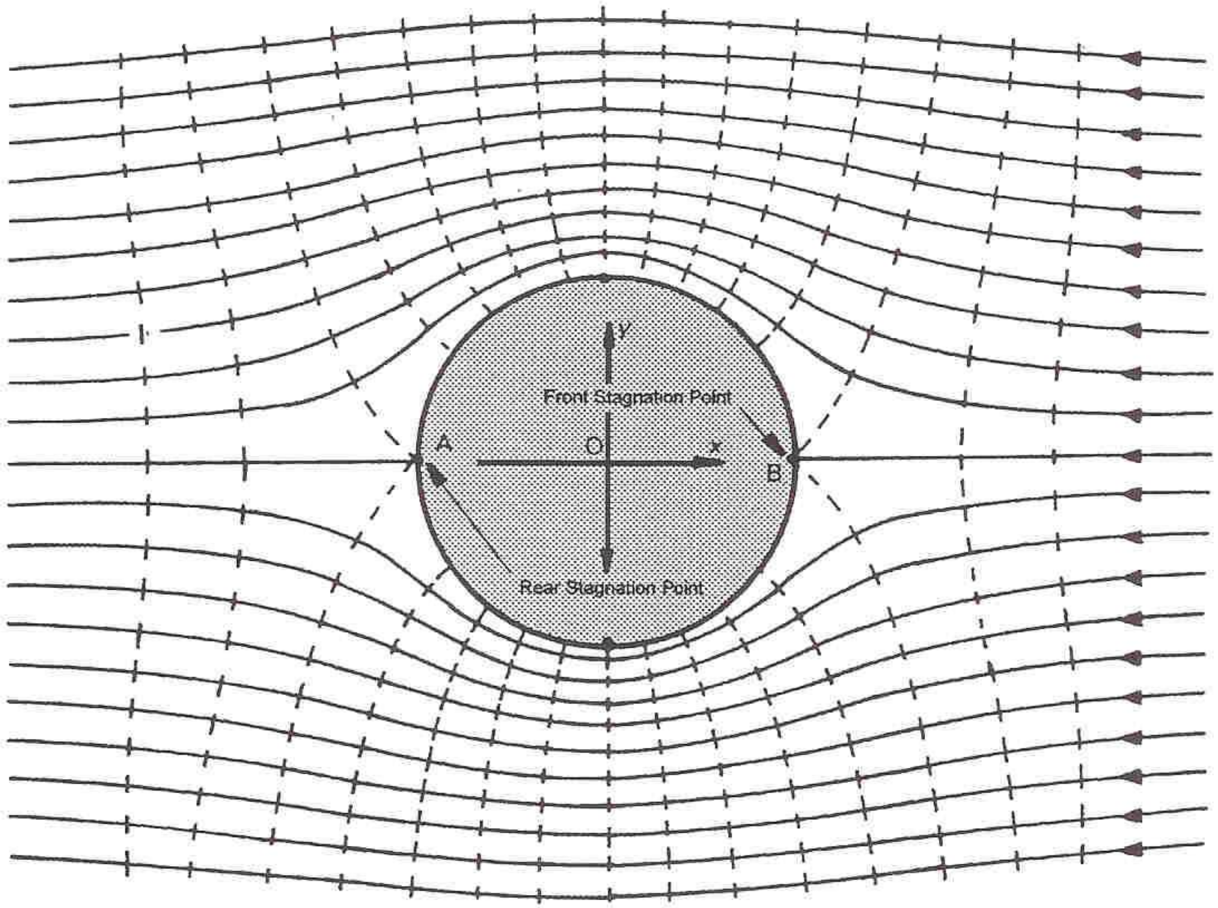
Dipole + uniform flow



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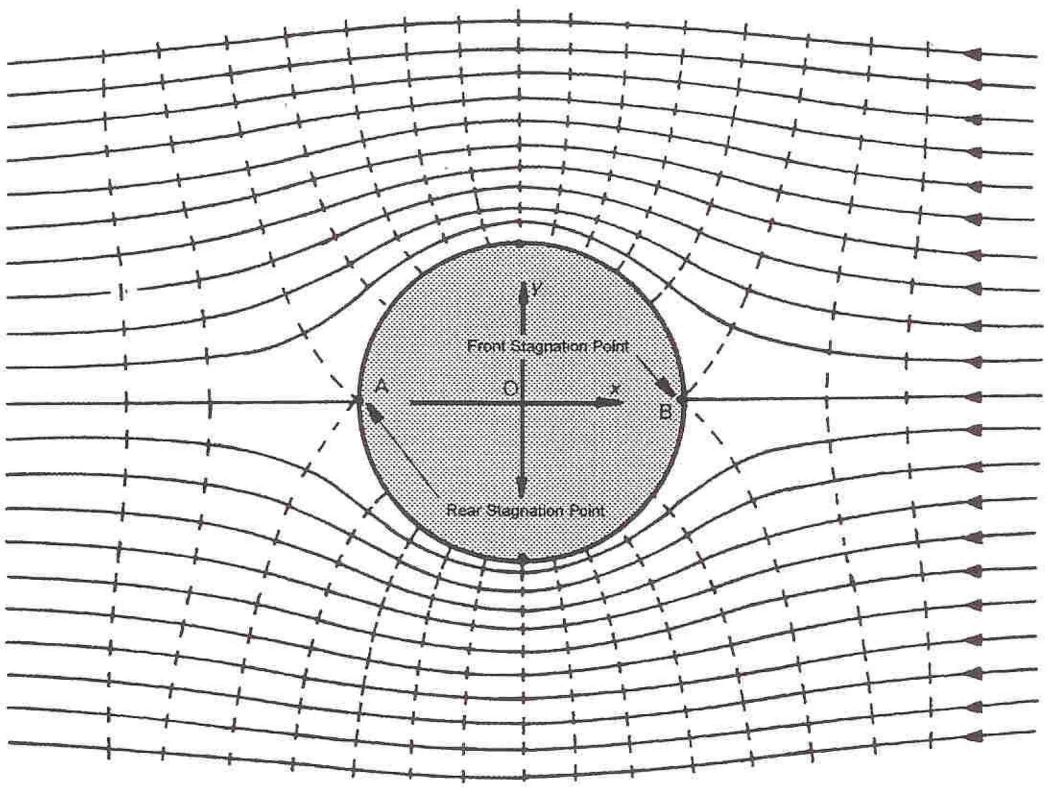
Uniform flow around a cylinder



Stagnation points on the X axis

V_{max} = 2 U at Y axis

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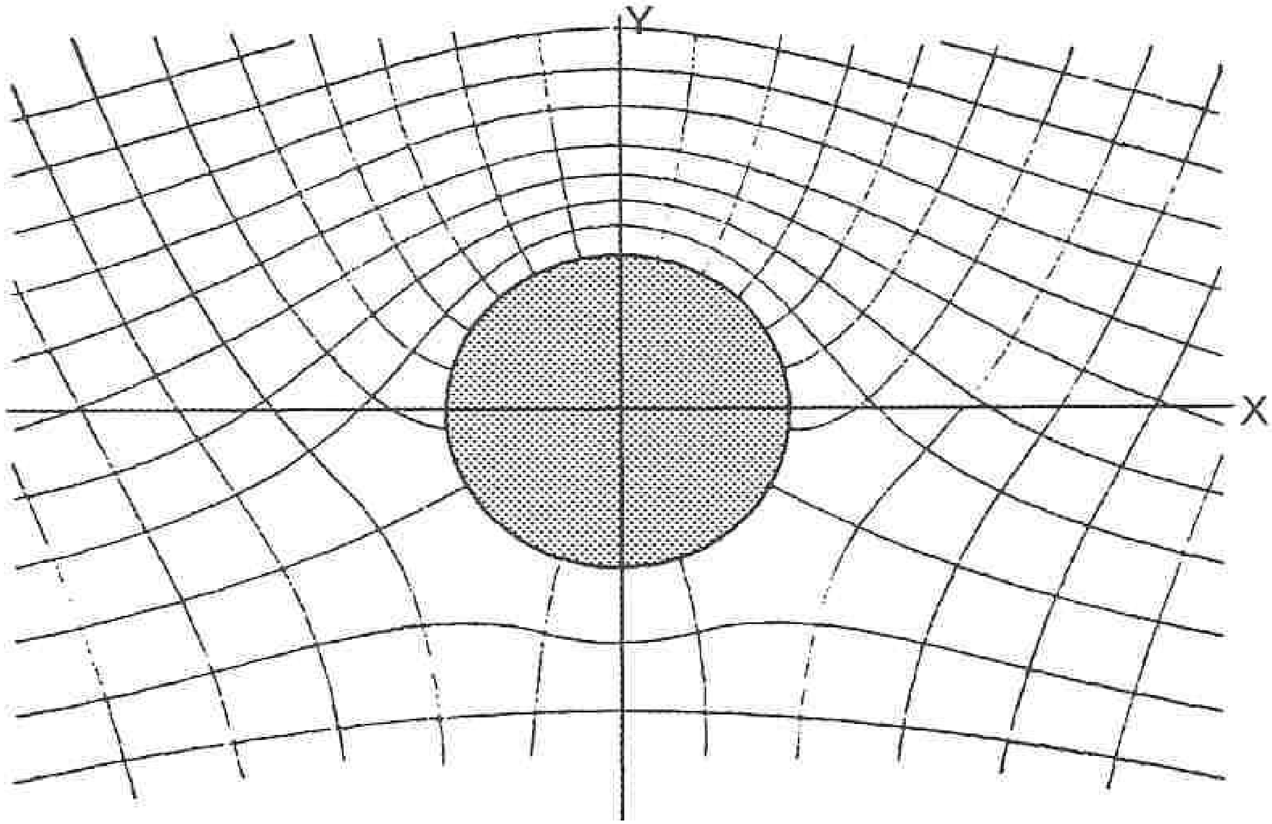
Uniform flow + circulation

Pressure asymmetry creates a lift force



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