4.1 Introduction

Network calculations assist in the analysis of flows en pressures in pressurised networks or open channel networks. The calculation of the flow through a single pipe or open channel is relatively simple and can be done with a manual calculation as is demonstrated in the previous chapters. In a network the calculation of flows and pressures is more complicated because of the magnitude of boundary conditions and the interconnection of the pipes or channels. Computerised calculations are necessary to analyse the behaviour of a network within a reasonable time limit.

The first network calculations were done by hand using methods like Newton Raphson and Hardy-Cross in pressure or volume flow equalisation. It is obvious that only networks with a moderate number of pipes and nodes can be analysed. A popular citation in that light is a quote ‘Every network hydraulic problem can be analysed with a model with 50 nodes and 50 pipes.’

Looking at the principle of hydraulic relevance (see paragraph XX) this quote holds true, but it takes a large amount of experience and hydraulic insight to be able to model networks to this size and to translate and interpret the results for the original network.

With the development of computers starting in the ’60-s from last century, the first calculation programs came available. Because of the costs of the programs and the CPU-time consumed these calculations were mostly used for design purposes for larger pipes and projects. From the early ’80-s of the last century the first calculation programs for personal computer were developed. These programs are best described as automated manual calculations and were capable of handling larger networks.

Rising popularity, availability and capacity of personal computers stimulated the development of calculation software. New techniques are developed to actually use the capacity of computers and combine this with an integral approach for network calculation. The linear programming method is made and forms the calculation core of much commercially available software.

What stayed over the years is that networks are complicated and that simulation is an approximation of reality. Assumptions are made to make mathematical modelling possible, continuous processes are transformed to discrete processes that in their turn

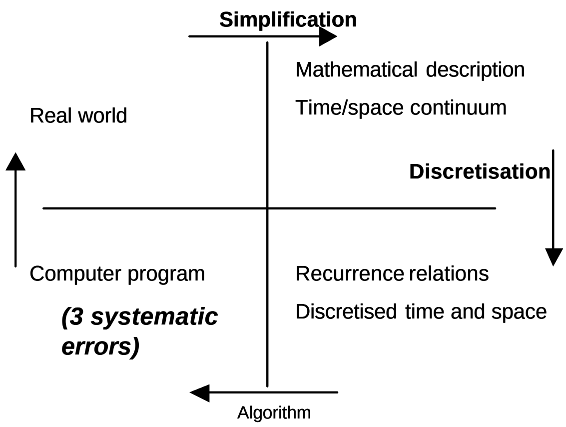

Fig. 4.1 - Stages between reaility and computer program

are translated to computer codes. The real world is translated in discrete input and calculations are made. Schematically this process is represented in figure 4.1

Computer aided modelling of reality gives better insight in the dynamics of real processes. They are however not more than an approximation of reality and common sense is indispensable to judge and use the results of calculations.

A ten-angular figure is a good mathematical approximation of a circle, but turned into a wheel it will give a bumpy ride.

In this chapter the background of commercially available models for drinking water systems (pressurised) and sewerage systems (open channels) is explained. Apart from the possibilities of these models the limitations will be demonstrated.

4.2 Pressurised pipes: Drinking water models

4.2.1 Basic equations

The basic equation for determining the pressure loss in a pipe is the Darcy-Weissbach equation (see XX)

$$\Delta H = H_2 - H_1 = l\,\frac{L}{D}\frac{u^2}{2g} = 0{,}0826\frac{l\,L}{D^5}Q^2$$

Together with the White-Colebrook formula to calculate the value of λ this is one equation describing the steady state of a network.

$$\frac{1}{\sqrt{l}} = -2\log\left[\frac{k_N}{3D} + \frac{1}{0{,}32\mathrm{Re}\sqrt{l}}\right]$$

The value of λ is dependent of the hydraulic situation in the pipe, characterised with the Reynolds number and the physical properties as the diameter and the roughness.

For the unique situation of the calculated steady state however the value is fixed, making the Darcy-Weissbach formula $\Delta H = H_2 - H_1 = a\,Q^2$ which is a one-dimensional relation between volume flow and pressure drop. The formula can graphically be represented in the so-called Moody diagram. (fig 4.2)

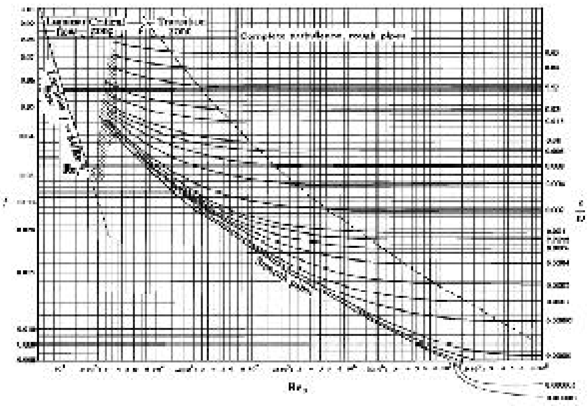

Fig. 4.2 - Moody diagram

4.2.2 Modelling the network

For analysis purposes a network is modelled as a system of pipes that are joined together at nodes. This makes a system of interconnected pipes that form loops. Friction losses in pipes can easily be calculated. The pipes are connected to the rest of the network at one end that automatically gives the pressure boundary at that side. The demand at the other side is given, so the pressure drop can be calculated end consecutively the pressure at the demand point.

All the pressure and demand boundaries are concentrated in the nodes and all the friction losses are concentrated in the pipes. Other losses are dealt with in the model as friction losses, which are velocity dependent.

With the help of Darcy-Weissbach and the laws of Kirchhoff a mathematical model can be made and solved.

4.2.3 Kirchhoff's laws

Kichhoff's laws are used to put together the system of equations to calculate the pressures and volume flows in a network. The laws are transcripted from the electrical analogy (see text box):

- Mass balance in a node is zero. The sum of flows towards a node equals the sum of flows leaving the node. $\sum_n Q = 0$
- The pressure losses in a loop of pipes equals zero. $\sum_{loep} H = 0$

Schematically the laws are given in figure 4.4

Applying the Kirchhoff laws to every node and every loop in a network gives the set of equations that

Fig 4.3 - Gustav Robert Kirchhoff

Gustav Robert Kirchhoff (1824-1887 laid the mathematical fundaments bearing the analysis of piped water networks. In 1845 he announced the laws that were named after him as student of the University of Königsberg. He formulated these laws to allow calculation of currents, voltages and resistance in electrical circuits with multiple loops, extending the work of Ohm. Kirchhoff's laws are also applicable for piped networks as they follow the electrical analogy.

Kirchhoff considered an electrical network consisting of circuits joined at nodes of the network and gave laws which reduce the calculation of the currents in each loop to the solution of algebraic equations. The first law states that the sum of the currents into a given node equals the sum of the currents out of that node. The second law states that the sum of electromotive forces in a loop in the network equals the sum of potential drops, or voltages across each of the resistances, in the loop.

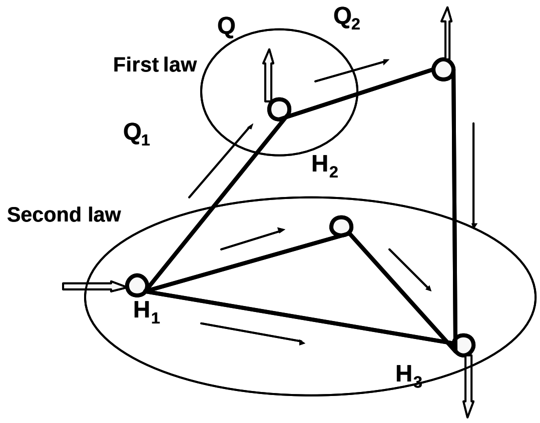

Fig. 4.4 - Scheme Kirchhoff's laws

solves the pressures in the nodes and the volume flows in the pipes, given a network and sufficient boundary conditions.

Consider a network with N nodes and X pipes.

- For every node in the network the mass balance can be established. (first law of Kirchhoff) This gives N equations with x unknown being the flows in X pipes (The demands in a node are known).
- The relation between the flows in the pipes and the pressures in the nodes is given by the Darcy Weissbach and White-Colebrook law: $Q^2_x = f(\Delta Hx) = Hn - H(n\text{-}1)$
- Every flow in X pipes can be transformed to an equation in one or more of the pressures in the nodes. This gives X equations with N unknown.
- The set of equations is reduced to N equations with N unknown, which can mathematically be solved.

Complication is that the relation between Q and H is non-lineair so the set equations can only be solved in an iterative way. Basically there are two methods:

- Hardy-Cross, pressure or volume flow equalisation
- Linear programming

Hardy-Cross equalisation

The Hardy-Cross methods are the conventional methods for solving non lineair systems. Originally this method was used in manual network calculation. Programming the method used very little computer memory. This accounts for the popularity of this method in the first generation of network calculation programs. The method still is the base for some networks programs because of the simplicity and the possibility to use the method on simple computers.

Pressure equalisation method

The method consists of a loop of steps:

- Assume/estimate pressures in all nodes
- Apply Kirchhof's first law: mass balance in every node is zero. Adjust the pressure in the node considered in such a way that the volume flows induced from the other nodes meet this law.
- Take the next node and apply Kirchhoff's first law and adjust the pressure until the flows meet the law.
- Repeat this cycle for all nodes until the largest adjustment of the pressure in the nodes in one cycle is below a certain threshold.

In appendix 4.1 this is illustrated in more detail.

The method is sensitive for the first estimation of the pressures and for large differences in connected pipes, for instance one node with a pipe of 500 mm and a pipe of 100 mm. An adjustment in pressure in the considered node causes a volume flow adjustment in the 500 mm pipe that is 3125 (5^5) times bigger as the adjustment in the 100 mm pipe.

Volume flow equalisation

This method also considers an iterative cycle of steps:

- Assume/estimate volume flow in all pipes;
- Apply Kirchhoff's second law to a loop. Adjust the volume flows with ΔQ (all flows same adjustment) until the second law is met.
- Take the next loop and apply Kirchhoff's second law, adjust until…etc.
- Calculate the pressures and repeat flow adjustments until the largest adjustment in one cycle is smaller than a certain threshold.

In appendix 4.2 this is illustrated in more detail.

This method is also sensitive for the first estimation. Less sensitivity is experienced towards nodes with larger and smaller pipes.

Linear programming

Because the relation between pressure drop and volume flow is non-linear it is impossible to solve the matrix of equations. The method of linear programming is based on a linearisation of the Darcy-Weissbach equation. The equation is written as

$$\Delta H = \frac{Q\left|Q^*\right|}{R}$$

With $R = \frac{p^2 g}{8L}\frac{D^5}{l}$ and Q* as an estimation of Q.

Applying the two laws of Kirchhoff makes it possible to establish the matrix for the whole network. The result is presented below. More detail is found in appendix 4.3

$$\begin{vmatrix} -\sum_{j=1}^{m} K_{j1} & K_{21} & . & . & K_{n1} \\ K_{12} & -\sum_{j=1}^{m} K_{j2} & . & . & K_{n2} \\ . & . & . & & . \\ . & . & & . & . \\ K_{1n} & K_{2n} & . & . & -\sum_{j=1}^{m} K_{jn} \end{vmatrix} \cdot \begin{vmatrix} H_1 \\ H_2 \\ . \\ . \\ H_n \end{vmatrix} = \begin{vmatrix} Q_1 \\ Q_2 \\ . \\ . \\ Q_n \end{vmatrix}$$

The calculation $K = \frac{R}{Q*}$ cycle is as follows:

- o Assume pressures or volume flows in all node or pipes
- o Calculate Q*
- o Solve the matrix
- o Calculate new pressures and restart cycle with calculating Q*
- o Repeat until the differences between the calculated pressures from the last step and the present step are below a certain threshold.

Advantage of the linear programming is the relative independence of initial estimations and the robustness of the calculation scheme.

Boundary conditions and input

The boundary conditions as demands and pressures induce the flows in a network. Normally the ‘downstream’ boundary is a demand and the ‘upstream’ boundary is an input or pressure delivered by a high level reservoir or a pump.

As demonstrated it is imperative that for every node one boundary condition is given, either be it a volume flow (supply to customers) or a pressure. Generally nodes in a network can be distinguished as supply nodes (outgoing flow) or as input node (incoming flow).

Looking at the Darcy-Weisbach formula presented as $H_1 - H_2 = R\,Q^2$ shows that three possibilities exist for an individual pipe:

- o The flow and one pressure are given; the other pressure is calculated. An example is a network fed by a high level reservoir. The flow are the supplies to customers, the pressure is the level in the high reservoir (Fig. 4.5).

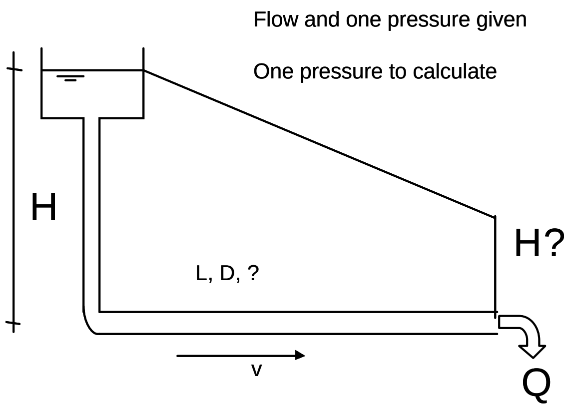

Fig. 4.5 - Flow and upstream pressure given; Down stream pressure to calculate

- o Two pressures are given; the flow is calculated. An example of this situation is for instance the pipe (or network) connecting two high level reservoirs (Fig. 4.6);

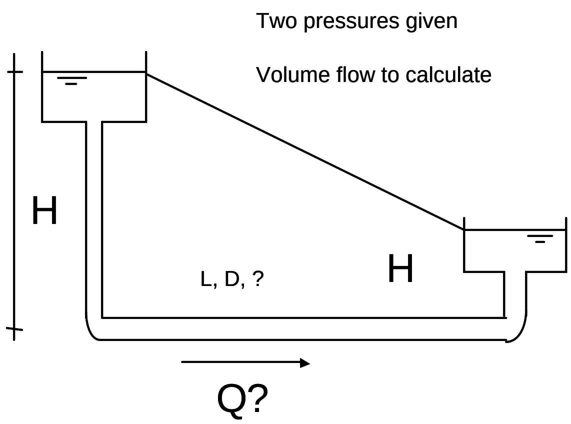

Fig. 4.6 - Two pressures given, flow to calculate

- o The flow and a Q-H relation. This is the most common network with a pump curve (see section 3.2.1) as input and flows as supply to customers (Fig. 4.7).

Following these obligatory boundaries there should be at least one pressure boundary (High level reservoir or pump curve) and one flow boundary (supply condition).

An input or supply for the network can be modelled as a pump. Basically a pump is treated in a network model as a pipe with a pressure increase instead of a pressure loss. The pump normally is modelled using a suction node and a pressure node. The con-

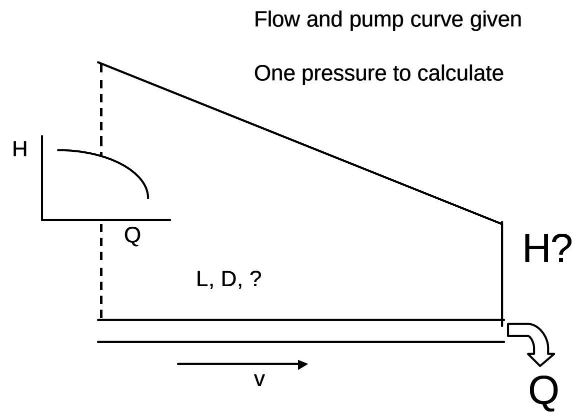

Fig. 4.7 - Flow and Q-H relation given pressure calculated from upstream QH relation, downstream pressure to calculate

necting pipe has the Q-H-relation in a function format.

Accuracy of the calculation

Lots of data are necessary to make a model of a (drinking water) network. All geometrical data as length, material, roughness, internal diameter, etc. will be drawn from registration systems. This can be either digital as computer databases or analogue as maps of the system.

Presently most drinking water companies have these data digitally available. The databases are made up from analogue maps, originally produced at the construction time of the pipes. This means that some of the information is decades or even a century old.

Besides the geometrical data the supply data needs special attention. As shown in lecture notes CT3420 it is difficult to get accurate data on supply, especially on a detailed level.

Although network calculations are static calculations: only one moment in time is calculated, the dynamics of the supply and demand is important. The situation in the night is completely different from the situation during the day.

In billing systems, the data on supply are available on an individual connection level but on a year basis. This is too detailed information and some way of data management is necessary to handle this into information on node level.

The governing formula (Darcy Weissbach) shows the relative importance of the data:

$$H_2 - H_1 = 0{,}0826\frac{l\,L}{D^5}Q^2$$

Assume the case that pressure H_1 has to be calculated. Input data are H_2, L, D, Q and λ (indirectly the roughness $k_{N)}$. Considered are the effects of a 10% error in the accuracy of each of the input data:

- 10% error in H2 gives a the same absolute error in H1:

$$1{,}1H_2 - 0{,}0826\frac{l\,L}{D^5}Q^2 =$$
$$H_2 - 0{,}0826\frac{l\,L}{D^5}Q^2 - 0{,}1H_2 = H_1 - 0{,}1H_2$$

$$0{,}9H_2 - 0{,}0826\frac{l\,L}{D^5}Q^2 =$$
$$H_2 - 0{,}0826\frac{l\,L}{D^5}Q^2 + 0{,}1H_2 = H_1 + 0{,}1H_2$$

- 10% error in L gives a 10% error in pressure drop over the pipe

$$H_2 - 0{,}0826\frac{l\,1{,}1L}{D^5}Q^2 =$$
$$H_2 - 0{,}0826\frac{l\,L}{D^5}Q^2 - 0{,}1*0{,}0826\frac{l\,L}{D^5}Q^2 = H_1 - 0{,}1\Delta H$$

$$H_2 - 0{,}0826\frac{l\,0{,}9L}{D^5}Q^2 =$$
$$H_2 - 0{,}0826\frac{l\,L}{D^5}Q^2 + 0{,}1*0{,}0826\frac{l\,L}{D^5}Q^2 = H_1 + 0{,}1\Delta H$$

- 10% error in internal diameter gives almost 50 - 70% error in pressure drop over the pipe

$$H_2 - 0{,}0826\frac{l\,L}{\left(1{,}1D^5\right)}Q^2 =$$
$$H_2 - \frac{1}{1{,}1^5}*0{,}0826\frac{l\,L}{D^5}Q^2 = H_1 + 0{,}48*\Delta H$$

$$H_2 - 0{,}0826\frac{l\,L}{\left(0{,}9D^5\right)}Q^2 =$$
$$H_2 - \frac{1}{0{,}9^5}*0{,}0826\frac{l\,L}{D^5}Q^2 = H_1 - 0{,}69*\Delta H$$

- 10% error in ë gives a 10% error in pressure drop over the pipe. A 10% error in ë is caused by an error in roughness of 20-30%

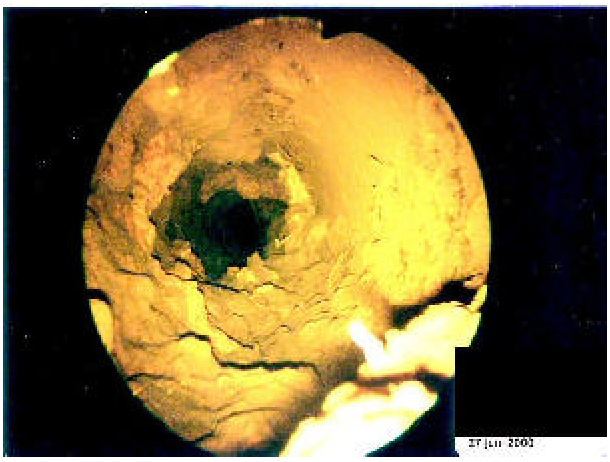

Fig. 4.8 - Encrusted cast iron pipe

Conclusion is that data on actual internal diameters is most important, followed by data on length and supply and on pressure boundaries. This is especially important in older cast iron pipes, as there is a large influence on the internal diameter of the voluminous encrustations.
As with many calculations the saying goes: garbage in = garbage out.

4.2.4 Time dependent calculations

The governing formula describing the flow in closed pressurised pipes is the Darcy-Weissbach equation. This formula has no time dependency in it, making it possible to make 'snap shot' calculations. In reality however there is a constant changing situation of demand boundary conditions. The demand varies over the day as shown in Fig. 3.21.The time scale of the changing is too short to take into account the inertia terms as is in the water hammer analysis (see chapter 2). The interest of water companies is to make an extended time analysis of the network on the time scale of hours or days. Most software allows for this type of analysis by automatically performing a series of calculations. The boundaries for the calculation are time dependent demands in nodes. The input boundaries are formed by pump curves and high-level reservoirs, allowing for changing conditions.
An extended time analysis is consequensive series of "snapshot" calculation. If supply boundaries are formed with pumpcurves no special measures are necssary.

A special place in these analyses is taken by high-level reservoirs. In the snap shot calculation the level of the reservoir is fixed and is treated as a fixed pressure point. The calculated volume flow in or out of the reservoir will change the level of the reservoir and thus the constant pressure. This changing level can be dealt with in several ways. Mostly a predictor-corrector method is used that calculates the situation at a certain calculation time point t0. The in- or outflow of the reservoir is calculated using the level of the reservoir at that time. During the time step, progressing to time t1, the in- or outflow will influence the level in the reservoir. Considering this in- or outflow to be constant over the time step this will give another level in the reservoir at the end of the time step: $H_1 = H_0 + \frac{Q\Delta t}{A}$ with Δt the time step and A the surface of the reservoir. Now a calculation can be made with an average of the H1 and H0. This is considered to be the actual height of the reservoir level over the time step Δt. Fig. 4.9 illustrates this.

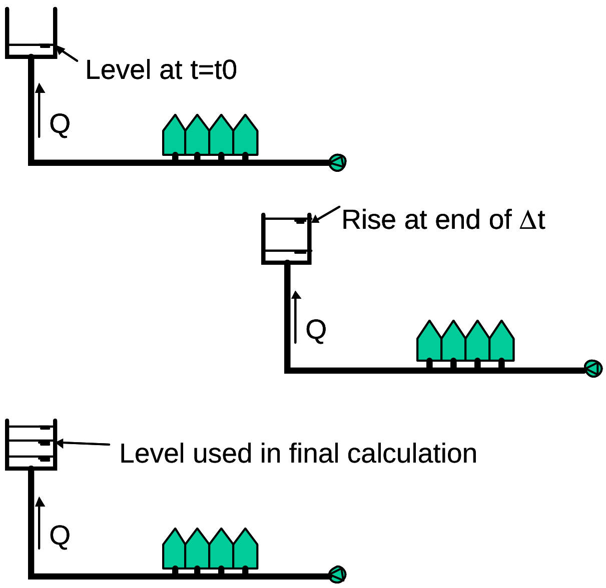

Fig. 4.9 - Water tower in extende time analysis

4.3 Models in urban drainage

4.3.1 Processes

As discussed earlier, a model is a description of reality, in this paragraph the focus is on mathematical models used in hydrodynamic calculations in urban drainage. This implies that the mathematical description of processes involved in the water movements in urban drainage systems is studied. When discussing these models a distinction has to be made in components constituting a model. The definitions as

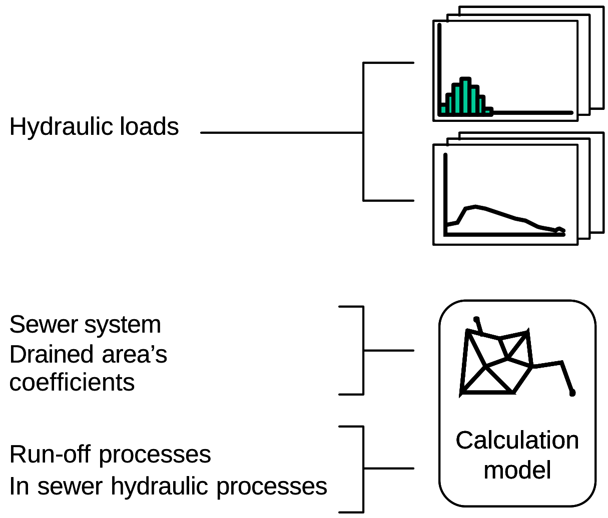

Fig. 4.10 - Components of a model in urban drainage

formulated by van Mameren & Clemens (1997) are used; a model is built up from three basic components (see Figure 4.10):

- A description of the hydraulic processes (mostly referred to as process model).
- A geometrical description of the system under study (the database or geometrical model).
- Hydraulic loads, making a distinction between dry weather conditions and storm conditions.

The hydraulic processes involved are usually distinguished into two main groups:

- The processes in the urban drainage system (the hydraulic model);
- The processes involved in the transformation from rain intensities to run-off (the hydrological model);

In fact, calculation models in urban drainage contain therefore two more or less separate process descriptions:

- The hydrological model (describing run-off processes).
- The hydraulic model.

This has implications for the calibration, since in most practical cases o nly water levels and/or discharges in the sewer system can be measured, implying that the runoff is not quantified as a separate quantity. In fact, the output of the hydrological model is the input for the hydraulic model, in this manner creating a time variable boundary condition.

Starting in the early 1970's software tools were developed in order to be able to handle the massive calculation effort involved in major hydrodynamic simulations on urban drainage networks.

4.3.2 Loads

When studying the processes taking place in an urban drainage system two modes are distinguished:

- Dry Weather Flow (DWF).
- Storm conditions.

In purely DWF systems, the DWF is the main contributing flow of water. Although in practical cases some storm water will enter the system due to either faulty connections or storm water from roofs connected to the system in order to create some regular flushing of the system, see e.g. Meijer (1998). On the other hand, in pure storm water systems only storm water will enter the system; in this case, also some DWF may enter the system due to mis-connections. In combined systems and improved separated systems, both sources of water are present. The DWF is a result of different sources of water:

- Domestic wastewater.
- Industrial waste water.
- Drain water due to leakage.

Domestic wastewater consists for the major part out of discharged drinking water after use for cooking, toilet flushing, washing etc. Therefore, the figures for water consumption are usually used in order to estimate the quantity of the DWF.
Drinking water consumption in households shows a periodic pattern in time, as does the actual DWF in a sewer system. However there is a phase shift and a difference in amplitude between them. This is because drinking water is temporarily stored in the households before it is discharged and because a certain portion of the drinking water does not enter the drainage system at all (due to e.g. evaporation and garden sprinkling). Therefore, the time patterns present in drinking water consumption should not be used for making accurate estimates of the DWF pattern.
Especially in older systems, the portion of drainage water can be a considerable fraction (up to 50%) of the DWF flow in a system. This is due to the combination of two factors:

- Leaking joints.
- High groundwater levels.

In cases in which leaking joints are present and the groundwater levels are beneath (part) of the system) DWF exfiltrates. This latter process is in practice hard to detect.

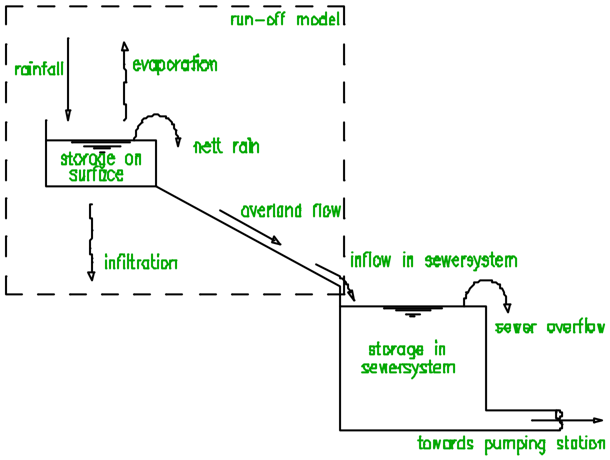

Fig. 4.11 - Processes involved in modelling urban drainage systems

Before entering the drainage system, storm water flows over the receiving areas like streets and roofs. The processes transforming rain intensity patters into inflow patterns in the drainage system are (see fig 4.11):

- Wetting of dry surface.
- Infiltration.
- Storage in local surface depressions.
- Evaporation.
- Flow over the receiving area.

This implies that the inflow intensity pattern deviates from the run-off intensity pattern, therefore when modelling an urban drainage system the pattern in the run-off intensities and the locations at which this storm water enters the systems must be known. To this end hydrological models are used. Run-off and its practical modelling is discussed later on.

4.4 Hydrological models

4.4.1 General

The transformation of rain falling on a surface into the actual amount of storm water being discharged into a drainage system is subject of extensive research over the last decades.
Theoretically, every process involved can be accurately described using a deterministic model. For practical use however, such an approach is inhibited due to the many model parameters involved and the large number of initial and atmospheric conditions that must be known. This will become clear in the next sections on the individual processes involved in the run-off process.

4.4.2 Storage in surface depressions and initial losses

When rain falls on a surface this will not result in an immediate discharge into the drainage system. When the surface is dry at the outset of the rain, initial losses will occur. These initial losses depend on the type of surface and the humidity and temperature of the surface at the start. A certain amount of the rain is caught in small local depressions in the pavement. Since it is virtually impossible to describe the geometry of the pavement for a whole catchment area in detail in the modelling practice, a constant value is used varying with the type of pavement.
Exact figures for initial losses and storage losses are scarcely found in literature. In Table results of various field data obtained from literature (see e.g. Pecher (1969), NWRW 4.3 (1989) and van de Ven (1989)) are summarised. Apart from the type of pavement, its state of maintenance is an important factor influencing the parameters for depression storage. Therefore, the magnitude of depression storage changes over time in a given catchment. Furthermore, it must be mentioned that water stored in depressions vanishes over time between successive storms due to evaporation and infiltration.
The available amount of surface storage at the start of an individual storm therefore depends on the history as well.

The state of maintenance of paved areas does have an influence of the surface storage capacity. In this sense, neglecting maintenance of roads is advantageous since it decreases the total hydraulic load on the receiving urban drainage system. Kidd (1978) related the surface storage to the terrain slope by:

$$b = 0.77 i_t^{-0.49}$$

Table 4.1 - Initial losses and surface storage

	Flat roofs	Tilted roofs	Impervious road areas	Semi-impervious roads
Initial losses	0-0.5 mm	0.1 mm	0.07-0.7 mm	0-1.5 mm
surface storage	2-2.5 mm	0.1 mm	0.3-1.7 mm	0.8-6.0 mm

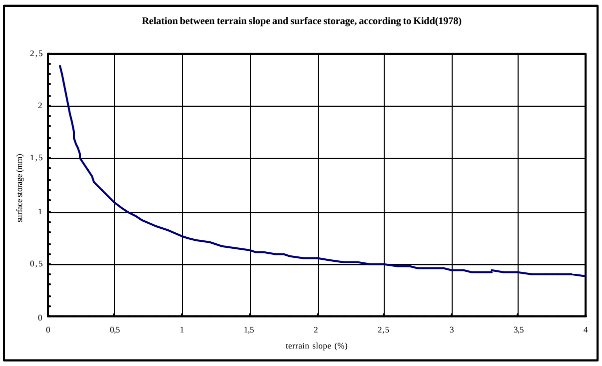

Fig. 4.12 - Relation between terrain slope and surface storage (Kidd 1978)

In which:

b surface storage (mm)

i_t terrainslope (%)

4.4.3 Evaporation

A relevant process in relation to run-off is evaporation. Surface storage is made available due to infiltration and evaporation. The evaporation rate depends on several variables:

- Temperature.
- Wind speed.
- Atmospheric humidity.
- Rate of heat influx.
- Intensity of sunshine.
- Colour of the surface.

The variables mentioned are in general not known in any detail when modelling an urban drainage system; therefore monthly average figures are normally applied (the so-called Penman evaporation (Raudkivi (1979)).

In his research into run-off models, Van de Ven (1989) concluded that it is impossible to quantify the evaporation term for individual storms. Therefore, a practical approach is usually adopted. This implies that the evaporation either is neglected during a storm or is set to a constant value equal to that of an open water surface. In fig 4.13 the monthly-averaged evaporation-rate values for De Bilt in the Netherlands are shown.

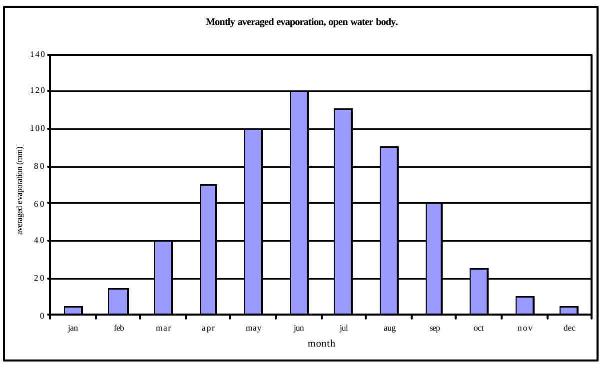

Fig. 4.13 - Monthly averaged evaporation values, de Bilt, period 1955-1979

4.4.4 Infiltration

Rain falling on pervious or semi-pervious area (e.g. grass, clinkers) will partly infiltrate into the groundwater. The process of infiltration is complicated; depending of the initial conditions the infiltration rate will decrease with time because the unsaturated zone is filled (see e.g. Mein & Larson (1973)). As soon as this zone has become saturated, the minimum infiltration rate is reached. The infiltration capacity of the soil increases only after the precipitation has stopped and the storage in the unsaturated zone has been emptied. In literature several values for infiltration rates are reported (see e.g. Ando (1984), Bebelaar & Bakker (1981), de Roo (1982), van Dam & Schotkamp (1983) and van de Ven (1989)), in table 4.2 the ranges found are shown.

Table 4.2 - Ranges for the infiltration capacity obtained from literature

	infiltration values (mm/h)
Concrete clinkers	7-353
Tiles	1-254
Grass	10-500
soil without vegetation	10-100

Several models are applied in practice to describe the infiltration process:

Hillel&Gardner (1970): $I_{cum} = \sqrt{at+b} - c$

In which:

I_{cum} cumulative infiltration since t=0 in mm

a,b,c parameters depending on transmissivity, humidity and crust resistance.

Philip : $I_{cum} = a\sqrt{t} + bt$

In which:

I_{cum} cumulative infiltration since t=0 in mm

a,b,c parameters depending on transmissivity, humidity and crust resistance.

A widely accepted model is the model suggested by Horton (1940). In this model, it is assumed that a maximum and minimum value limit the infiltration rate. When infiltration starts at a given rate it will decrease with time due to saturation of the soil. Eventually it will reach a minimum value when the storage capacity of the pores is filled. As soon as the surface area is dried up (due to evaporation and infiltration), this storage capacity becomes available again. This implies the infiltration rate is increasing again with time. These processes are described by the following formulas

Decrease: $$f(t) = f_e + (f_b - f_e)e^{-k_d t}$$

Increase: $$f(t) = f_b - (f_b - f_e)e^{-k_d t}$$

This model contains four parameters, the maximum infiltration capacity f_b (assuming the unsaturated zone is fully available), a minimum infiltration capacity f_e (the storage in the unsaturated zone is filled) and the recession factors k_a and k_d. Basically, these values depend on the type of soil and the momentary groundwater level.

A very simple model is the constant infiltration rate model. In this model the infiltration rate is set to a constant; the value depends only on the characteristics of the particular surface.
Van de Ven (1989) made a comparison of several infiltration models based on in-situ measurements. He concluded that the models as defined by Hillel & Gardner (1970), Horton (1940) and Philip did not show significant differences. Furthermore, he concluded that the constant infiltration rate model was less accurate than the other models. However, when studying the reported experimental results, the constant rate model could be used for practical purposes according to van de Ven. This is mainly of importance since this simple model calls for only one parameter to be estimated, making calibration in practice simpler and enhancing the reliability of the parameter values obtained in a calibration. Intable 4.3, the value for R^2 (model efficiency in a comparison with field measurements) is tabulated for the infiltration models mentioned. As can be seen the Hillel&Gardner model gives the best results, the differences however are relatively small.

Furthermore, it is argued that since exact values for evaporation cannot be given, a very refined model for infiltration is of academic value only. In relation to the hydraulic load on an urban drainage system, it is only of importance to have an estimate for the net-rain and an estimate for the available storage capacity on surface areas.

4.4.5 Run-off

Once the amount of rain that has fallen becomes larger than the sum of the initial losses, the losses in local depressions and the evaporation, run-off occurs. So, the moment at which after the start of the storm water starts to run-off (t_s) is calculated from the integral equation:

$$\int_{t=0}^{t=t_s} (r(t) - i(t) - e(t))dt = S + W$$

In which:

S	the surface storage in mm
W	the initial loss in mm
r(t)	the rain intensity as a function of time
i(t)	the infiltration rate as a function of time
e(t)	the evaporation as a function of time

So, for $t>t_s$ run-off to the drainage system occurs if $r(t)>i(t)+e(t)$. The amount of rain resulting in run-off is defined as the net-rain-intensity $p_n(t)=r(t)-i(t)-e(t)$.

Several models are developed to describe the transformation from $p_n(t)$ into the actual discharge entering a drainage system q(t). Some of the most widely used models will be discussed briefly in the annex 4.4.

A practical comparison of some of the models discussed are summarised in figure 1

Table 4.3 - R^2 values for several infiltration models reported by van de Ven (1989)

	Hillel&Gardner	Philip	Horton	Constant
Copper slug	0.993	0.988	0.977	0.966
Concrete clinker	0.993	0.992	0.997	0.954
Concrete tiles	0.995	0.991	0.989	0.97

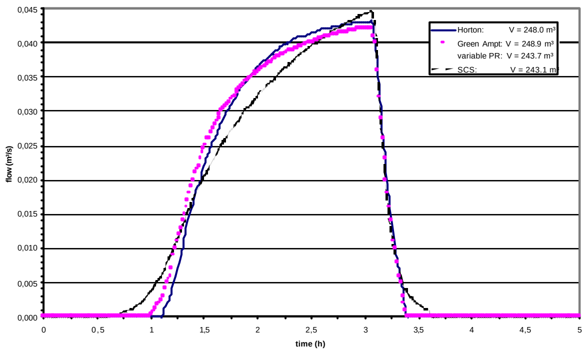

Fig. 4.14 - Some results for different run-offs models imposing a block storm

From these results the following practical conclusions are draw:

- All models show a more or less equal characteristic response to the storm imposed
- Simple models (like the NWRW 4.3 model) perform reasonably well, while posing the advantage of being simple to implement.

When comparing several complete run-off models (i.e. surface storage model + infiltration model + Routing model) with respect to the number of parameters to be specified per area the following figures are obtained (see table 4.4)

From a practical point of view, a model with the smallest number of parameters is to be preferred. This is even more so when one realises that some parameters (like e.g. 'n' and 'k' for the Nash cascade) cannot be quantified from simple measurements since they represent no physically meaning-full parameters. In this sense run-off models are to be regarded as black or grey box models. The notion of using preferably measurable parameters and models with a minimum number of parameters is also supported when calibrating models. A reduction of the number of parameters in a calibration process increases the quality of the calibrated result. Furthermore, when using models having physical meaningful parameters it is possible to crosscheck the parameter values obtained from a calibration with independent separate measurements.

Table 4.4 - Number of parameters in several models

	Model parameters	Numerical parameters	Empirical parameters
Horton	4	1	
Nash	2	1	
Kidd	1		2
Hillel&Gardner	3	1	
Philip	2	1	
Desbordes	9	2	2

Annex 4.1

Method Hardy-Cross: pressure equalisation method

Consider a node called '1' with four pipes (2-5). The demand at node 1 is Q_1.

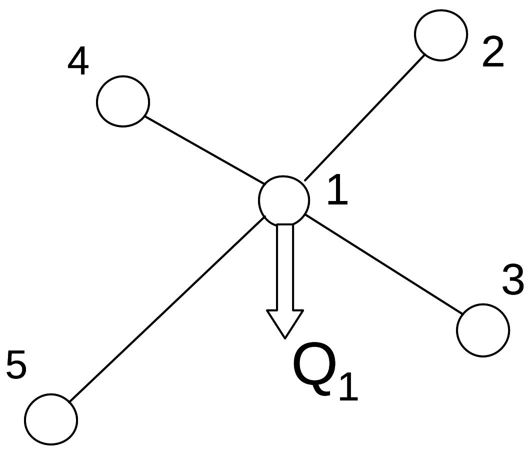

Fig. A4.1 Node 1

Applying the first Kirchhoff law gives:

$$\sum_{j=2}^{5} (Q_{1j}) - Q_1 = 0$$

Darcy Weissbach gives $\Delta H = RQ|Q|$

Rewriting Darcy Weissbach gives

$$Q = sign(\Delta H) \bullet R^* \bullet \sqrt{|\Delta H|}$$

With $R^* = \sqrt{\frac{p^2 g}{8L} \frac{D^5}{l}}$

R^* is a factor dependant of the characteristics of the pipe as length, diameter and roughness and must be determined iteratively.

Assume now a start value for a pressure in every node. The pressure in node 1 has to be corrected with Δp.
Substitution of the rewritten Darcy-Weissbach in the Kirchhoff equation gives

$$\sum_{j=2}^{5} \left[sign(\Delta H) \bullet R^*_{1j} \bullet \sqrt{\left(|H_1 - H_j| - \Delta p\right)} \right] - Q = 0$$

From this equation Äp can be solved using the Taylor series. In general terms this is written as

$$f(x + \Delta x) = f(x) + \Delta x \bullet f'(x) = \frac{(\Delta x)^2}{2!} f''(x)$$

If $f(x + \Delta x) = \sqrt{(x + \Delta x)}$

than $\sqrt{(x + \Delta x)} = \sqrt{x} + \Delta x \frac{1}{2\sqrt{x}}$

Applied for the Kirchhoff equation this gives

$$\sum_{j=2}^{5} \left[sign(\Delta H) \bullet R^*_{1j} \bullet \sqrt{|H_1 - H_j|} - \Delta p \frac{sign(\Delta H) \bullet R^*_{1j}}{2\sqrt{|H_1 - H_j|}} \right] - Q_1 = 0$$

or

$$\Delta p = \frac{\sum_{j=2}^{5} \left[sign(\Delta H) \bullet R^*_{1j} \bullet \sqrt{|H_1 - H_j|} \right] - Q_1}{\sum_{j=2}^{5} \left[\frac{sign(\Delta H) \bullet R^*_{1j}}{2\sqrt{|H_1 - H_j|}} \right]}$$

With this calculated Äp a new estimation can be made and the new cycle can start until the value of Δp drops below a certain threshold.

Annex 4.2

Method Hardy-Cross: volume flow equalisation method

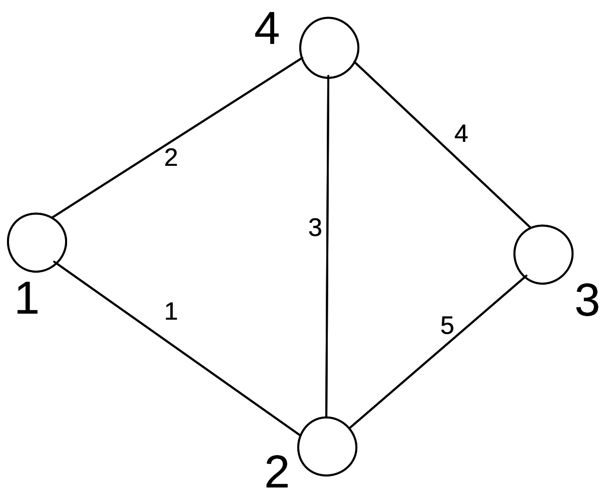

Fig. A4.2 - Loop

Consider a network with 4 nodes and 5 pipes.

In the loop 1-2-3 the second law of Kirchhoff states

$$\sum_{i=1}^{3} [\text{pressure drops}] = 0$$

$$\sum_{i=1}^{3} \left[\mathrm{R_i} Q_i \left| Q_i \right| \right] = 0$$

with

$$R_i = \frac{8 \bullet L_i}{p^2 \bullet g} \frac{l_i}{D_i^5}$$

Assume now a start value for a pressure in every node. Following these pressures the volume flow in each pipe can be calculated. This flow has to be corrected with ΔQ.
The second law of Kirchhoff than becomes

$$\sum_{i=1}^{3} \left[\mathrm{R_i} \left(Q_i - \Delta Q \right) \left| Q_i - \Delta Q \right| \right] = 0$$

This gives two possibilities for further evaluation

$$(Q_i - \Delta Q_i) \bullet |Q_i - \Delta Q_i| = Q_i^2 - 2Q_i \Delta Q + (\Delta Q)^2 \text{ when } Q_i \geq \Delta Q$$

$$(Q_i - \Delta Q_i) \bullet |Q_i - \Delta Q_i| = -Q_i^2 - 2Q_i \Delta Q - (\Delta Q)^2 \text{ when } Q_i \leq \Delta Q$$

Only the first equation is further elaborated in the second law of Kirchhoff. The second one can be evaluated similarly.

$$R_1 Q_1^2 - 2R_1 Q_1 \Delta Q + R_1 (\Delta Q)^2 + R_2 Q_2^2 - 2R_2 Q_2 \Delta Q + R_2 (\Delta Q)^2 +$$
$$R_3 Q_3^2 - 2R_3 Q_3 \Delta Q + R_3 (\Delta Q)^2 = 0$$

When the iteration process have progressed, the terms $(\Delta Q)^2$ will become negligible, leaving the equation as:

$$\sum_{i=1}^{3} R_i Q_i^2 - 2 \bullet \Delta Q \bullet \sum_{i=1}^{3} R_i Q_i = 0$$

and

$$\Delta Q = \frac{\sum_{i=1}^{3} R_i Q_i^2}{2 \sum_{i=1}^{3} R_i Q_i}$$

In general terms this becomes

$$\Delta Q_j = \frac{\sum_{i=1}^{n} R_{ij} Q_{ij}^2}{2 \sum_{i=1}^{n} R_{ij} Q_{ij}}$$

Nota bene:
this elaboration is only valid for $Q_{ij} \geq \Delta Q_j$.
For $Q_{ij} \leq \Delta Q_j$ the elaboration is similar.

With the calculated ΔQ_j for each pipe the new estimations for Q can be made and the process can start again. The cycle is repeated until the largest ΔQ_j is smaller than a certain threshold.

Annex 4.3: Linear programming

Darcy-Wessbach's equation can be written as

$$\Delta H_{jn} = \frac{Q_{jn}\left|Q_{jn}\right|}{R}$$

with

$$\Delta H_{jn} = H_j - H_n \text{ and } R = \frac{p^2 g}{8L}\frac{D^5}{l}$$

If an estimation is made for a value for Q^*_{jn} the Darcy Weissbach formula will be linearalised in

$$\Delta H_{jn} = Q_{jn}\frac{\left|Q^*_{jn}\right|}{R}$$

or

$$Q_{jn} = \Delta H_{jn} K_{jn} \text{ with } K_{jn} = \frac{R}{Q^*_{jn}}$$

This linearised Darcy Weissbach equation can be solved with matrix techniques. To elaborate this we consider the node n surrounded with m pipes and a demand Qn.
The first law of Kirchhoff states:

$$\sum_{j=1}^{m}\left(Q_{jn}\right) - Q_n = 0 \text{ with}$$

Qjn : Volume flow from node j to considered node n
Qn : Demand (or supply) in considered node n
m : number of pipes connected to node n

Substituting the linearised Darcy Weissbach in the first law of Kirchhoff gives

$$\sum_{j=1}^{m}\left(\Delta H_{jn} \bullet K_{jn}\right) - Q_n = 0$$

with $\Delta H_{jn} = H_j - H_n$ this derives to (see text box)

$$\sum_{j=1}^{m}\left(H_j \bullet K_{jn}\right) - H_n \sum_{j=1}^{m} K_{jn} = Q_n$$

For each node this equation can be written up in the general form:

$$\begin{vmatrix} -\sum_{j=1}^{m} K_{j1} & K_{21} & . & . & K_{n1} \\ K_{12} & -\sum_{j=1}^{m} K_{j2} & . & . & K_{n2} \\ . & . & . & & . \\ . & . & & . & . \\ K_{1n} & K_{2n} & . & . & -\sum_{j=1}^{m} K_{jn} \end{vmatrix} \cdot \begin{vmatrix} \Delta H_1 \\ \Delta H_1 \\ . \\ . \\ \Delta H_1 \end{vmatrix} = \begin{vmatrix} Q_1 \\ Q_2 \\ . \\ . \\ Q_3 \end{vmatrix}$$

The lines in the matrix are only filled at m points, being the characteristics of the connecting pipes to the adjacent nodes. If a node is a connection point of 5 pipes, the line is filled at 5 points and the diagonal.
The matrix is symmetric in the diagonal because K_{jn} equals K_{jn}:

$$K_{jn} = \frac{R_{jn}}{\left|Q^*_{jn}\right|} \text{ and } R_{jn} = \frac{p^2 g}{8L}\frac{D^5}{l_{jn}} = R_{jn}$$

Consider a node n with four pipes. The value of m is 4.
The first Kirchhoff law becomes

$$Q_{1n} + Q_{2n} + Q_{3n} + Q_{4n} - Q_n = 0$$

Linerarisation gives

$$\Delta H_{1n} \bullet K_{1n} + \Delta H_{2n} \bullet K_{2n} +$$
$$\Delta H_{3n} \bullet K_{3n} + \Delta H_{4n} \bullet K_{4n} = Q_n$$

With $\Delta H_{jn} = H_j - H_n$ this becomes

$$\left(H_1 - H_n\right) \bullet K_{1n} + \left(H_2 - H_n\right) \bullet K_{2n} +$$
$$\left(H_3 - H_n\right) \bullet K_{3n} + \left(H_4 - H_n\right) \bullet K_{4n} = Q_n$$

Ordering to H_1, H_2, H_3 and H_4 gives:

$$H_1 \bullet K_{1n} + H_2 \bullet K_{2n} + H_3 \bullet K_{3n} + H_4 \bullet K_{4n} -$$
$$H_n\left(K_{1n} + K_{2n} + K_{3n} + K_{4n}\right) = Q_n$$

or

$$\sum_{j=1}^{4}\left(H_j \bullet K_{jn}\right) - H_n \sum_{j=1}^{4} K_{jn} = Q_n$$

Because $Re_{jn} = Re_{nj}$ and thus $I_{jn} = I_{nj}$ gives

$$\left|Q_{jn}^{*}\right| = \left|Q_{nj}^{*}\right|$$

This all leads to $K_{jn} = K_{jn}$

The symmetry of the matrix is convenient because of the simplicity of solving methods. Also the matrix can be used to check the hydraulic validity of the network. If a line in the matrix is only filled at the diagonal, than the corresponding node is not connected to the system.

Annex 4.4

The Nash-model

The Nash model or Nash-cascade, (see Nash & Sutchiffe (1970)) is a cascade of n identical linear reservoirs (Figure), the momentary value for q(t) is calculated using the following convolution between net-rain intensity and a transfer function h(t):

$$q(t) = \int_0^t h(t-t\,)\,p_n(t\,)\,dt$$

The transfer function h(t) is defined as:

$$h(t) = \frac{1}{k\Gamma(n)}\left[\frac{t}{k}\right]^{n-1} e^{\frac{-t}{k}}$$

In which:

k	reservoir constant
n	number of reservoirs
t	time
p(t)	net-rain intensity as function of time
q(t)	run-off discharge as function of time
G(n)	the gamma function of n (if n is a integer then G(n)=n!)

This model contains two parameters, due to the definition it is possible to define a cascade with a non-integer number of reservoirs.

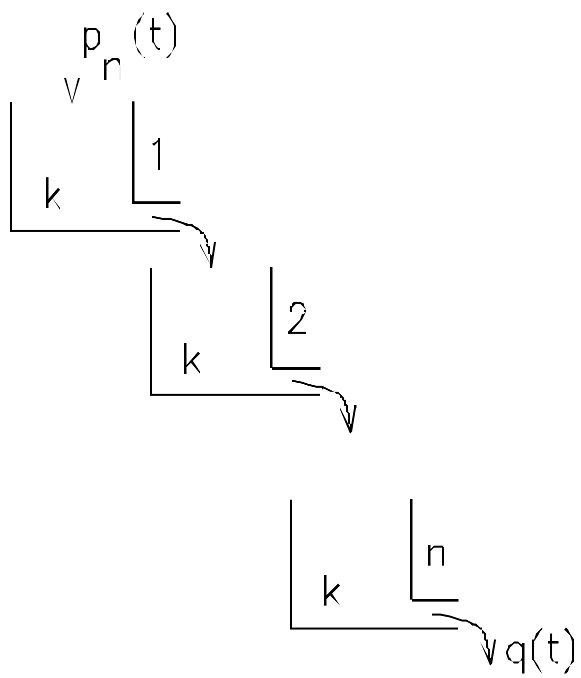

Fig. A4.3 - The Nash cascade

Non-linear reservoir model (NLR-model)

The non-linear reservoir model is defined by:

$$q(t) = Kp_n(t)^b$$

In which

q(t)	Run-off
K	Reservoir constant
$p_n(t)$	Netto-rain
b	Power

If the parameter b is equal to unity then this model is equal to a Nash-model with n=1. The non-linear model is often applied with a value for b of 2/3. This is based on the application of the stationary, uniform equation of water motion in one dimension. This parameter value however, is only of theoretical importance and does most of the time not apply to practical cases.

The values for the constants in the family of reservoir models have been subject to a wide spectrum of researches. As an example, a model known as the Desbordes model is briefly discussed. This model has been proposed by Desbordes and is linear due to that fact that b=1. The value of K however is defined as:

$$K = K_{Desb}\; A_r^{\,0.18}\; P_{nt}^{\,-0.36}\; (1+C)^{-1.9}\; T_3^{\,0.21}\; L^{0.15}\; H_{pe}^{\,-0.07}$$

In which:

K_{Desb}	proportionality constant depending on the type of surface	(-)
A_r	sub-catchment area	(ha)
P_{nt}	sub-catchment slope	(%)
C	the proportion of sub-catchment area that is impermeable (between 0 and 1)	(-)
T_3	the duration of the rainfall sub event	(s)
L	sub-catchment length	(m)
H_{pe}	total accumulated effective rainfall for the rainfall sub-event	(m)

The value for K depends on characteristics of the area under consideration but also on the characteristics of the storm event in terms of duration and accumulated rain.

Apart from these generally applied models, the Volterra model and the Laguerre model are also known, these model a detailed description of these models is found in van der Kloet & van de Ven (1981).

These models however, pose the problem for practical applications that a large number of model parameters are to be estimated (in some cases up to 10). As will be seen in the chapter on calibration, an increase in the number of the model parameters has a negative effect on the process of calibration as well as on the reliability of the calibrated model parameters. Therefore the Volterra en Laguerre model are not discussed in this thesis.

An intensive research done by van de Ven (van de Ven, 1989)) has shown that the differences between run-off models are only marginal in practice. In table A4.1some of his results obtained from field measurement are shown.

The power b in the non-linear reservoir model is close to unity, implying a linear model can be used. In the Netherlands a standard model (NWRW 4.3 (1989)) has been chosen for practical implementation. This model is in its essence built up from:

- A surface depression storage and initial losses model.
- An infiltration model according to Horton.
- A single linear reservoir model.
- A simple evaporation model using an average evaporation value varying per month (the so called Penmann evaporation).

Table A4.1 -Some parameter values for the Nash model and the non-linear model (NLR) b in mm, k for the NLR in mm^{1-b} min^{-1}

	Nash 'n'	Nash 'k'	NLR b	NLR k
Municipal area	0.65	415	1.02	5.6
Parking lot	1.05	225	1.07	4.0