OE4625 Dredge Pumps and Slurry Transport

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October 13, 2004

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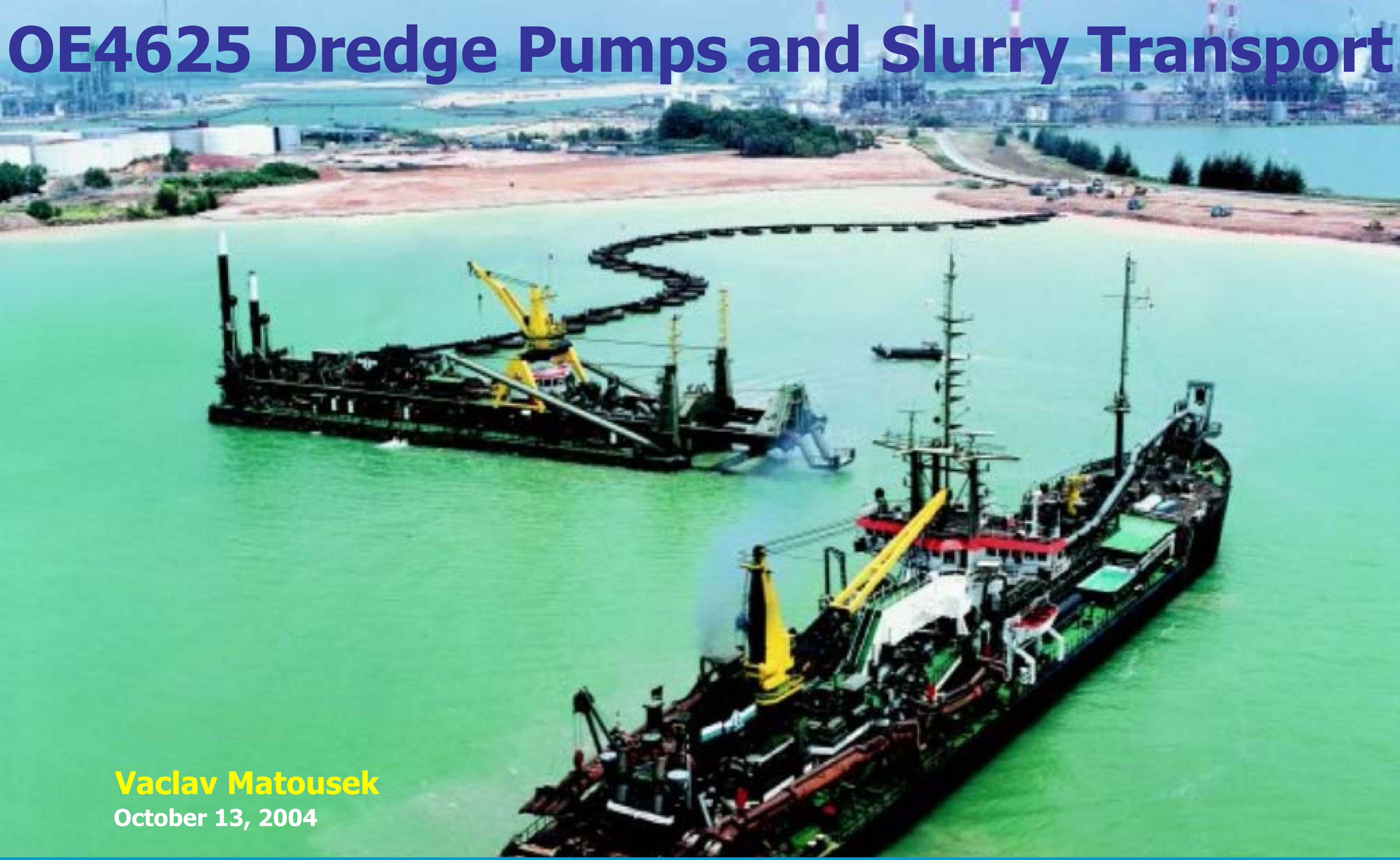

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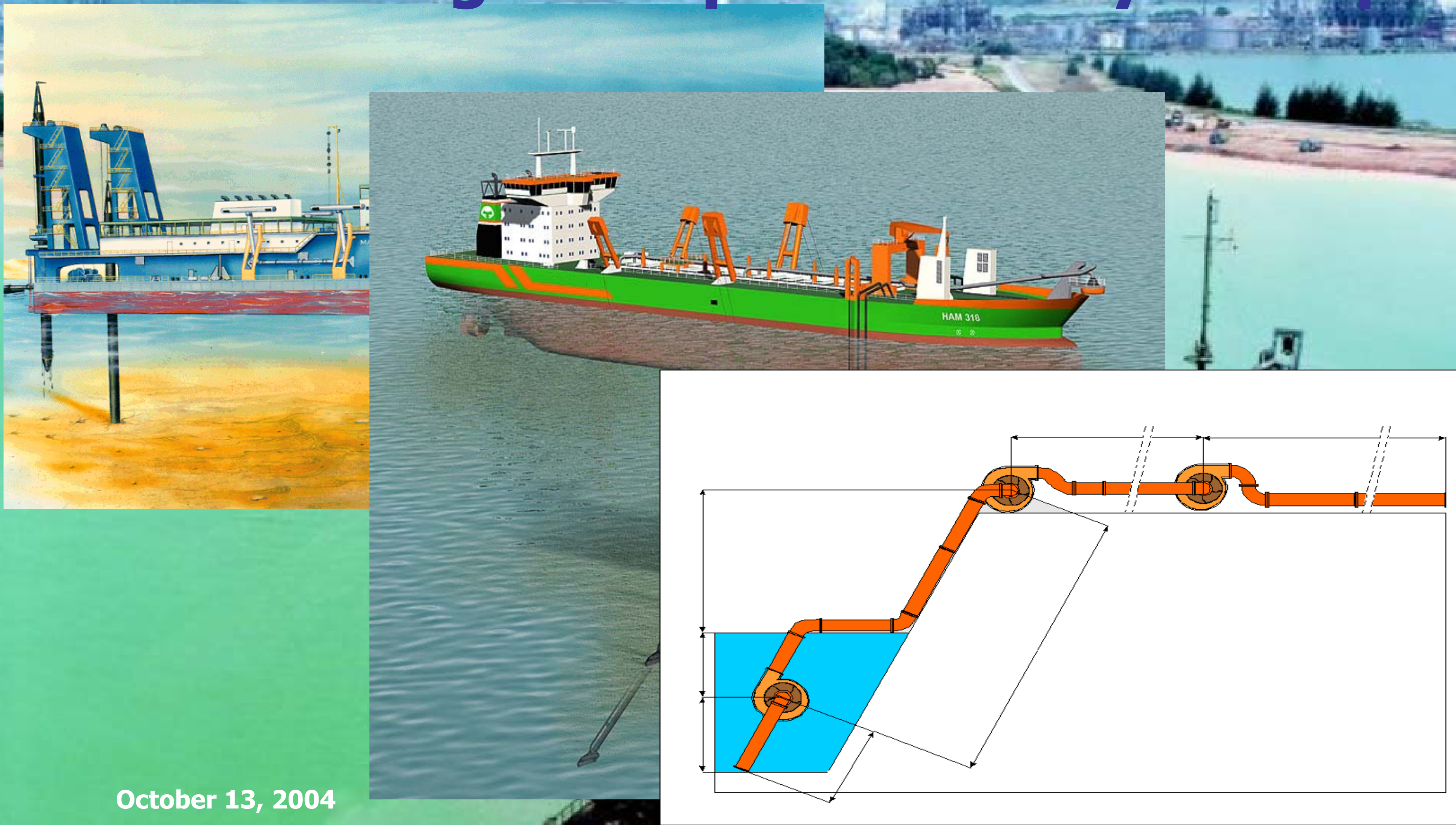

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Keywords:

Transport (*Horizontal, Vertical, Inclined*)

Pipe (*Length, Diameter*)

Pump (*Type, Size*)

Goals:

Design a pipe of appropriate size

Design pumps of appropriate size;
Determine a number of required pumps

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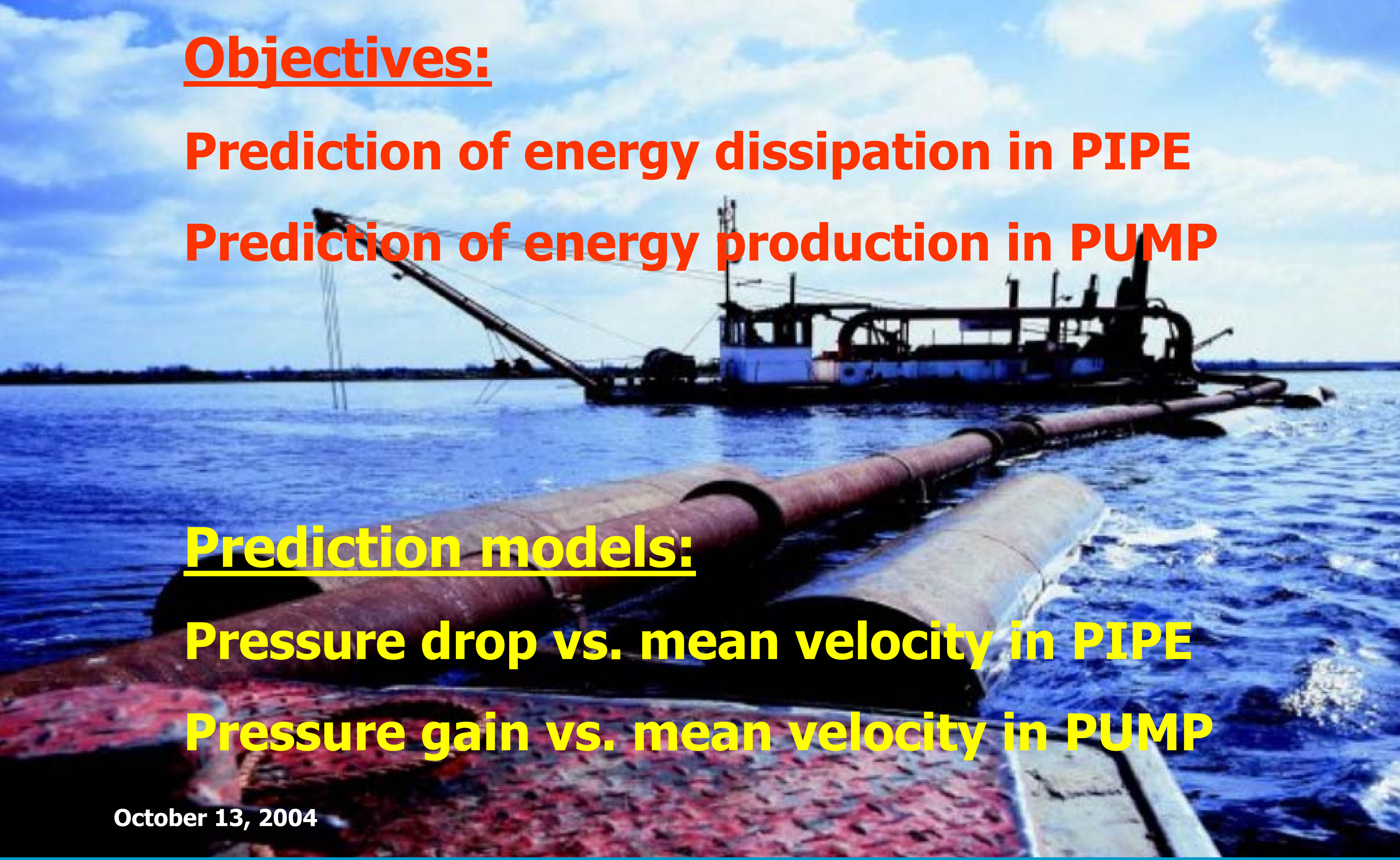

Objectives:

Prediction of energy dissipation in PIPE

Prediction of energy production in PUMP

Prediction models:

Pressure drop vs. mean velocity in PIPE

Pressure gain vs. mean velocity in PUMP

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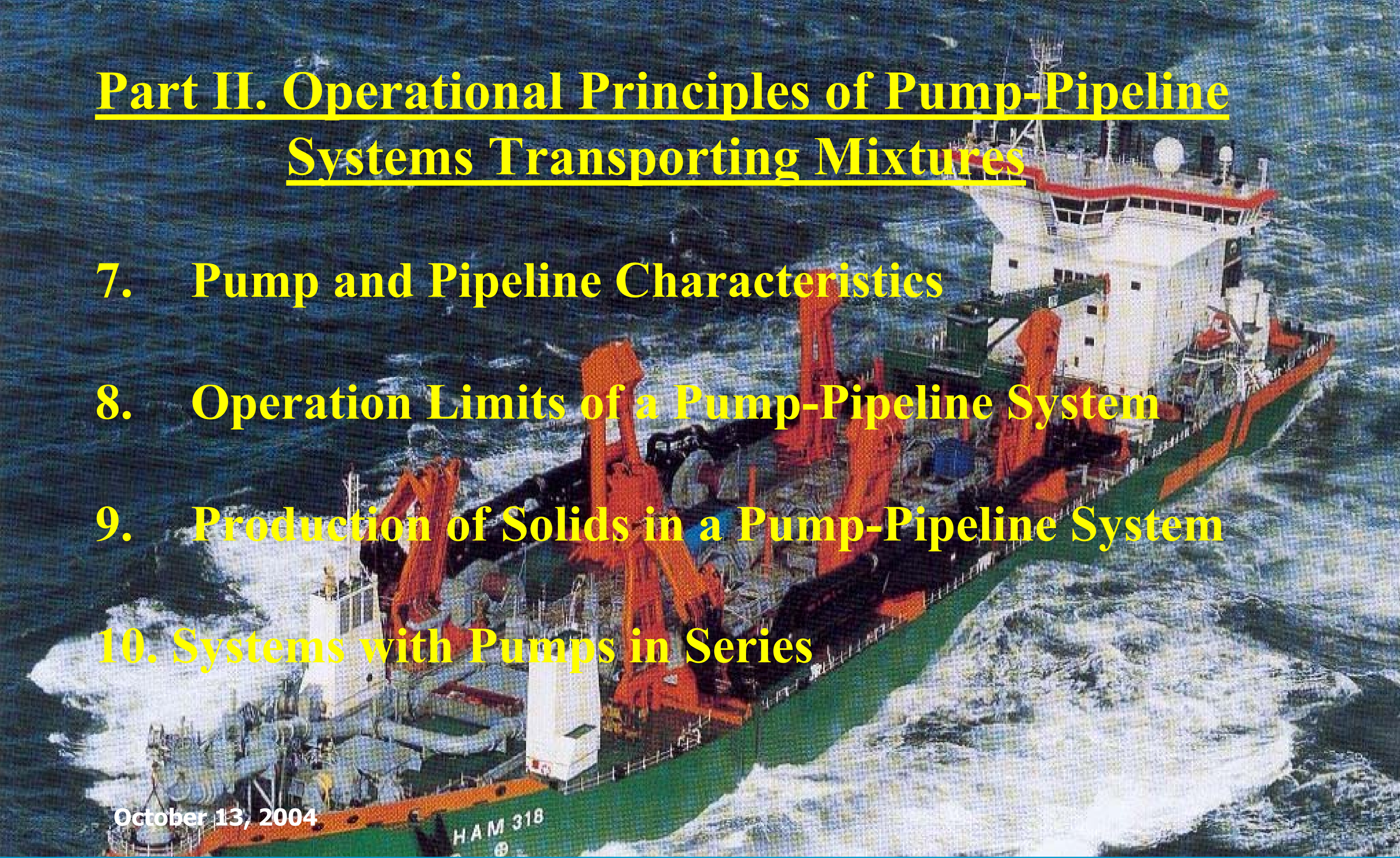
Part I. Principles of Mixture Flow in Pipelines

1. Basic Principles of Flow in a Pipe
2. Soil-Water Mixture and Its Phases
3. Flow of Mixture in a Pipeline
4. Modeling of Stratified Mixture Flows
5. Modeling of Non-Stratified Mixture Flows
6. Special Flow Conditions in Dredging Pipelines



Part II. Operational Principles of Pump-Pipeline Systems Transporting Mixtures

7. Pump and Pipeline Characteristics
8. Operation Limits of a Pump-Pipeline System
9. Production of Solids in a Pump-Pipeline System
10. Systems with Pumps in Series

1. BASIC PRINCIPLES OF FLOW IN PIPE

CONSERVATION OF MASS

CONSERVATION OF MOMENTUM

CONSERVATION OF ENERGY

Conservation of Mass

Continuity equation for a control volume (CV):

$$\frac{d\left(mass\right)}{dt}=\sum\left(q_{outlet}-q_{inlet}\right) \qquad [kg/s]$$

q [kg/s] ... Total mass flow rate through all boundaries of the CV

Conservation of Mass

Continuity equation in general form:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.\left(\rho \vec{V}\right) = 0$$

For incompressible (ρ = const.) liquid and steady flow (ə/ət = 0) the equation is given in its simplest form

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Conservation of Mass

The physical explanation of the equation is that the mass flow rates $q_m = \rho VA$ [kg/s] for steady flow at the inlets and outlets of the control volume are equal.

Expressed in terms of the mean values of quantities at the inlet and outlet of the control volume, given by a pipeline length section, the equation is

$$\mathbf{q_m = \rho VA = const.}\ \text{[kg/s]}$$

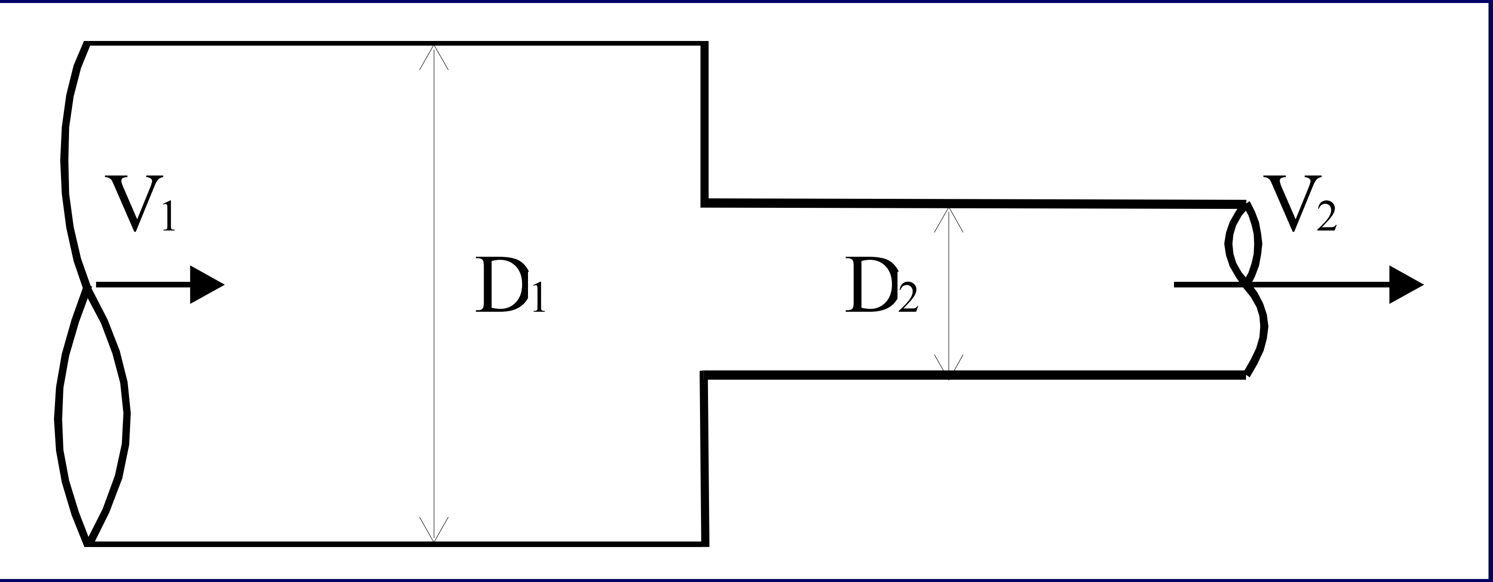
thus

$$\mathbf{(\rho VA)_{inlet} = (\rho VA)_{outlet}}$$

Conservation of Mass in 1D-flow

For a circular pipeline of two different diameters D_1 and D_2

$$V_1 D_1^2 = V_2 D_2^2 \text{ [m}^3\text{/s]}$$



Conservation of Momentum

Newton's second law of motion:

$$\frac{d\left(momentum\right)}{dt} = \sum F_{external}$$

The *external forces* are

- body forces due to external fields (gravity, magnetism, electric potential) which act upon the entire mass of the matter within the control volume,
- surface forces due to stresses on the surface of the control volume which are transmitted across the control surface.

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Conservation of Momentum

In an *infinitesimal control volume* filled with a substance of density the force balance between inertial force, on one side, and pressure force, body force, friction force, on the other side, is given by a differential linear momentum equation in vector form

$$\rho \frac{D\vec{V}}{Dt} = \frac{\partial}{\partial t}\left(\rho \vec{V}\right) + \rho \vec{V}.\vec{\nabla}\vec{V} = -\vec{\nabla}P - \rho g \vec{\nabla}h - \vec{\nabla}.\vec{T}$$

Claude-Louis Navier

George Stokes

Conservation of Momentum

Navier-Stokes' Equations (in Vector Form):

Mass Conservation (Continuity):

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \left(\rho \vec{V} \right) = 0$$

Momentum Conservation:

$$\frac{\partial}{\partial t}\left(\rho \vec{V} \right) + \vec{V} \cdot \vec{\nabla}\left(\rho \vec{V} \right) = -\vec{\nabla} p + \mu \vec{\nabla}^2 \vec{V} + \rho \vec{a}$$

Transient — Advective — Pressure Gradient — Diffusion — External Acceleration

Navier-Stokes' Equations in Cartesian Co-ordinates:

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

x-Momentum:

$$\frac{\partial (\rho u)}{\partial t} + u\frac{\partial (\rho u)}{\partial x} + v\frac{\partial (\rho u)}{\partial y} + w\frac{\partial (\rho u)}{\partial z} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho a_x$$

y-Momentum:

$$\frac{\partial (\rho v)}{\partial t} + u\frac{\partial (\rho v)}{\partial x} + v\frac{\partial (\rho v)}{\partial y} + w\frac{\partial (\rho v)}{\partial z} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \rho a_y$$

z-Momentum:

$$\frac{\partial (\rho w)}{\partial t} + u\frac{\partial (\rho w)}{\partial x} + v\frac{\partial (\rho w)}{\partial y} + w\frac{\partial (\rho w)}{\partial z} = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \rho a_z$$

Conservation of Momentum in 1D-flow

In *a straight piece of pipe of the differential distance dx (1D-flow)*, quantities in the equation are averaged over the pipeline cross section:

$$\rho\left(\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial h}{\partial x}\right)+\frac{\partial P}{\partial x}+4\frac{\tau_o}{D}=0$$

Conservation of Momentum in 1D-flow

For *additional conditions* :

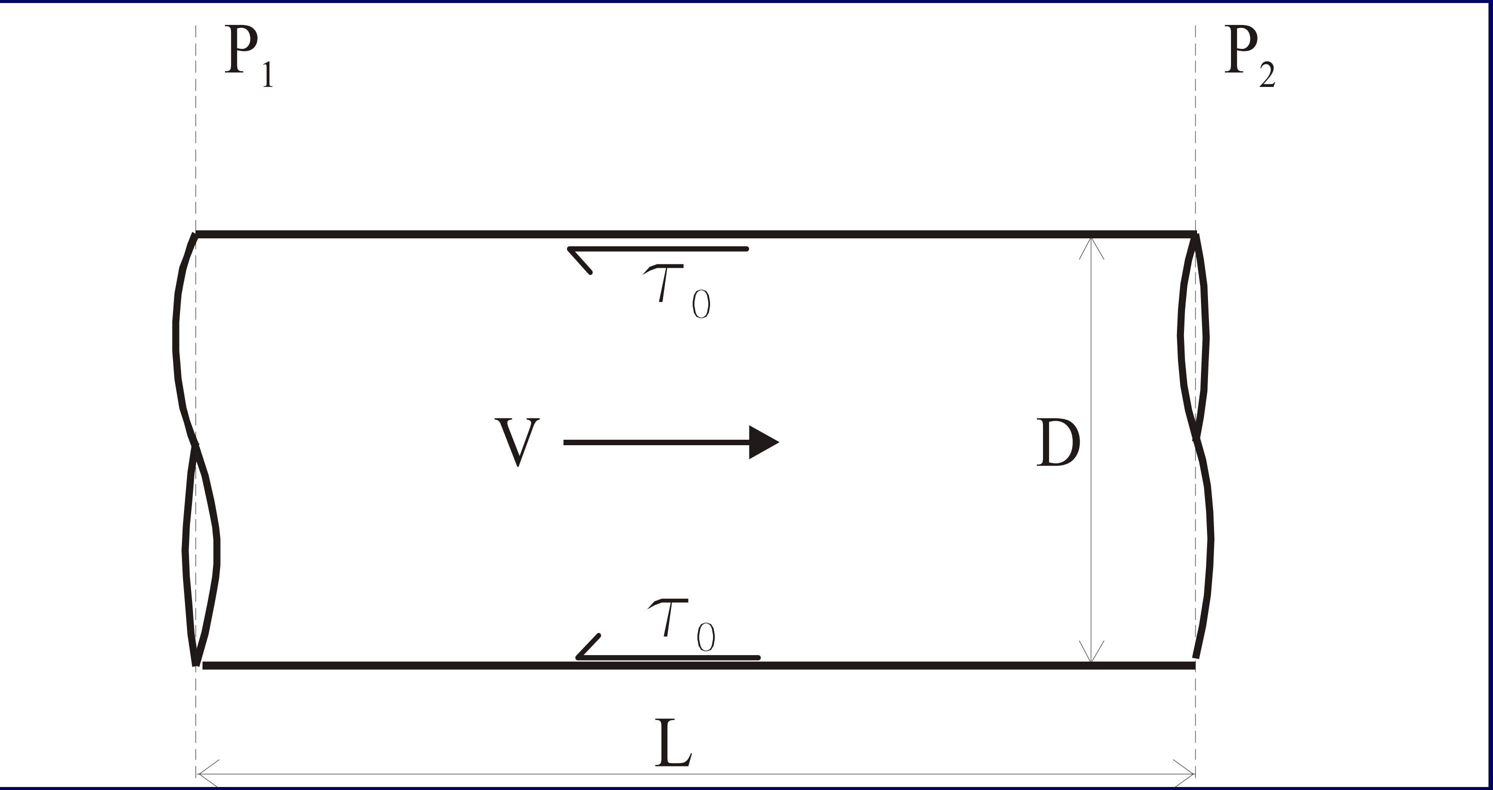
- incompressible liquid,
- steady and uniform flow in a horizontal straight pipe

$$-\frac{dP}{dx}A = \tau_o O \,, \quad \text{i.e.} \quad -\frac{dP}{dx} = \frac{4\tau_o}{D}$$

for *a pipe of a circular cross section* and internal diameter D.

Conservation of Momentum in 1D-flow

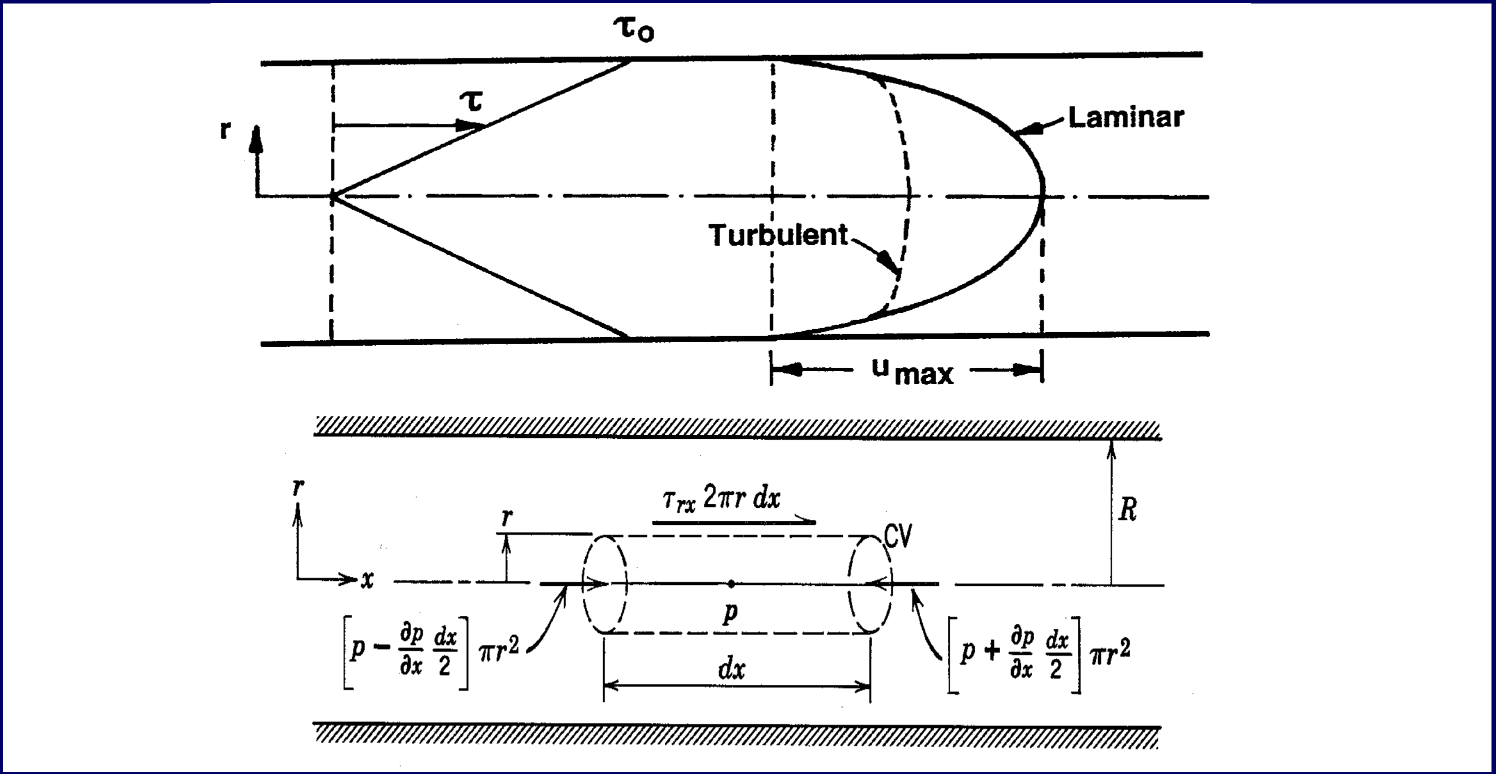
For a straight horizontal circular pipe $$\frac{P_1 - P_2}{L} = \frac{4\tau_o}{D}$$



Liquid Friction in 2D Pipe Flow

The force-balance equation generalized for 2D-flow gives the shear stress distribution in a cylinder:

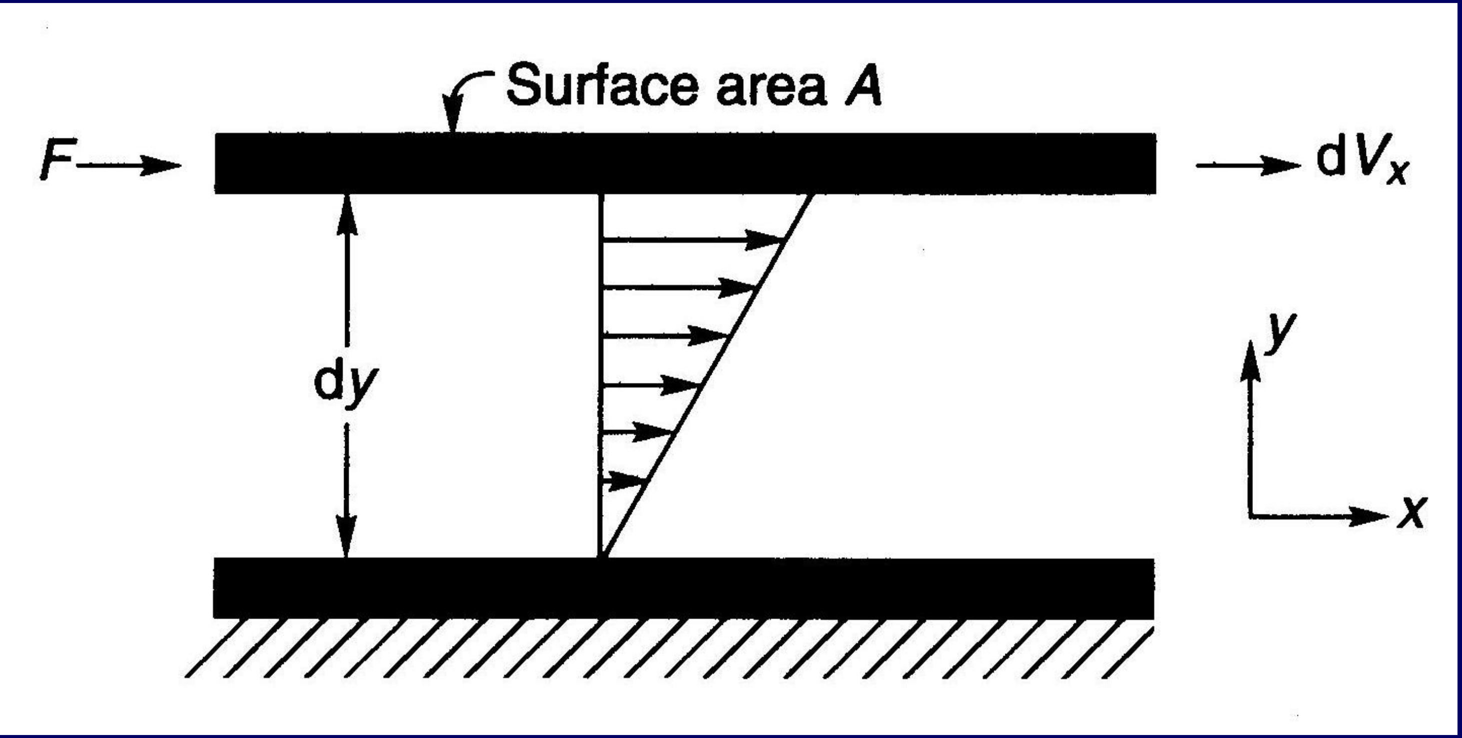
$$-\frac{dP}{dx} = \tau \frac{2}{r}$$



Liquid Friction in 2D Pipe Flow

Newton's law of liquid viscosity (valid for laminar flow):

$$\tau = \frac{F}{A} = \mu_f \left(-\frac{dV_x}{dy} \right)$$



Liquid Friction in 2D Laminar Flow in Pipe

The generalized force-balance equation for the 2D-flow in a cylinder

$$-\frac{dP}{dx} = \tau \frac{2}{r}$$

+

Newton's law of liquid viscosity (valid for laminar flow)

$$\tau = \mu_f \left(-\frac{dv_x}{dr} \right)$$

=

Velocity distribution in laminar flow in a pipe

$$\frac{dv_x}{dr} = \frac{dP}{dx} \frac{r}{2\mu_f}$$

Liquid Friction in 1D Laminar Flow in Pipe

The integration of the velocity gradient equation gives a value for mean velocity in pipe

$$V_f = \frac{1}{A}\iint_A v_x dA = \frac{8}{D^2}\int_0^{D/2} v_x r dr$$

and thus a relationship between pressure drop and mean velocity

$$V_f = \frac{D^2}{32\mu_f}\left(\frac{dP}{dx}\right)$$

which is the required pressure-drop model for laminar flow in pipe

$$\frac{dP}{dx} = \frac{32\mu_f V_f}{D^2}$$

Liquid Friction in 1D Laminar Flow in Pipe

A comparison of the pressure-drop model for laminar flow in pipe

$$\frac{dP}{dx} = \frac{32\mu_f V_f}{D^2}$$

+

with the general force balance (driving force = resistance force) for pipe flow

$$\frac{dP}{dx} = \frac{4\tau_o}{D}$$

=

gives the equation for the shear stress at the pipe wall in laminar flow

$$\tau_o = \mu_f \frac{8V_f}{D}$$

Liquid Friction in 1D Turbulent Flow in Pipe

The wall shear stress for ***turbulent flow*** cannot be determined directly from the force balance and Newton's law of viscosity (it does not hold for turbulent flow). Instead, it is formulated by using dimensional analysis.

A function **$\tau_0 = fn(\rho_f, V_f, \mu_f, D, k)$** is assumed. The analysis provides the following relationship between dimensionless groups

$$\frac{\tau_o}{\frac{1}{2}\rho_f V_f^2} = fn\left(\mathrm{Re}, \frac{k}{D}\right)$$

Liquid Friction in 1D Turbulent Flow in Pipe

The dimensionless group Re, ***Reynolds number***, is a ratio of the inertial forces and the viscous forces in the pipeline flow

$$\mathrm{Re} = \frac{V_f D \rho_f}{\mu_f} = \frac{inertial.force}{viscous.force}$$

Remark: The Reynolds number determines a threshold between the laminar and the turbulent flows in a pipe.

Liquid Friction in 1D Turbulent Flow in Pipe

The dimensionless parameter on the left side of the dimensional-analysis equation is called the friction factor.

It is the ratio between the wall shear stress and kinetic energy of the liquid in a control volume in a pipeline.

Fanning friction factor

$$f_f = \frac{\tau_o}{\frac{1}{2}\rho_f V_f^2}$$

Darcy-Weisbach friction coefficient

$$\lambda_f = \frac{8\tau_o}{\rho_f V_f^2}$$

$$\lambda_f = 4 f_f$$

Liquid Friction in 1D Flow in Pipe

A comparison of the Darcy-Weisbach friction coefficient equation

$$\lambda_f = \frac{8\tau_o}{\rho_f V_f^2}$$

+

with the linear momentum eq. (driving force = resistance force) for pipe flow

$$-\frac{dP}{dx} = \frac{4\tau_o}{D}$$

=

gives the general pressure-drop equation for the pipe flow

(Darcy-Weisbach equation, 1850)

$$-\frac{dP}{dx} = \frac{\lambda_f}{D}\frac{\rho_f V_f^2}{2}$$

Liquid Friction in 1D Laminar Flow in Pipe

A comparison of the general pressure-drop equation

$$-\frac{dP}{dx} = \frac{\lambda_f}{D}\frac{\rho_f V_f^2}{2}$$

+

with the pressure-drop eq. for laminar flow in pipe

$$-\frac{dP}{dx} = \frac{32\mu_f V_f}{D^2}$$

=

gives the pipe-wall friction law for laminar flow in pipe

$$\lambda_f = \frac{64\mu_f}{\rho_f D V_f} = \frac{64}{\mathrm{Re}}$$

Liquid Friction in 1D Turbulent Flow in Pipe

In ***turbulent flows*** there is no simple expression linking the velocity distribution with the shear stress (and so with the pressure gradient) in the pipe cross section.

The dimensional analysis provides the following relationship between dimensionless groups

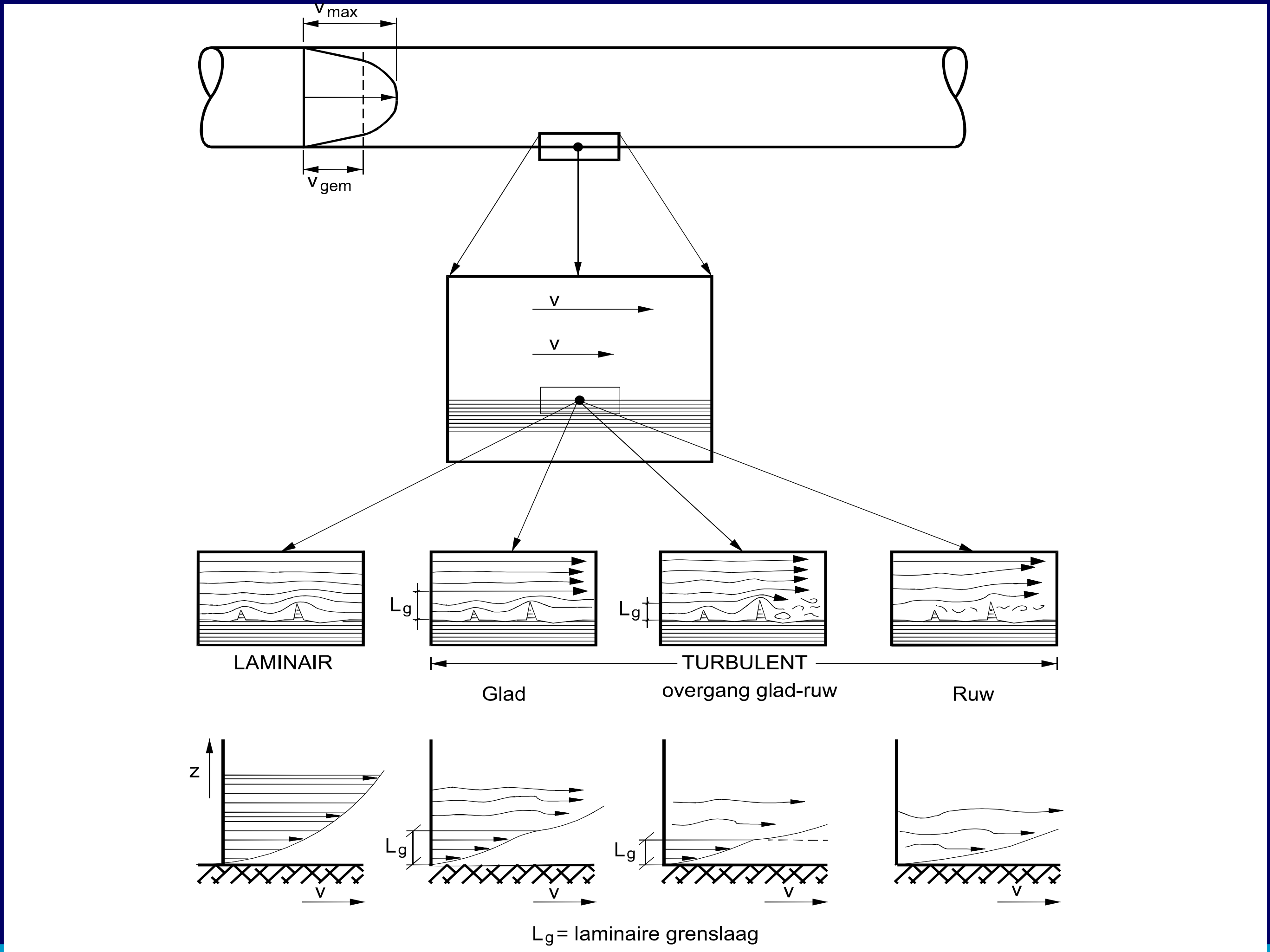
$$\lambda_f = fn\left(\mathrm{Re}, \frac{k}{D}\right)$$

Liquid Friction in 1D Turbulent Flow in Pipe

There are three different regimes with the different wall friction laws:

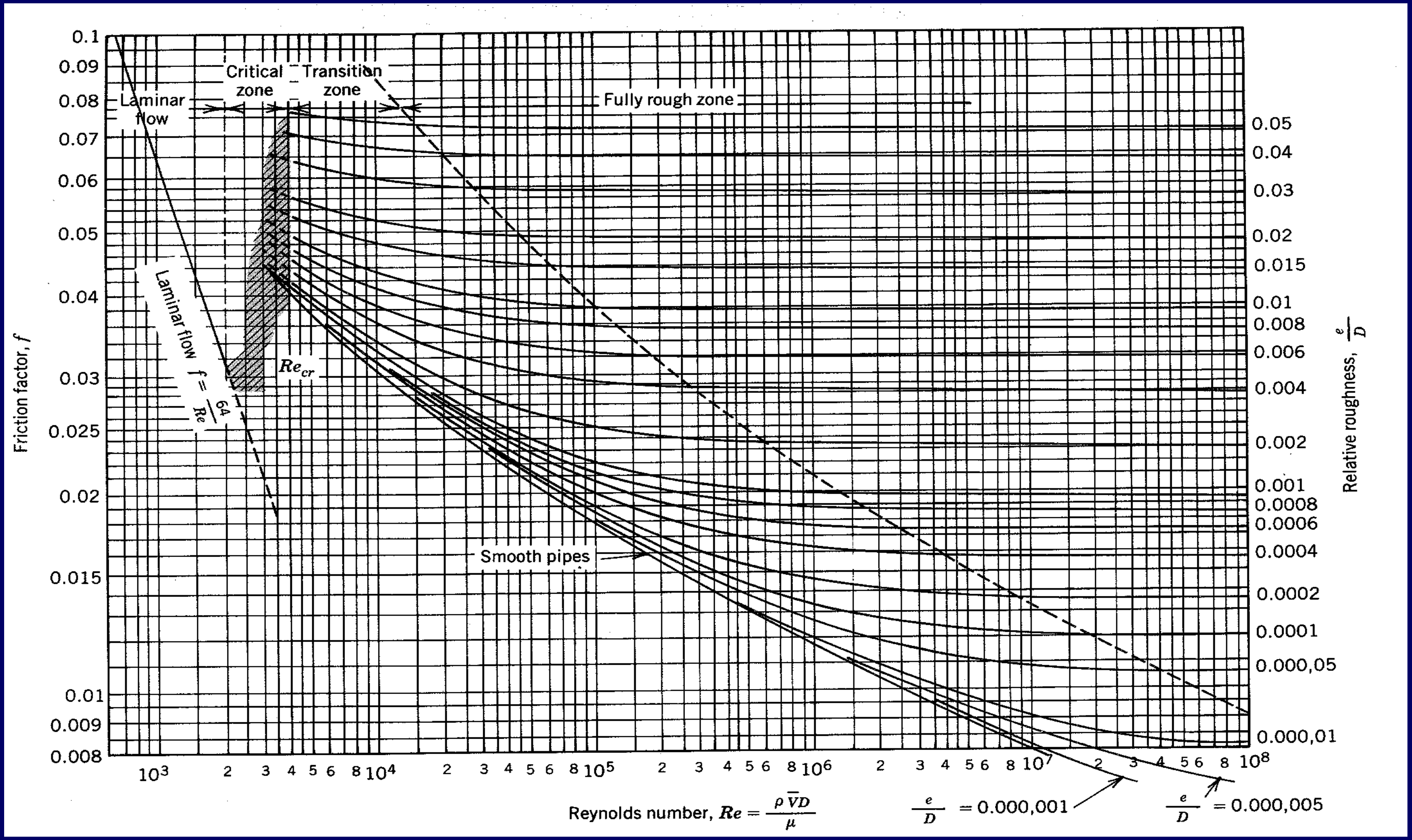
Hydraulically smooth	Transitional	Hydraulically rough
$\lambda_f = fn(Re)$	$\lambda_f = fn(Re, k/D)$	$\lambda_f = fn(k/D)$

Liquid Friction in 1D Turbulent Flow in Pipe



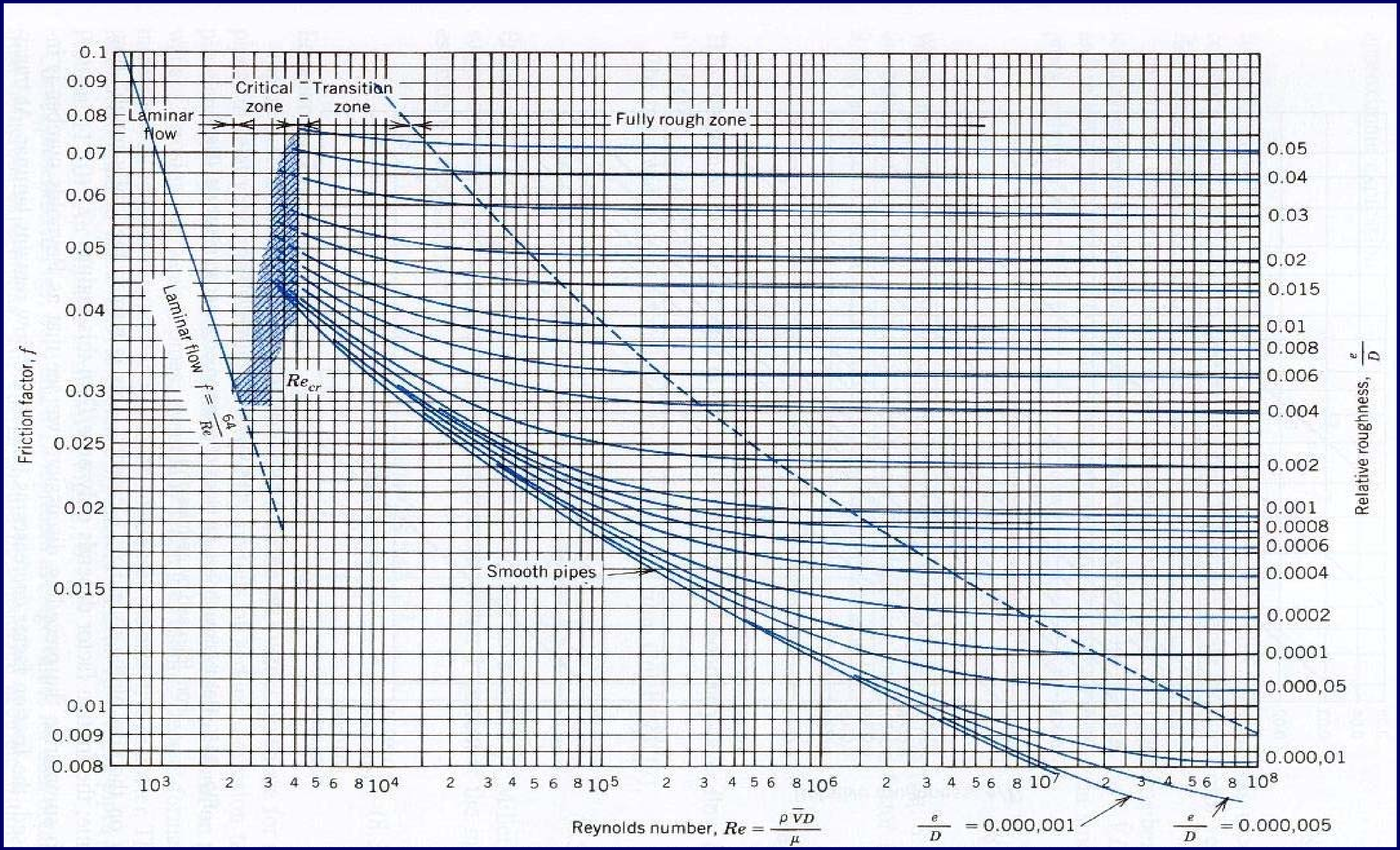
Hydraulically smooth	Transitional	Hydraulically rough
λ_f=fn(Re)	λ_f=fn(Re, k/D)	λ_f=fn(k/D)

Liquid Friction: Moody Diagram



Hydraulically smooth	Transitional	Hydraulically rough
$\lambda_f = fn(Re)$	$\lambda_f = fn(Re, k/D)$	$\lambda_f = fn(k/D)$

Liquid Friction: Moody Diagram



Hydraulically smooth	Transitional	Hydraulically rough
λ_f=fn(Re)	λ_f=fn(Re, k/D)	λ_f=fn(k/D)


Daniel Bernoulli

Conservation of energy

$$\frac{P_1}{\rho g}+\frac{V_1}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2}{2g}+z_2+\frac{\lambda L V^2}{2gD},$$

where λ is the friction factor, for our case:

$$\lambda=\frac{2gD^3}{v^2 R_e^2}$$

The Colebrook formula for λ is:

$$\sqrt{\lambda}=\frac{1}{2\log_{10}\left(\frac{2.51}{R_e\sqrt{\lambda}}\right)},\ R_e\geq 4000,$$

where R_e is Reynolds number

$$R_e=\frac{VD}{v}$$