oe4625 Dredge Pumps and Slurry Transport

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4. MODELING OF STRATIFIED MIXTURE FLOWS (Heterogeneous Flows)

EMPIRICAL MODELING

THEORETICAL MODELING

EMPIRICAL MODELING

DURAND

FÜHRBÖTER

JUFIN-LOPATIN

WILSON-GIW

MTI

Empirical Model by Durand et al

A. Experimental observations

- \- a wide range of slurry flow conditions including several pipeline sizes and sorts of sand and gravel
- \- low concentrated slurries (C_{vd} up to 22%)

B. Construction of the Durand correlation for I_m:

- \- the solids effect I_m-I_f decreases gradually with increasing mean slurry velocity V_m in flow of constant delivered volumetric concentration of solids C_{vd}
- \- the solids effect I_m-I_f increases approximately linearly with increasing C_{vd} at constant V_m.

Empirical Model by Durand et al

B. Construction of the correlation for I_m (cont'):

- The latter condition is written as $I_m - I_f = const.C_{vd}$ and generalized in the dimensionless group Φ,

$$\Phi = \frac{I_m - I_f}{I_f C_{vd}} = const.$$

The flow coefficient Φ is not constant for slurry flows of different pipeline size D, solids size d, or flow velocity V_m.

Empirical Model by Durand et al

B. Construction of the correlation for I_m (cont'):

The flow coefficient **Φ** varies with **D**, **d**, and $\mathbf{V_m}$.

An effect of these parameters is introduced to the correlation using the dimensionless groups

- the Froude number for mixture flow $$Fr^2 = \frac{V_m^2}{gD}$$

and

- the Froude number for a solid particle $$Fr_{vt}^2 = \frac{v_t^2}{gd}$$

Intermezzo: Froude Number

The dimensionless group **Fr, Froude number**, is a ratio of the inertial forces and the gravitational forces acting on a certain control volume (this is here a pipeline flow)

$$Fr^2 = \frac{V^2}{gD} = \frac{inertial.force}{gravitational.force}$$

The *Froude number* is a criterion of dynamic similarity in different flow conditions for flows with a dominant effect of inertia and gravity.

Empirical Model by Durand et al

The new <u>dimensionless group Ψ</u> (covering the effect of D,d,V_m):

$$\Psi = Fr^2 Fr_{vt}^{-1} = \frac{V_m^2}{gD} \frac{\sqrt{gd}}{v_t}$$

A general empirical relationship was established by Durand et al for pressure drop due to friction in pipeline flow of slurry

$$\Phi = K\Psi^n$$

where **K** and **n** are *empirical coefficients*. Their values have to be found by experiments.

Empirical Model by Durand et al

The **Φ - Ψ relationship** is determined using

- the hyperbolic curve in the Φ-Ψ plot by Durand & Condolios (see next slide) or
- the curve approximation giving K = 180 and n = -1.5.

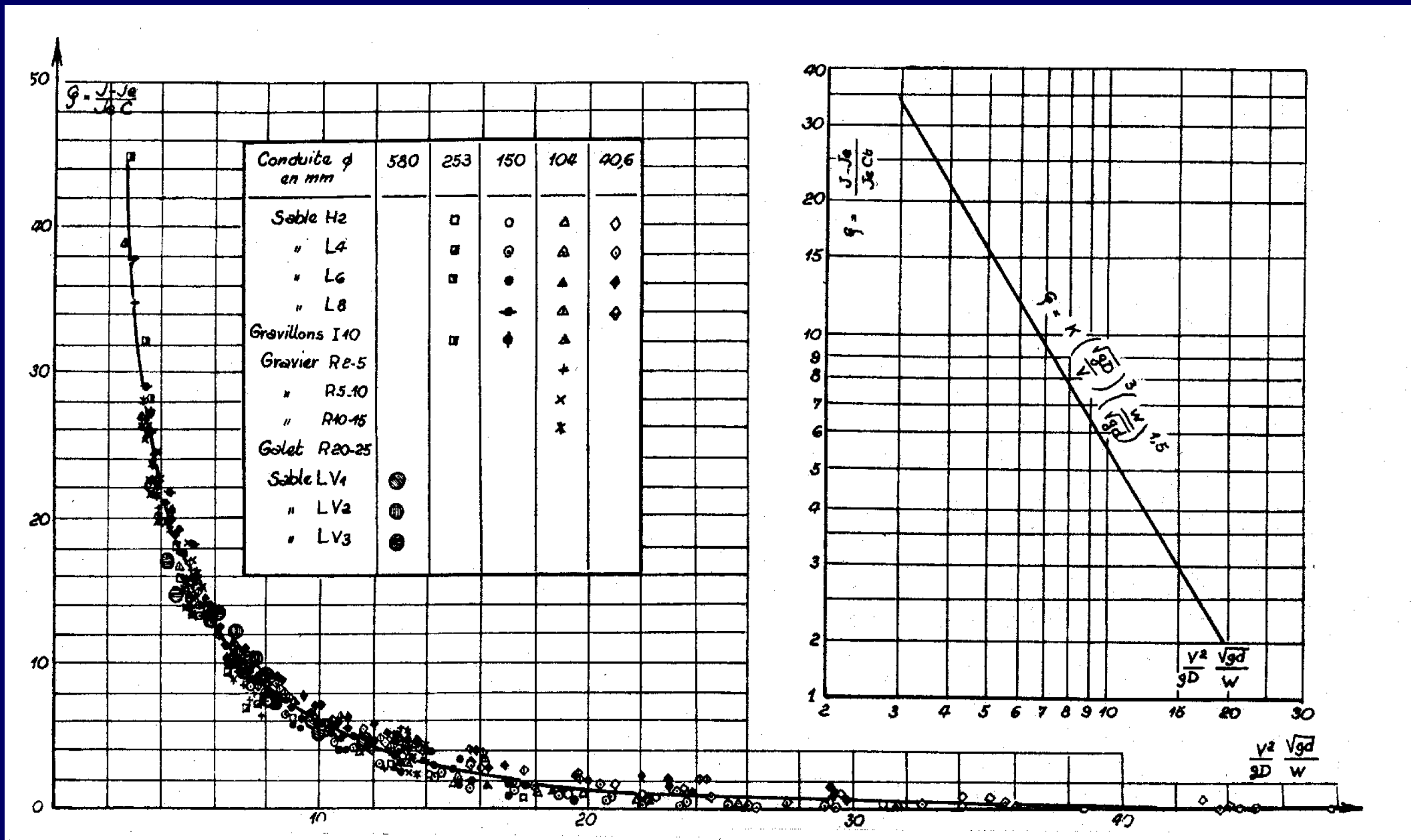
The ultimate correlation obtained by Durand et al:

$$\frac{I_m - I_f}{I_f C_{vd}} = 180 \left(\frac{V_m^2}{gD} \frac{\sqrt{gd}}{v_t} \right)^{-1.5}$$

The equation is recommended for the region 4 < Ψ < 15 (medium and medium to coarse sand).

Empirical Model by Durand & Condolios

Dimensionless group Φ [-]



Dimensionless group Ψ [-]

Empirical Model by Durand et al

A. Experimental observations for V_{dl}:

- visual observations of the initial formation of a stationary bed in pipelines for different mixture flow conditions.

B. Construction of the Durand correlation for V_{dl}:

- the Froude number for flow above the stationary bed remained constant when *a stationary bed was formed and gradually became thicker under decreasing V_m* in a pipeline.

Empirical Model by Durand et al

B. Construction of the V_{dl} correlation (cont'):

- The Froude number is based on the velocity above a stationary bed, V_e, and on the hydraulic radius, R_h, of discharging area above the stationary bed.

$$Fr^2 = \frac{V_e^2}{gR_h}$$

- For flow conditions at the beginning of the stationary bed ($V_e = V_m = V_{dl}$ and $D = 4R_h$) this condition is written as

$$Fr^2 = \frac{V_{dl}^2}{gD} = const.$$

Empirical Model by Durand et al

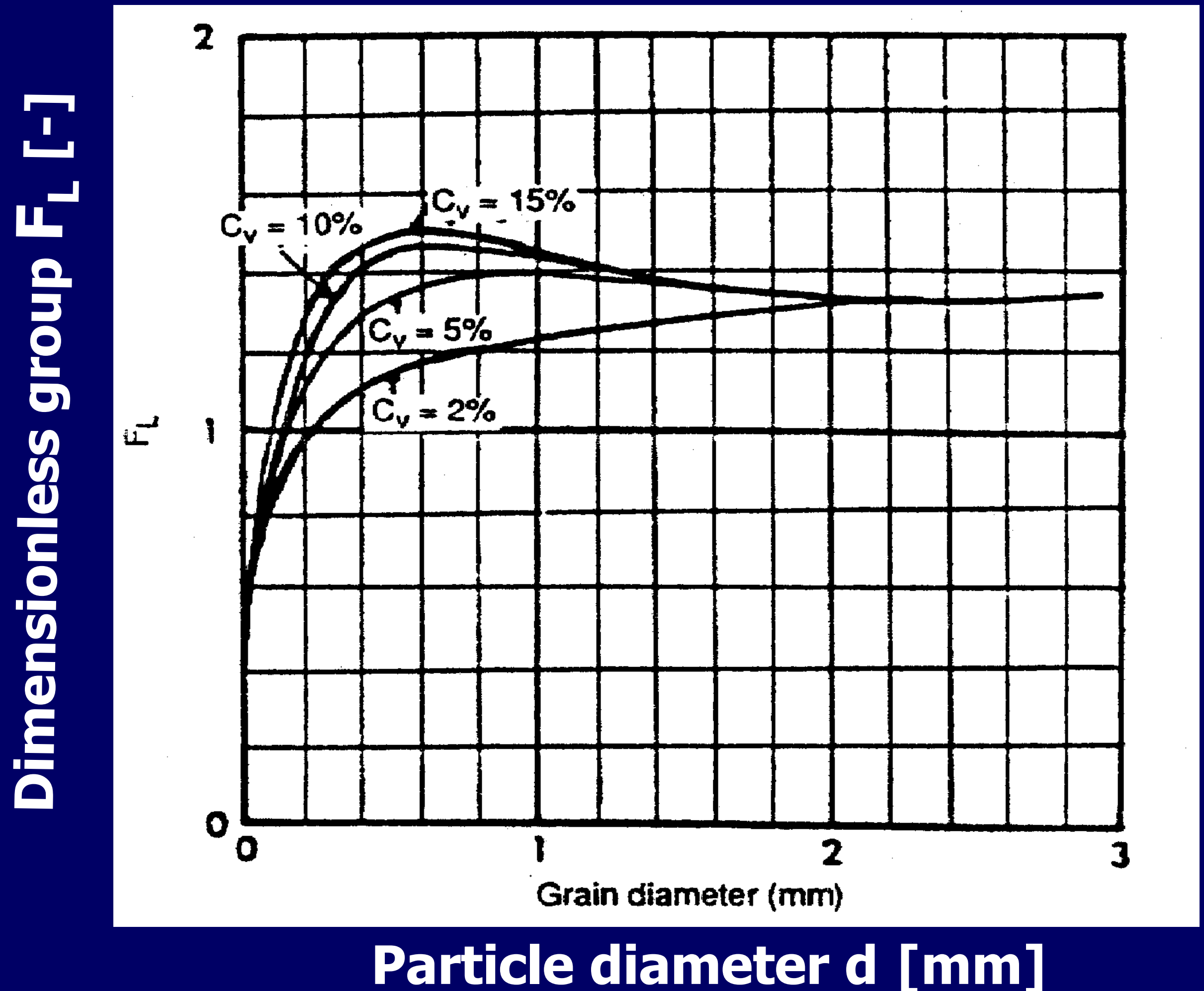
B. Construction of the V_{dl} correlation (cont'):

- An effect of various particle diameters **D** and delivered concentrations $\mathbf{C_{vd}}$ on the value of the V_{dl} was expressed in the empirical relationship $\mathbf{F_L = f(d, C_{vd})}$ presented as a graph (see next slide).

- The correlation for the deposition-limit velocity by Durand et al is written as

$$V_{dl} = F_L \sqrt{2g\left(S_s - 1\right)D}$$

Empirical Model by Durand et al



Empirical Model by Führböter

A. Experimental observations

- slurry flow conditions in a 300 mm laboratory pipeline for sand and gravel of particle size range between 0.15 mm and 1.8 mm.

B. Construction of the Führböter correlation for I_m:

- The correlation was found

$$I_m - I_f = S_k \frac{C_{vi}}{V_m}$$

in which $\mathbf{S_k}$ was *the empirical coefficient* dependent on solids properties.

Empirical Model by Führböter

B. Construction of the correlation for I_m (cont'):

- Practical calculations are done for $\mathbf{C_{vd}}$ instead of C_{vi}, thus the <u>slip effect</u> is incorporated to obtain C_{vd} in the model equation. The constant value of the slip ratio C_{vd}/C_{vi} = 0.65 is considered to hold for all mixture flow conditions. The *<u>transport factor</u>* $\mathbf{S_{kt}}$ is obtained by $\mathbf{S_{kt} = S_k C_{vi}/C_{vd}}$.
- The Führböter correlation is $$I_m - I_f = S_{kt}\frac{C_{vd}}{V_m}$$

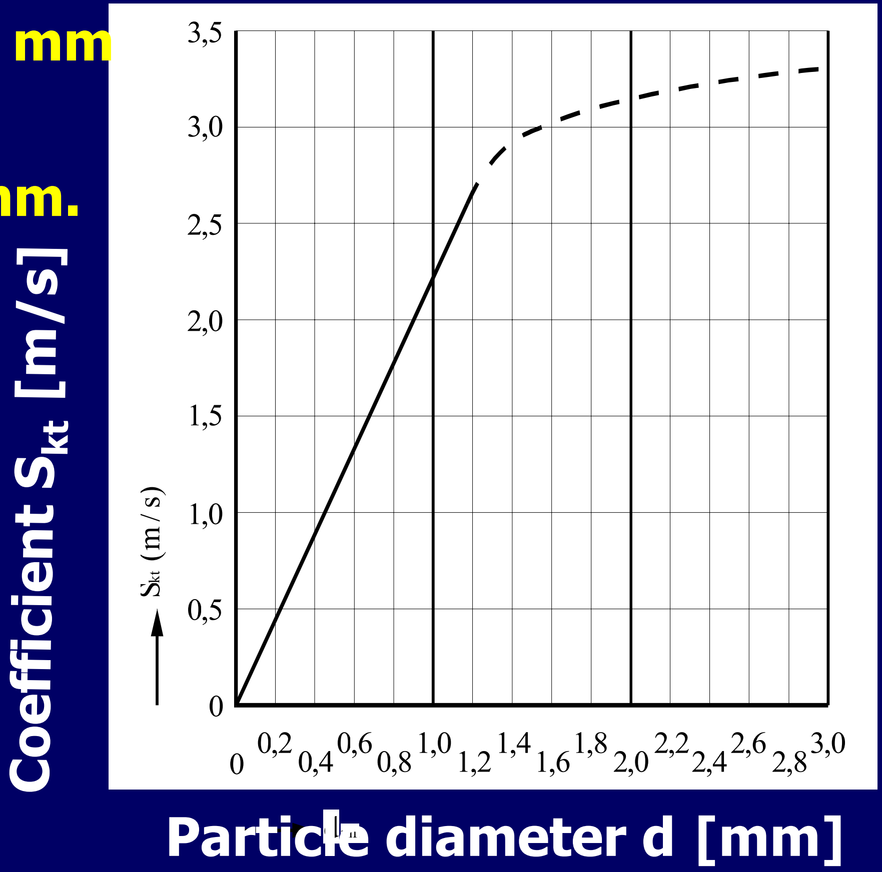
Values of the factor $\mathbf{S_{kt}}$ are determined empirically (by experiment).

Empirical Model by Führböter

S_{kt} = 2.59 d_m-0.037 for 0.2<d_m<1.1 mm

S_{kt} from graph for 1.1<d_m<3.0 mm

S_{kt} is approximately 3.3ford$_m$>3.0 mm.



Empirical Model by Jufin and Lopatin

A. Experimental observations

- broad data base including data from dredging installations.

B. Construction of the Jufin correlation for I_m:

- The assumption based on experiments sounds: the hydraulic gradient I_m at the minimum velocity V_{min} equals $\mathbf{I_m = 3I_f}$.
- The general correlation is

$$I_m = I_f \left(1 + 2 \left[\frac{V_{\min}}{V_m} \right]^3 \right)$$

Empirical Model by Jufin and Lopatin

C. The minimum velocity by Jufin:

The *minimum velocity V_{min}* is determined using the empirical correlation

$$V_{\min} = 5.3\left(C_{vd}.\psi^{*}.D\right)^{\frac{1}{6}}$$

in which the *particle settling parameter* **ψ* = f(d)** is either from Table or calculated as the modified Froude number of a solid particle:

$$\psi^{*} = Fr_{vt}^{1.5} = \left(\frac{v_t}{\sqrt{gd}}\right)^{1.5}$$

Empirical Model by Jufin and Lopatin

D. The deposition-limit velocity by Jufin:

The *deposition-limit velocity V_{dl}* is given by the empirical correlation

$$V_{dl} = 8.3D^{\frac{1}{3}}\left(C_{vd}.\psi^{*}\right)^{\frac{1}{6}}$$

in which the *particle settling parameter* **ψ* = f(d)** is either from Table or calculated as modified Froude number of a solid particle:

$$\psi^{*} = Fr_{vt}^{1.5} = \left(\frac{v_t}{\sqrt{gd}}\right)^{1.5}$$

Empirical Model by Wilson & GIW

A. Experimental observations

- data from the GIW laboratory circuits (diameters 200 mm and 440 mm, for medium to coarse sands in mixtures of delivered concentrations up to 0.16.

B. Construction of the correlation for I_m:

The model considers the *heterogeneous flow* as a transition between two extreme flows governed by different mechanisms for support of a solid particle in the carrying-liquid flow:

- the *fully-stratified flow* (all particles in a contact load) and
- the *fully-suspended flow* (all particles in a suspended load).

Empirical Model by Wilson & GIW

B. Construction of the correlation for I_m (cont'):

The theoretical consideration:

- The energy dissipation due to the presence of solid particles in a carrier flow is predominantly due to *mechanical friction* between contact-load particles and a pipeline wall.
- A resisting force of the contact bed against the carrier flow is related to the *submerged weight of the bed* via the coefficient of mechanical friction.

Empirical Model by Wilson & GIW

B. Construction of the correlation for I_m (cont'):

- Thus at $V_m = V_{50}$ (the mean slurry velocity at which one half of the transported solid particles contribute to a suspended load and one half to a contact load velocity) the solids effect is due to the submerged weight of the moving bed containing one half of the total solid fraction multiplied by the friction coefficient, μ_s,

$$I_m - I_f = 0.5\,\mu_s\,C_{vd}(S_s - 1).$$

Empirical Model by Wilson & GIW

B. Construction of the correlation for I_m (cont'):

The experimental experience:

- A linear relationship between measured values of the ratio

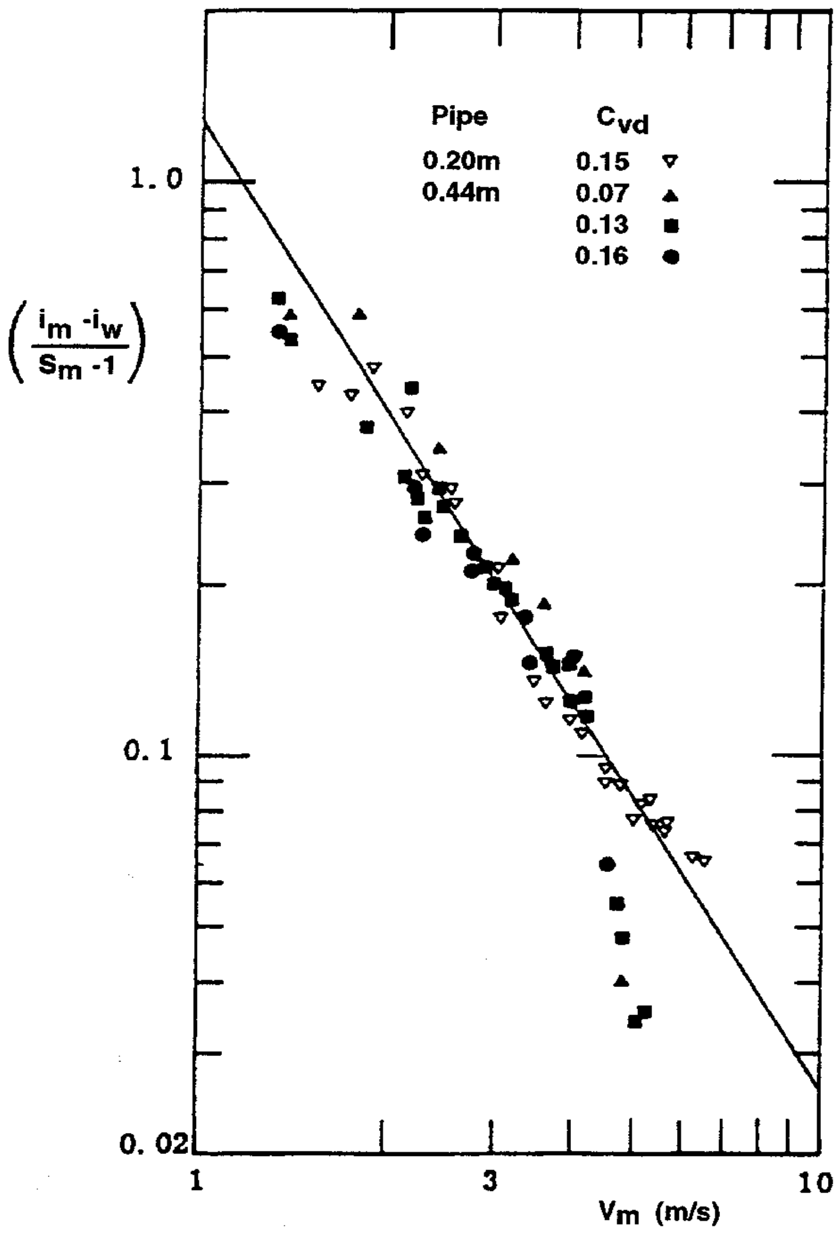
$$\frac{I_m - I_f}{C_{vd}\left(S_s - 1\right)}$$

and the mean mixture velocity V_m in plotted within the log-log co-ordinates.

The relationship was found the same for flows of different concentrations in pipes of different sizes.

Wilson & GIW

$$\frac{I_m - I_f}{C_{vd}\left(S_s - 1\right)} versus. V_m$$



Masonry sand mixture
(d_{50} = 0.42 mm),
after Clift et al. (1982).

Empirical Model by Wilson & GIW

B. Construction of the correlation for I_m (cont'):

Generalization:

The correlation for various mean mixture velocities:

$$\frac{I_m - I_f}{C_{vd}\left(S_s - 1\right)} = 0.5\,\mu_s\left(\frac{V_m}{V_{50}}\right)^{-M} = 0.22\left(\frac{V_m}{V_{50}}\right)^{-M}$$

Empirical Model by Wilson & GIW

B. Construction of the correlation for I_m (cont'):

Generalization (cont'):

V_{50} should be obtained experimentally or estimated roughly by the approximation:

$$V_{50} \approx 3.93(d_{50})^{0.35}\left(\frac{S_s - 1}{1.65}\right)^{0.45}$$

in which **d_{50} [mm]** and **V_{50} [m/s]**.

The exponent M is given by:

(M should not exceed 1.7, the value for narrow-graded solids, nor fall below 0.25).

$$M \approx \left[\ln\left(\frac{d_{85}}{d_{50}}\right)\right]^{-1}$$

Empirical Model for Critical Velocity (MTI)

A. Definition of critical velocity according to MTI Holland

The ***critical velocity*** is the threshold velocity between the "fully suspended heterogeneous flow" regime and the regime of "flow with the first particles settling to the bottom" of a pipeline.

This velocity is suggested to be the lowest acceptable velocity for a economic and safe operation of a dredging pipeline.

$$V_{cr} \sim V_{min}$$

Empirical Model for Critical Velocity (MTI)

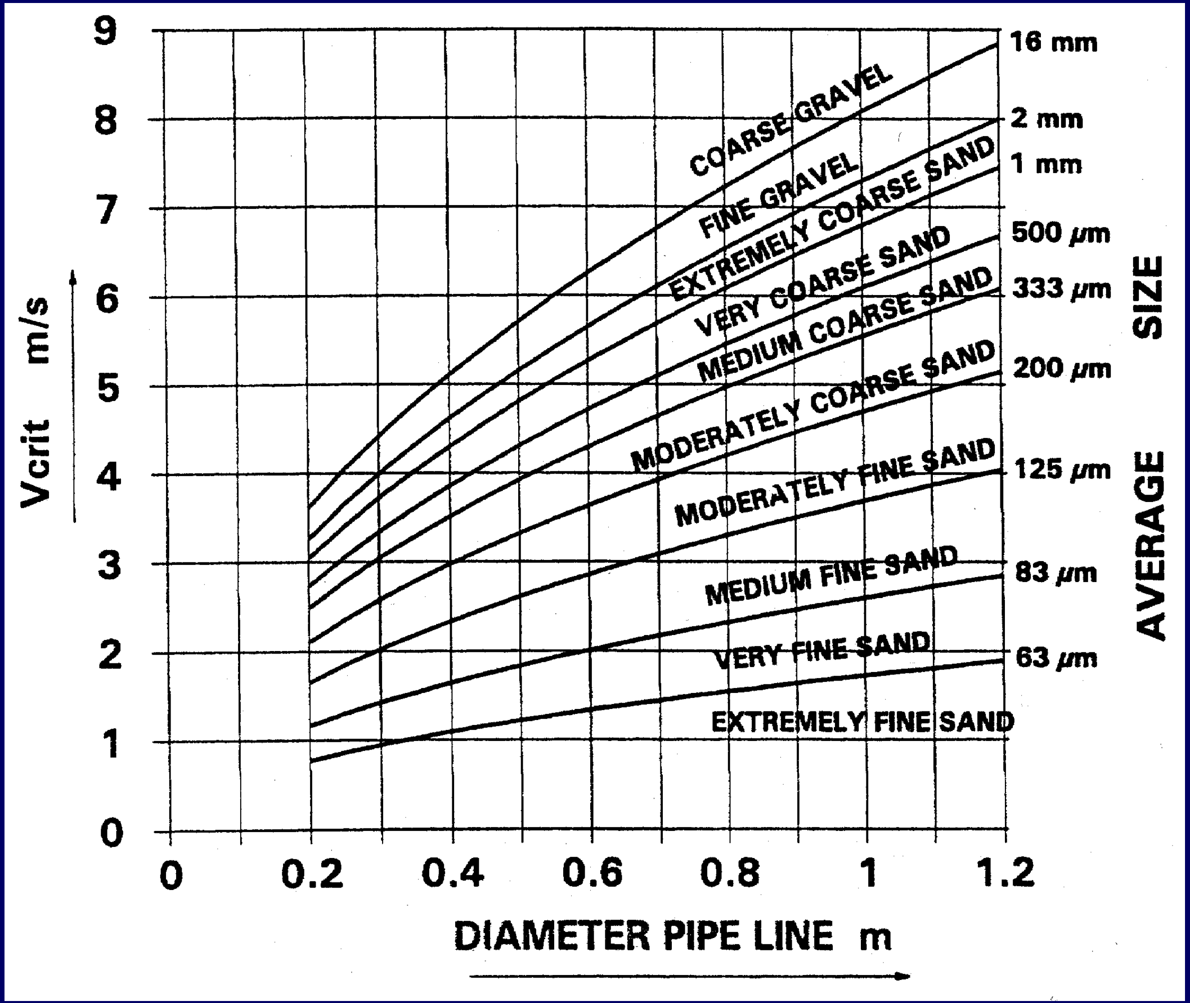
B. Correlation for V_{cr}:

$$V_{crit} = 1.7\left(5 - \frac{1}{\sqrt{d_{mf}}}\right)\sqrt{D}\left(\frac{C_{vd}}{C_{vd}+0.1}\right)^{\frac{1}{6}}\sqrt{\frac{S_s - 1}{1.65}}$$

In the equation **d_{mf} [mm]**, **D [m]** and **V_{cr} [m/s]**.

Empirical Model for Critical Velocity (MTI)

C. Diagram for V_{cr}:



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