oe4625 Dredge Pumps and Slurry Transport

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4. MODELING OF STRATIFIED MIXTURE FLOWS (Heterogeneous Flows)

EMPIRICAL MODELING

THEORETICAL MODELING

THEORETICAL MODELING

MACROSCOPIC

(Large Control Volume)

MICROSCOPIC

(Infinitesimal Control Volume)

MACROSCOPIC MODEL

A TWO-LAYER MODEL

FLOW COMPOSED OF TWO LAYERS

EACH LAYER = CONTROL VOLUME

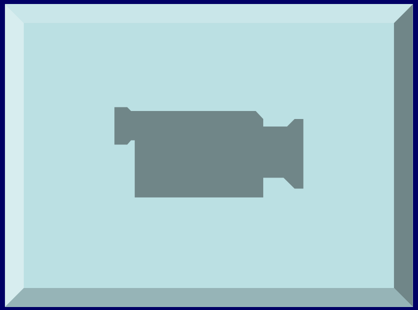
CONSERVATION LAWS FOR EACH LAYER:

Conservation of mass

Conservation of momentum

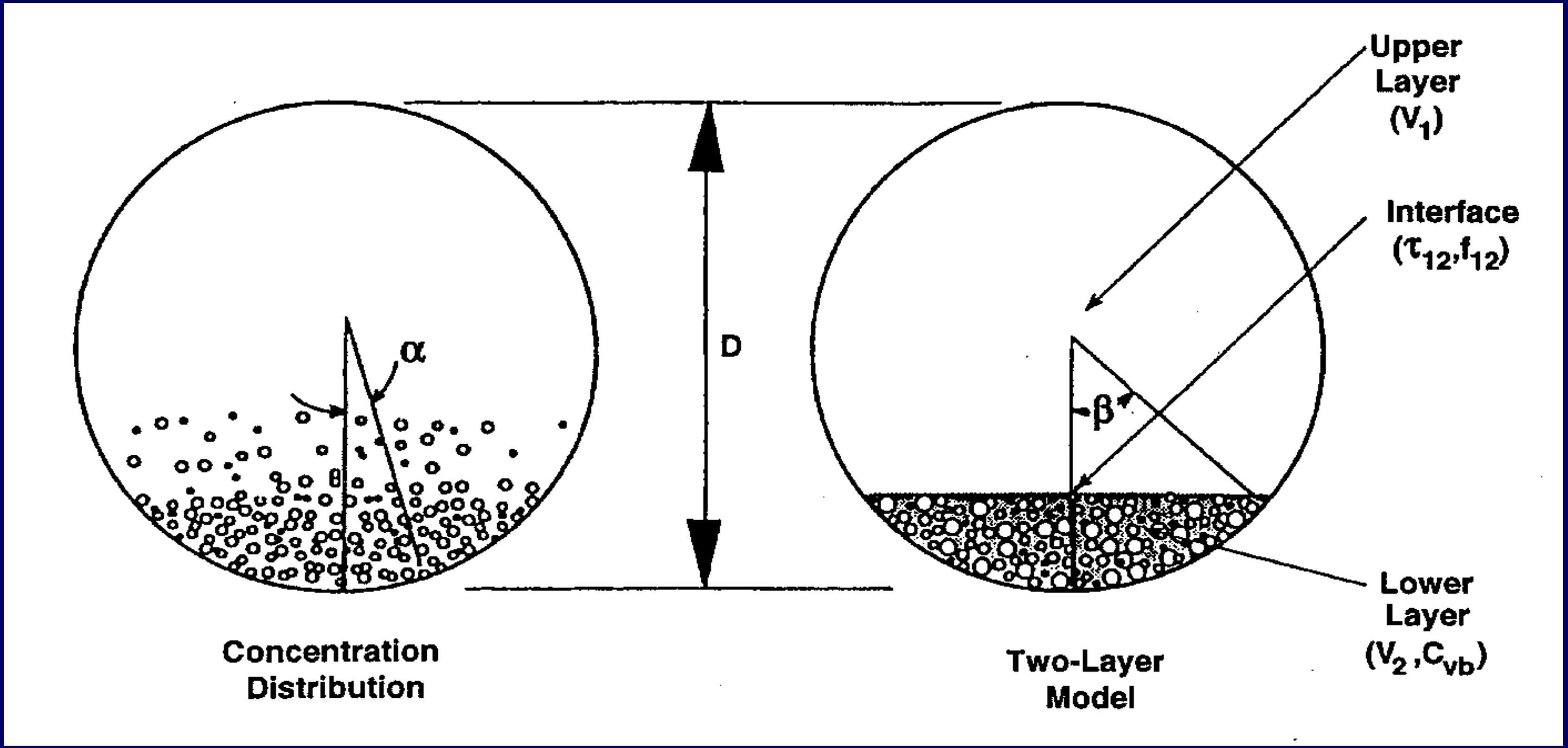
Stratified flows

Example of stratified flow



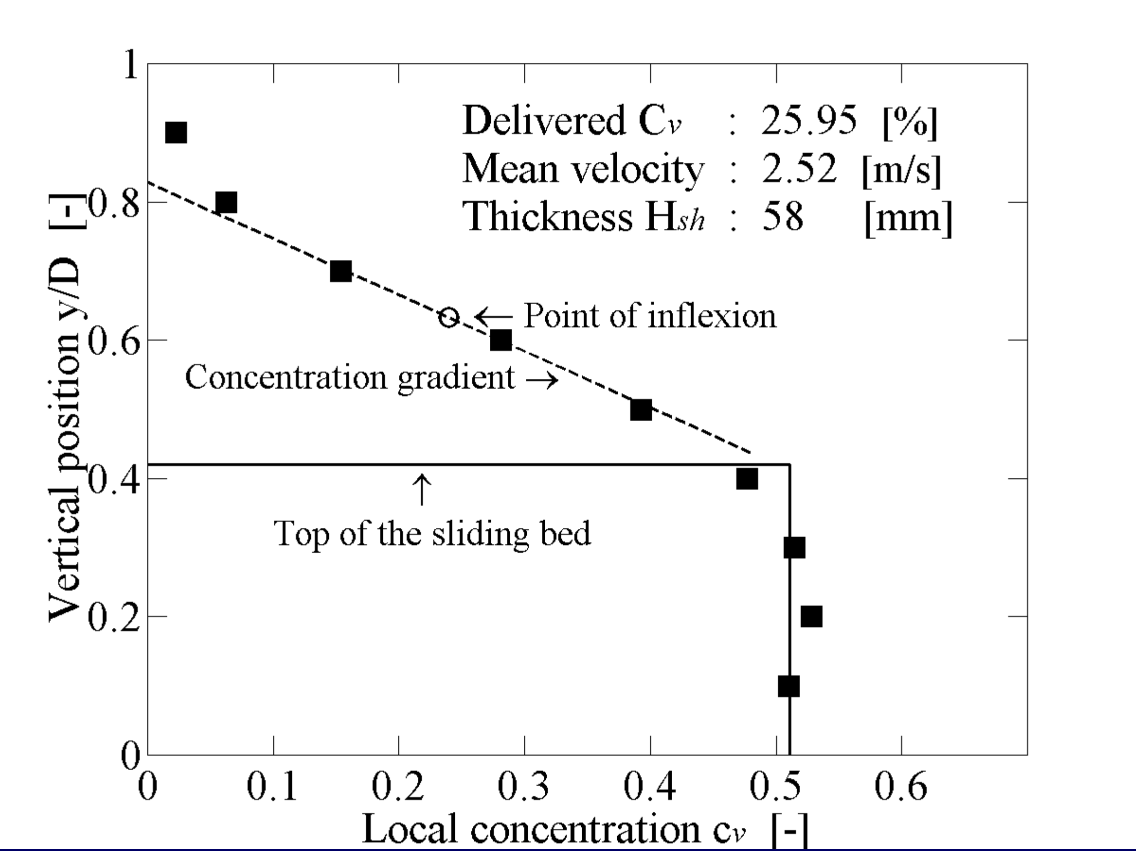
2LM: Flow Pattern

A. Simplified Flow Pattern



2LM: Flow Pattern

A. Real Flow Pattern



2LM: Flow Pattern

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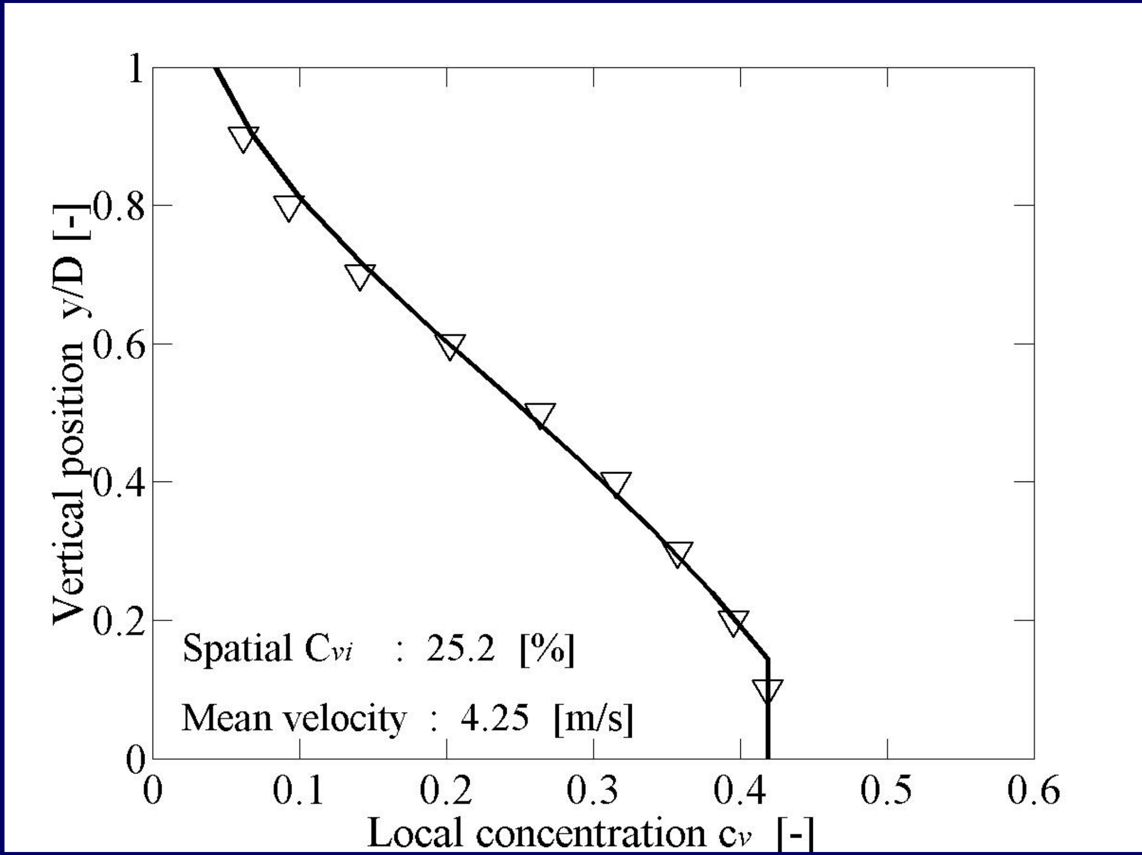
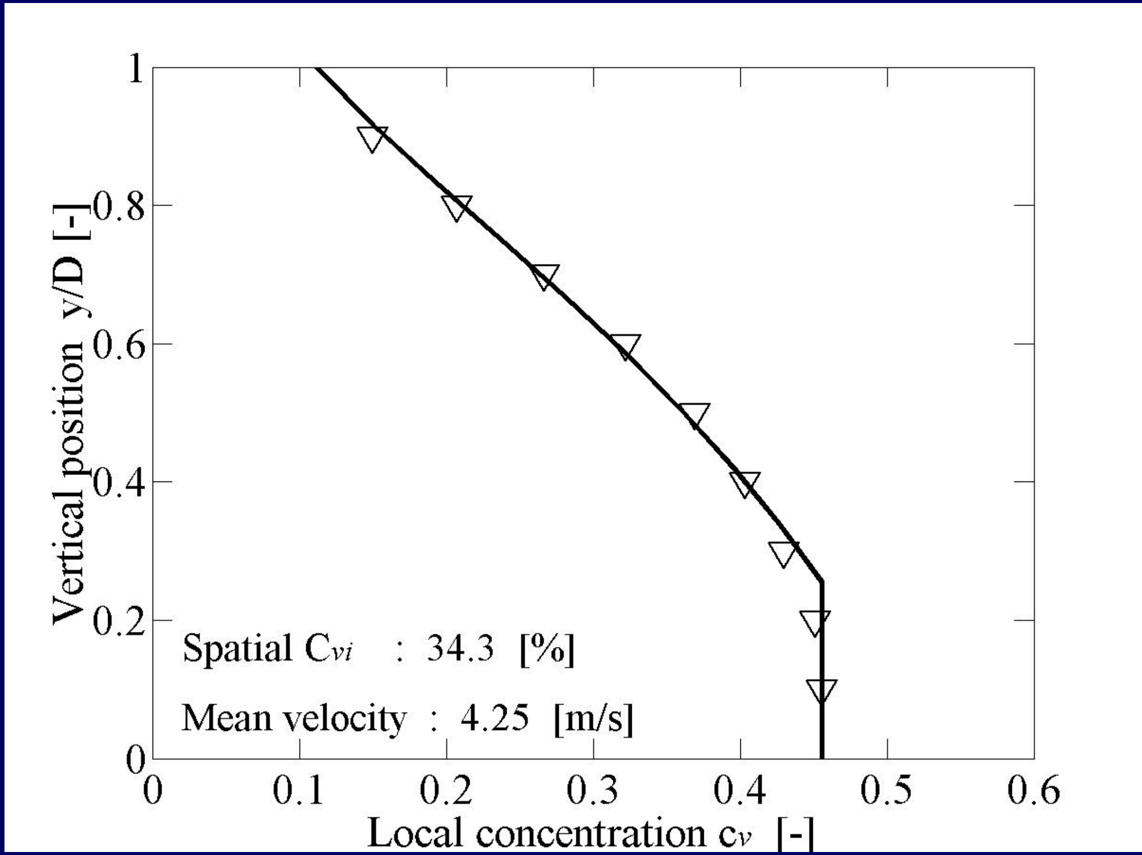


2LM: Flow Pattern

A. Real Flow Pattern



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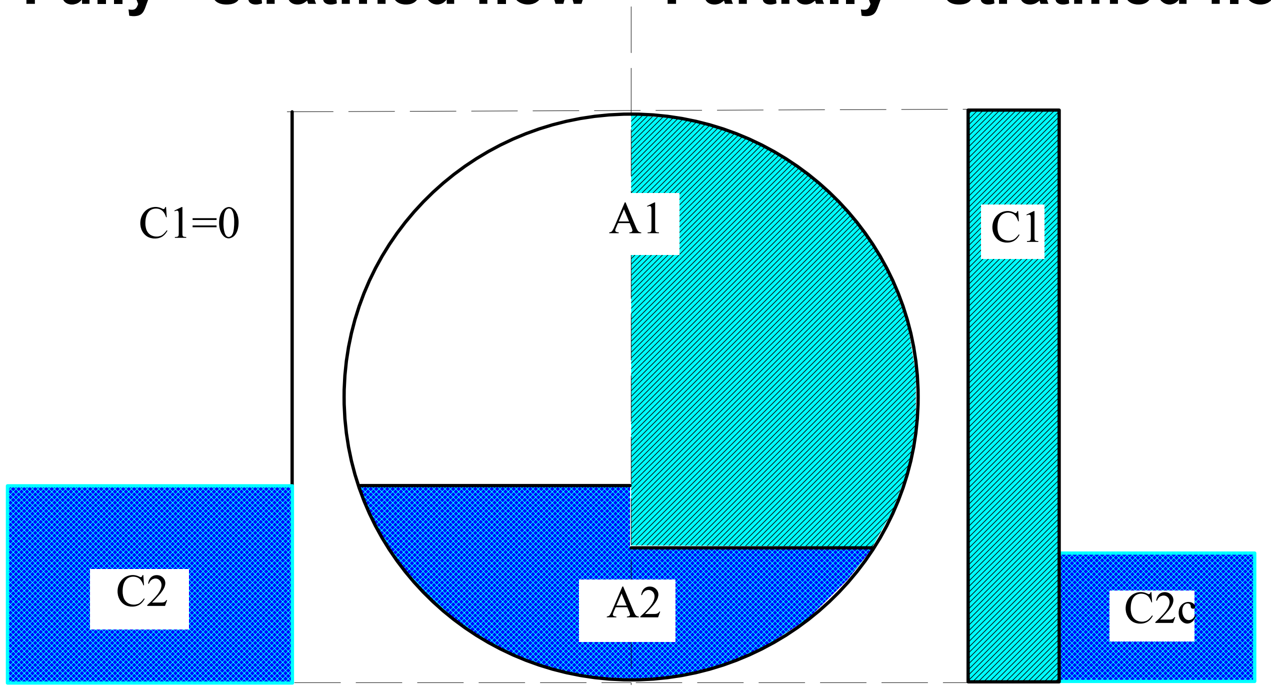
2LM: Flow Pattern

Fully - stratified flow **Partially - stratified flow**



Particle-free layer Suspended load Contact load

Figure: Schematic cross-section for two-layer model.

2LM: Principles

A. Simplified Flow Pattern

- STRATIFICATION.

B. Particle Support Mechanism

- the CONTACT
- the SUSPENSION = NO CONTACT.

2LM: Principles

A. Simplified Flow Pattern

- the real/virtual interface at the top of a contact layer
- the homogeneous distribution of velocity (V) and concentration (C) within each layer

  (C1, C2, V1, V2)

- no slip between phases within a layer

2LM: Principles

B. Particle Support Mechanism

<u>**INTERGRANULAR CONTACT**</u> **&** <u>**PARTICLE SUSPENSION**</u>

(contact load) **(suspended load)**

Contacts: ***continuous*** (Coulombic contacts within a stationary or sliding granular bed)

or ***sporadic*** (collisions within a transition 'shear' layer)

2LM: Principles

B. Particle Support Mechanism

MECHANISMS FOR SOLID PARTICLE SUSPENSION

Diffusive effect of ***carrier turbulence*** (no interparticle contacts within suspended layer)

Dispersive effect of ***repulsion forces*** due to interparticle collisions (within shear layer)

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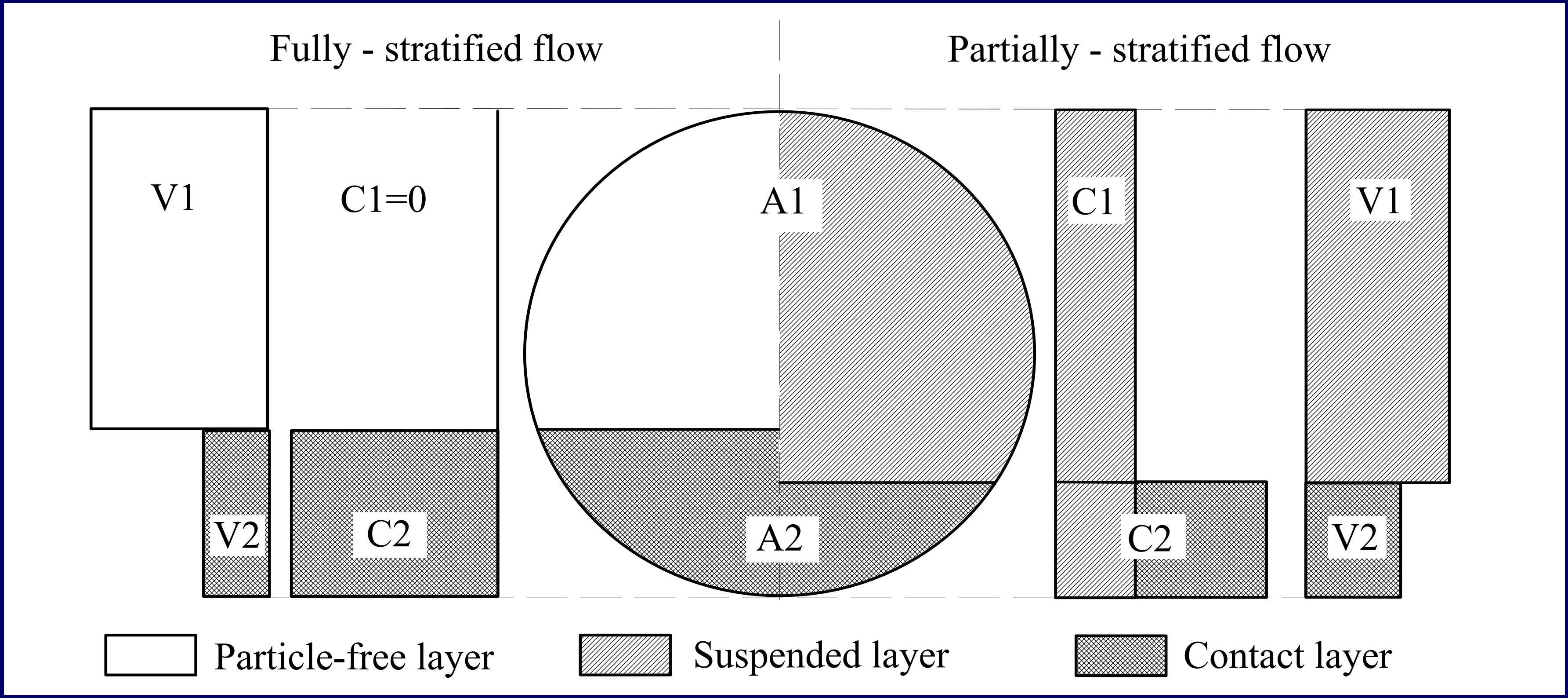
2LM: Flow Pattern

A. Simplified Flow Pattern



2LM: Flow Pattern

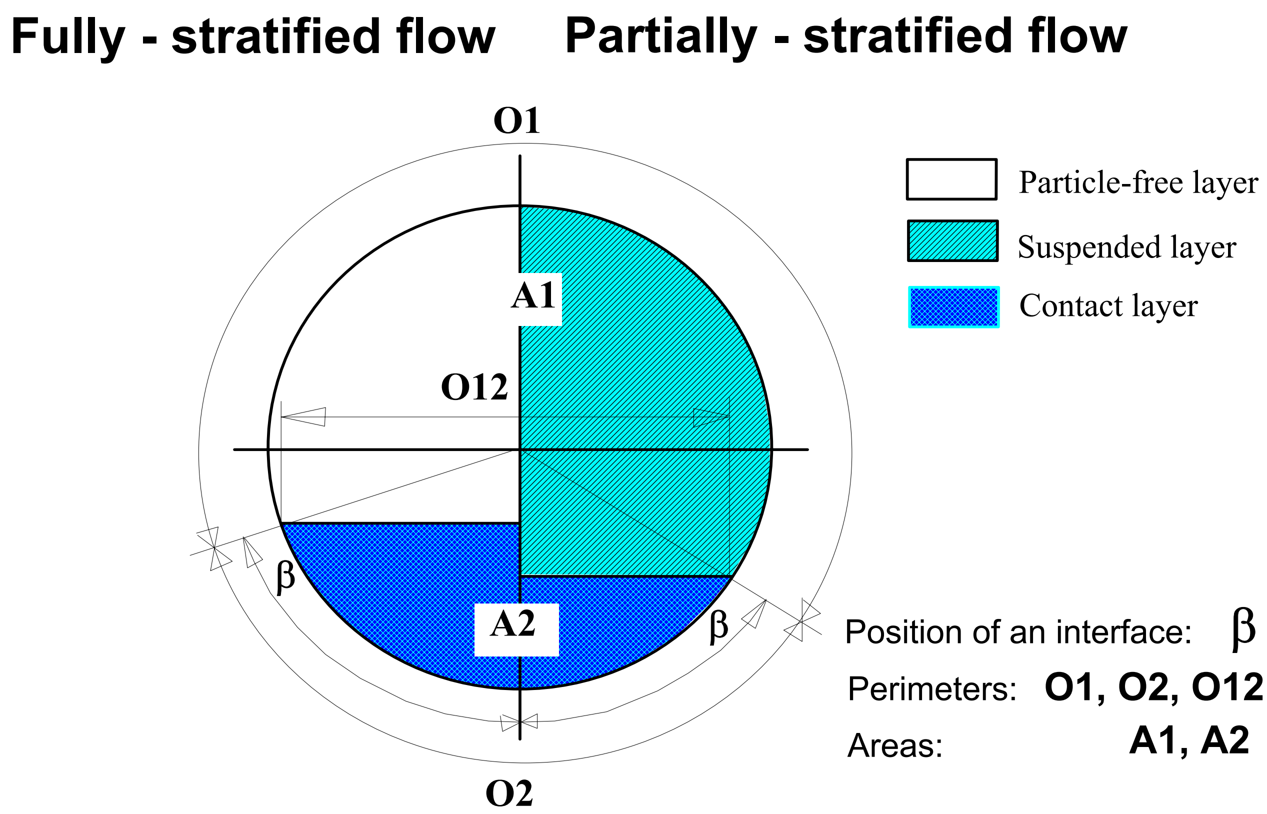


Figure: Geometry of schematic cross-section.

2LM: Concentration distribution

A. Simplified Flow Pattern

Solids fractions in both layers: **$C_{vi}.A = C1.A1 + C2.A2$**

Solids fraction in contact layer: **$C_c.A = C2.A2$**

Solids fraction in suspension layer: **$C_s.A = C1.A1$**

2LM: Conservation of mass

A. Continuity equations

Slurry flow rate: $A.V_m = A1.V1 + A2.V2$

Solids flow rate: $A_s.V_s = C_{vi}.A.V_s$

$C_{vi}.A.V_s = C_{vd}.A.V_m$

$$C_{vd}.A.V_m = C1.A1.V1 + C2.A2.V2$$

Flow-Stratification Parameter

The overall relationship

(for both turbulent suspension and shearing action)

$$\frac{C_c}{C_{vi}} = \exp\left(-Coeff\,\frac{V_m}{v_t}\right)$$

Coeff = const = 0.018 (Gillies et al, 1991)

0.024 (D=150 mm; Matousek, 1997)

0.0212 (Gillies and Shook, 2000)

Repetition: Conservation of Momentum in 1D-flow

For

- incompressible liquid,
- steady and uniform flow in a horizontal straight pipe

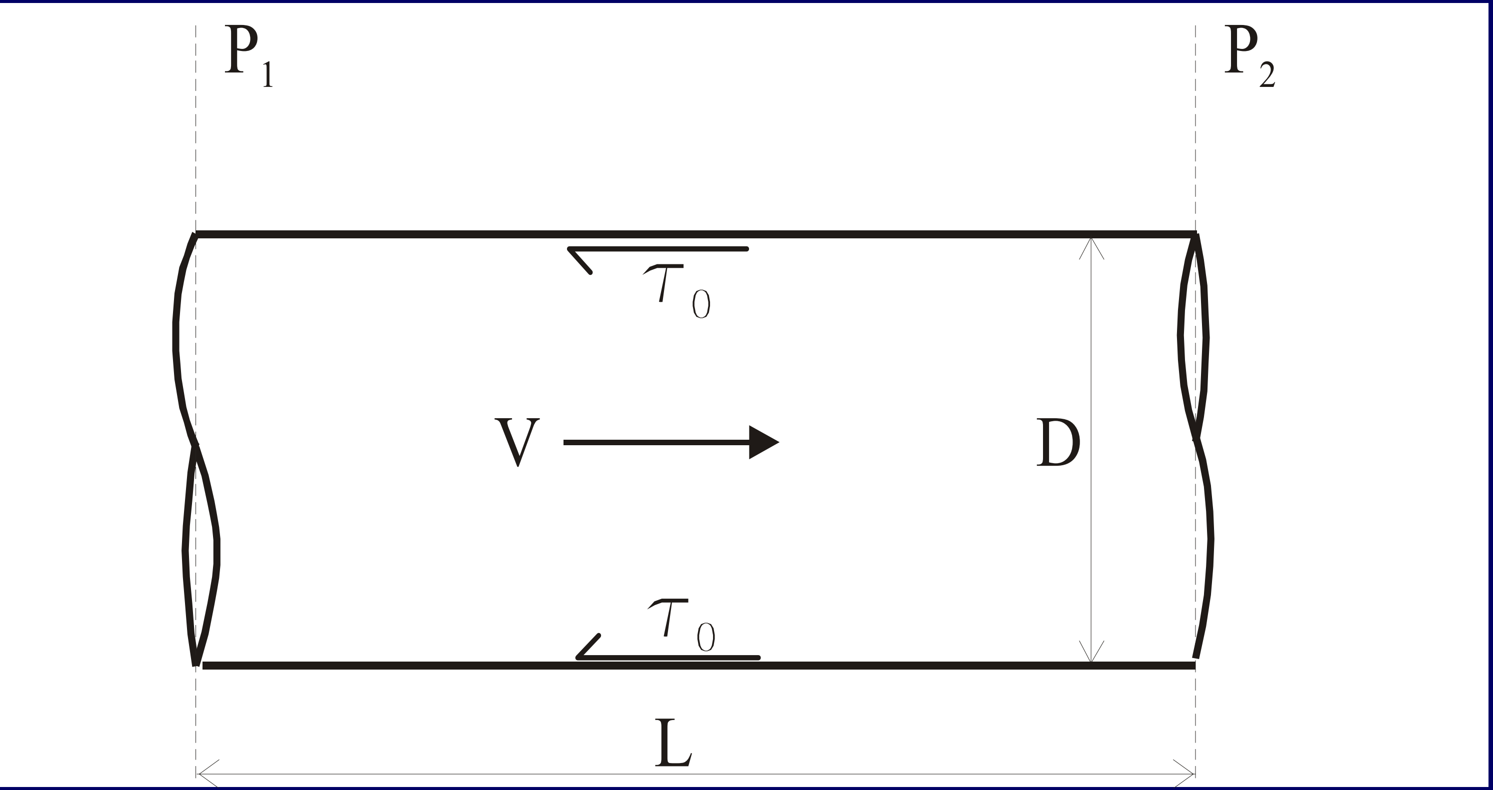
$$-\frac{dP}{dx}A = \tau_o O \,, \quad \text{i.e.} \quad -\frac{dP}{dx} = \frac{4\tau_o}{D}$$

for *a pipe of a circular cross section* and internal diameter D.

Repetition: C. of Momentum in 1D-flow

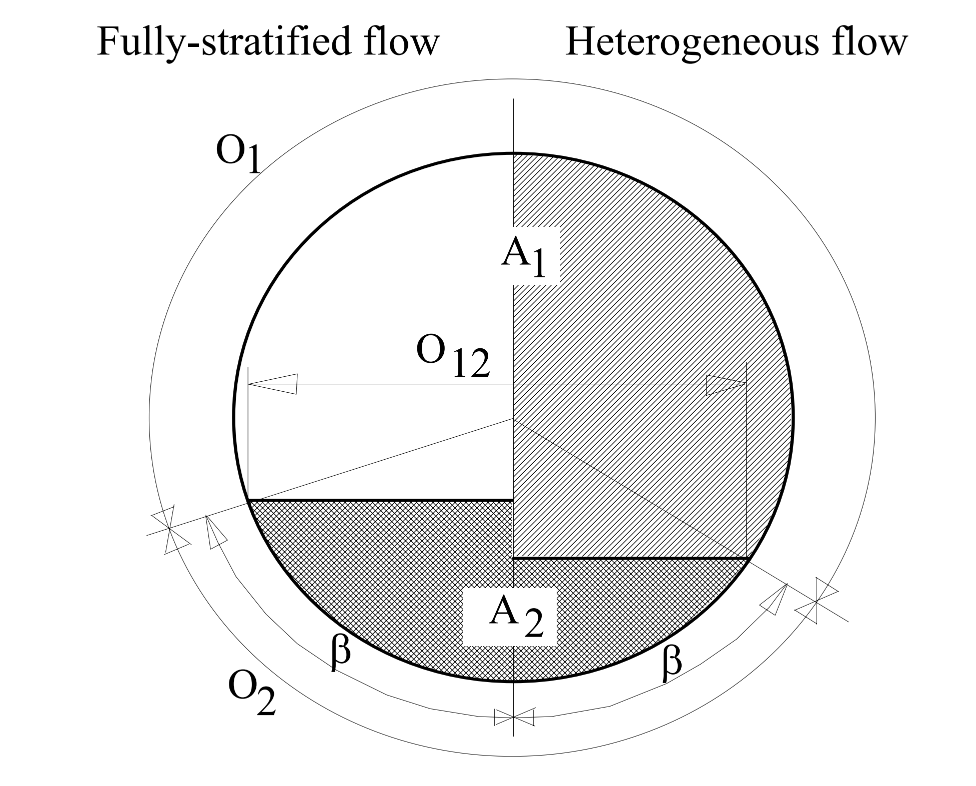
For a straight horizontal circular pipe

$$\frac{P_1 - P_2}{L} = \frac{4\tau_o}{D}$$



2LM: Flow Pattern

A. Simplified Flow Pattern



2LM: Conservation of momentum

A. Force-balance equations

(for the unit length L of a pipe)

Upper layer: **$dP.A1 = \tau 1.O1 + \tau 12.O12$**

Lower layer: **$dP.A2 = -\tau 12.O12 + \tau 2.O2$**.

$\tau 2.O2 = \tau 2_f.O2 + \tau 2_s.O2$ and **$\tau 2_s.O2 = \mu_s.F_N$**.

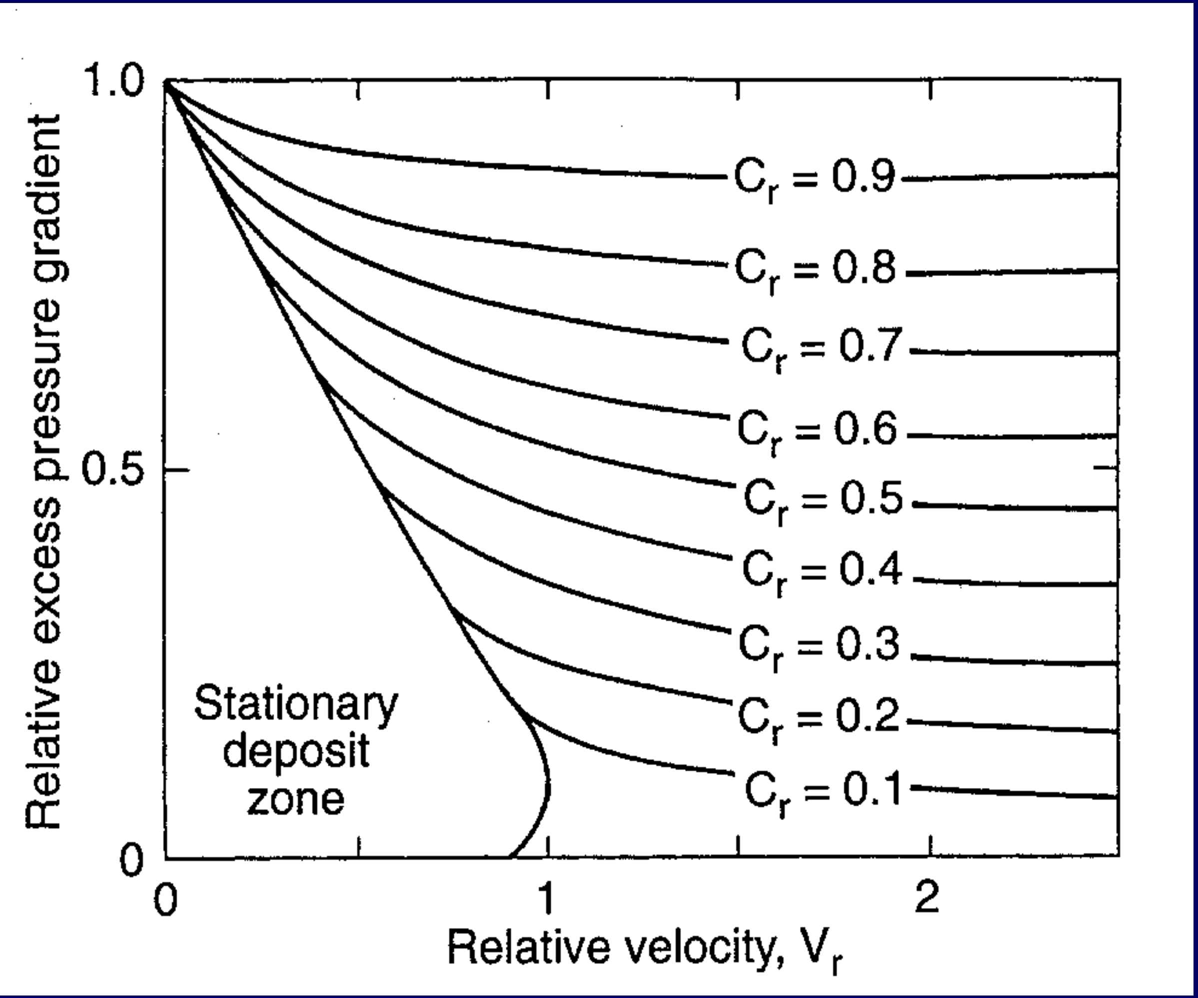
$\tau 2_f$ is the shear stress due to flow at a pipe wall of perimeter O2 (velocity-dependent viscous friction)

$\tau 2_s$ is the shear stress due to sliding at a pipe wall of the solids occupying a contact layer (velocity-independent mechanical friction).

2LM: Applications

Fully Stratified Flow

Prediction of frictional pressure drop (hydraulic gradient)



2LM: Applications

Fully Stratified Flow

Relative excess pressure gradient:

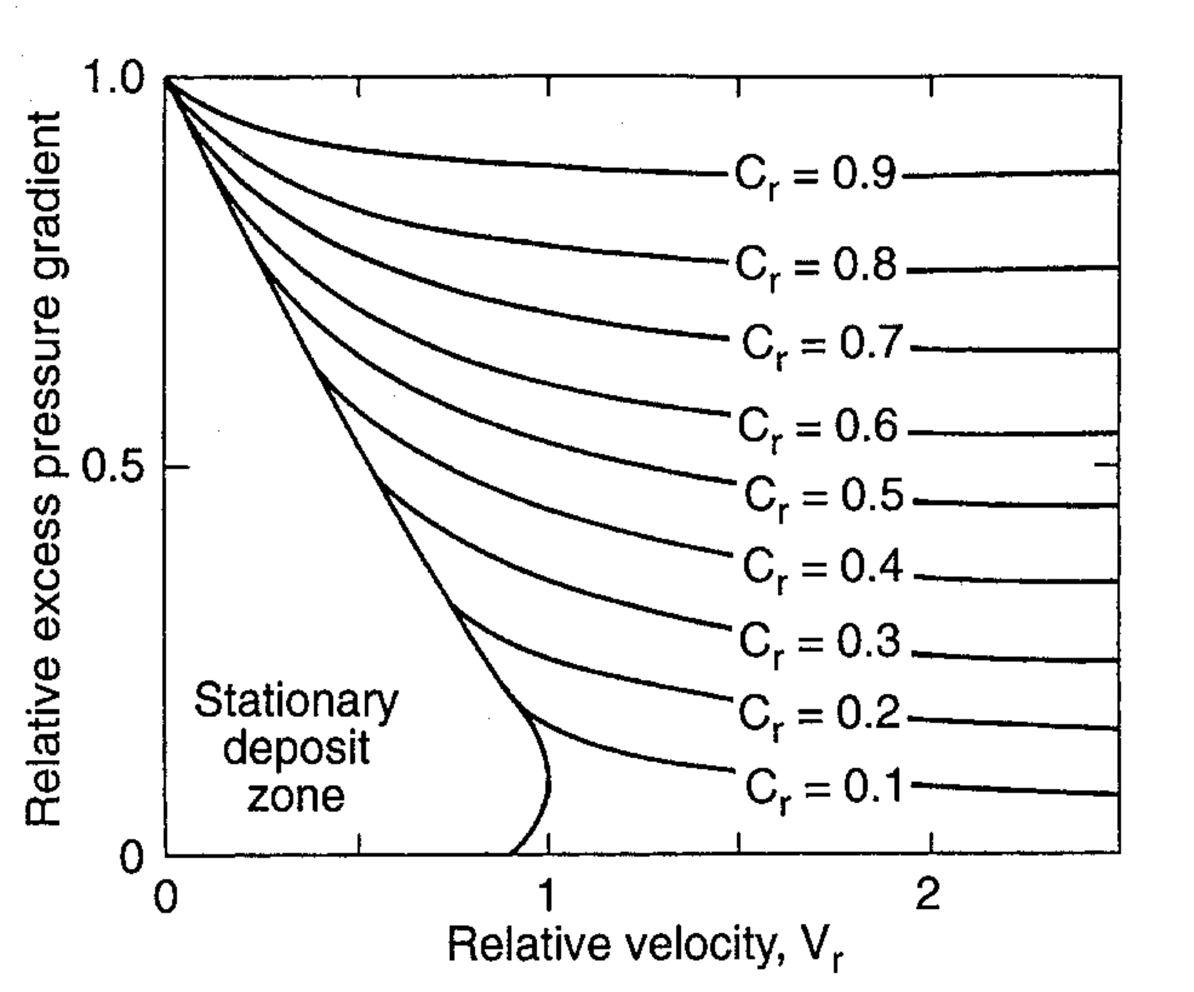
$$\frac{I_m - I_f}{I_{pg}} = \frac{I_m - I_f}{2\mu_s \left(S_s - S_f\right) C_{vb}}$$

Relative velocity:

$$V_r = \frac{V_m}{V_{sm}}$$

Relative concentration:

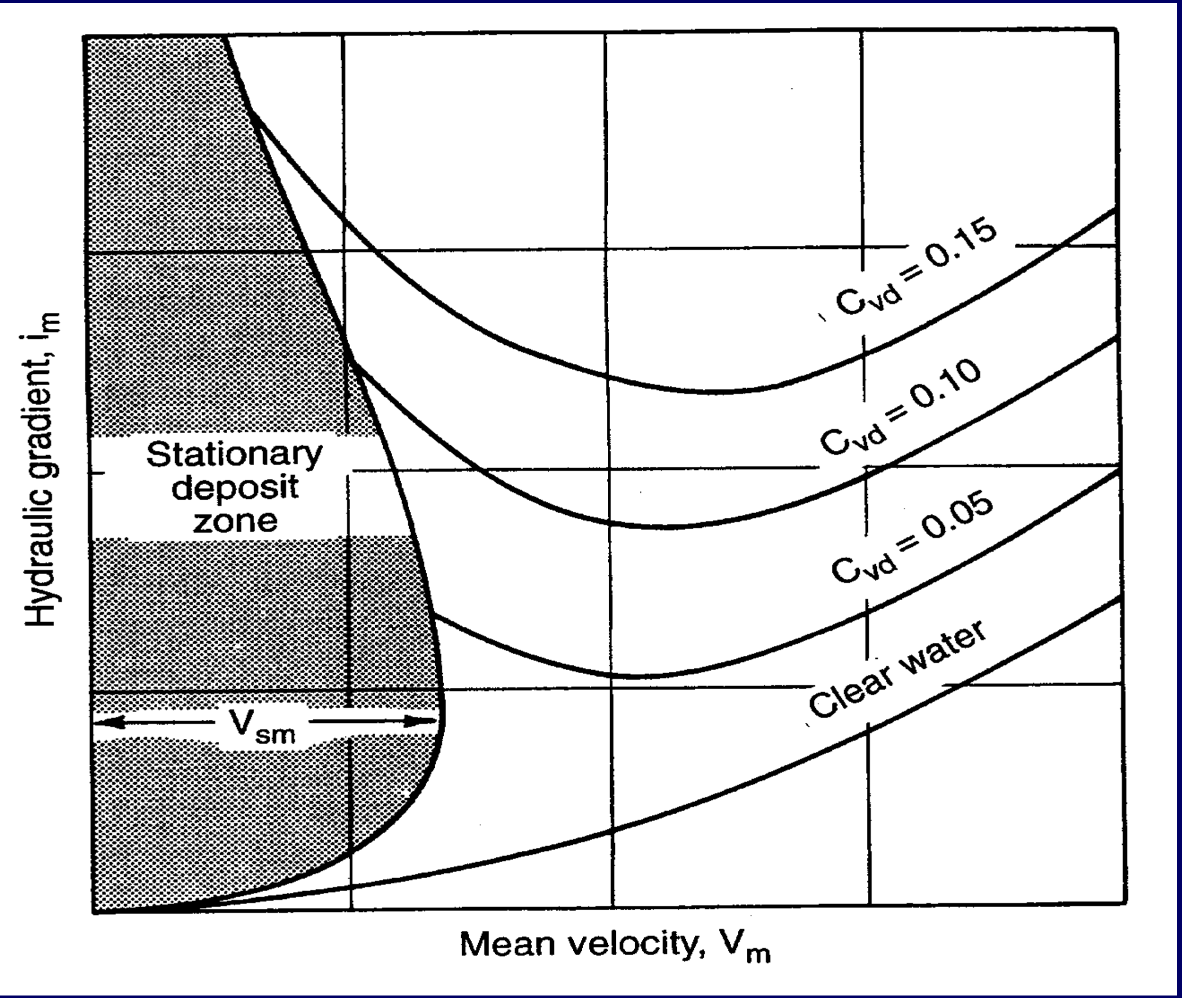
$$C_r = \frac{C_{vd}}{C_{vb}}$$



2LM: Applications

Fully Stratified Flow

The result of prediction of frictional pressure drop (hydraulic gradient)



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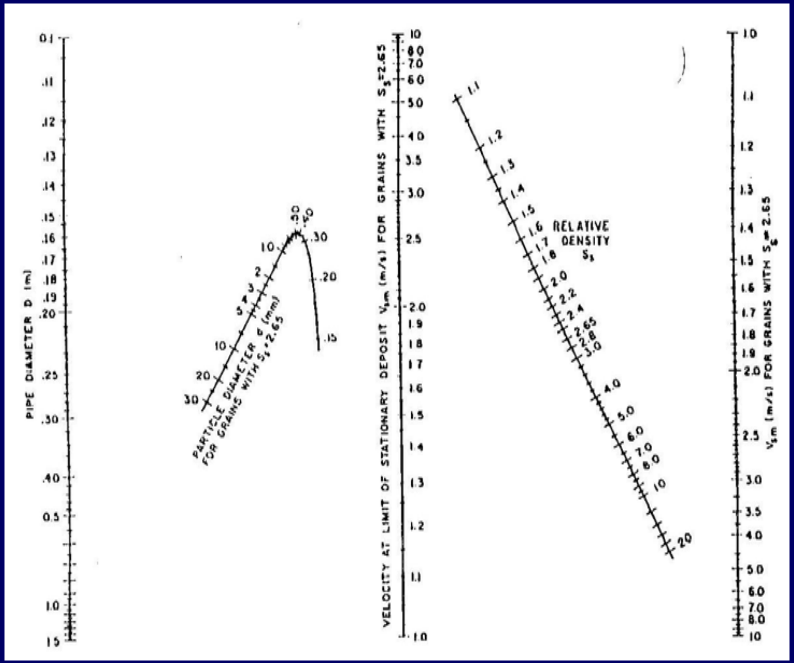
2LM: Applications

Fully & Partially Stratified Flows

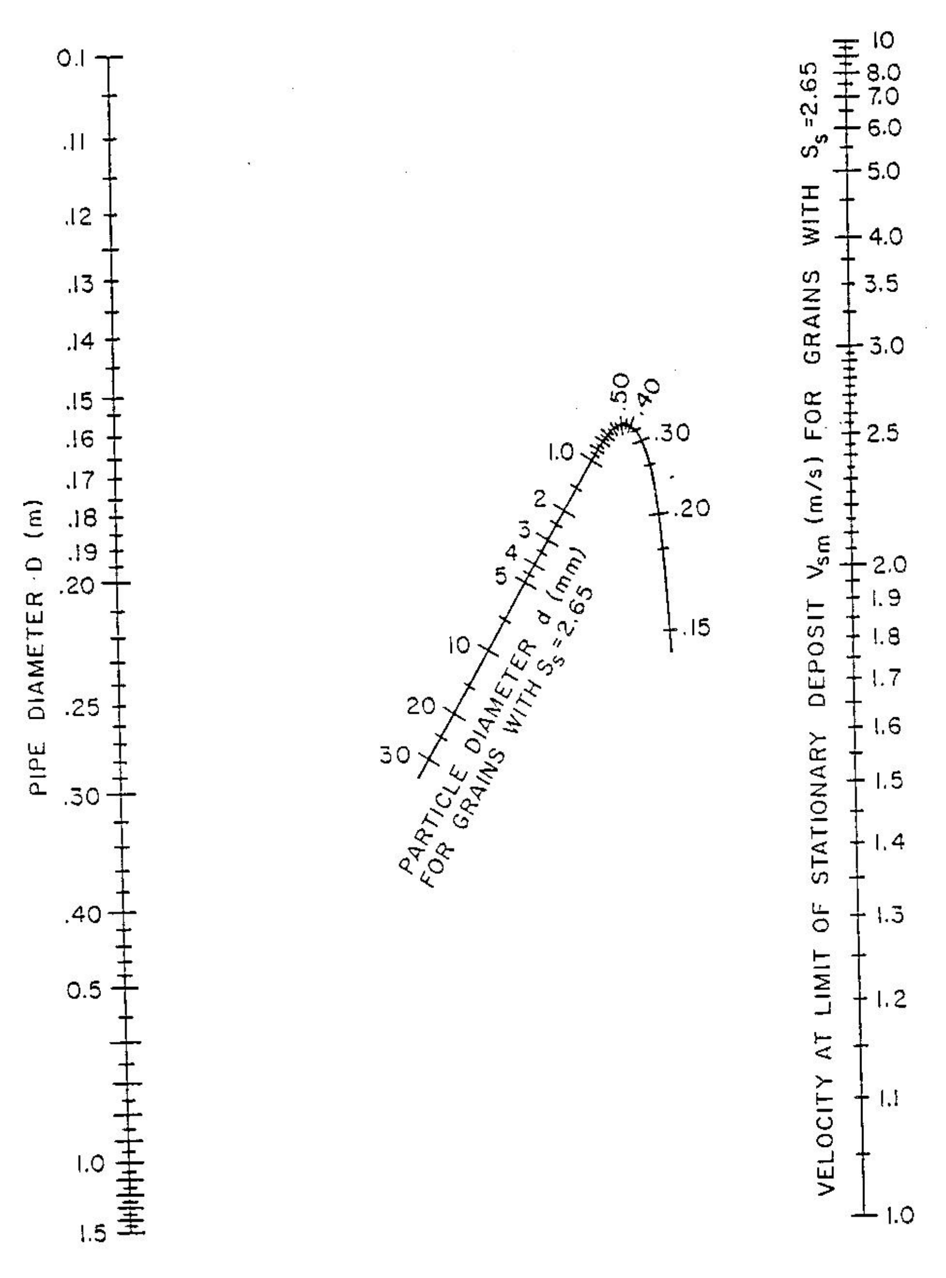
Prediction
of
the maximum velocity at the limit of stationary deposition
(the demi-McDonald's diagram)



2LM: Applications

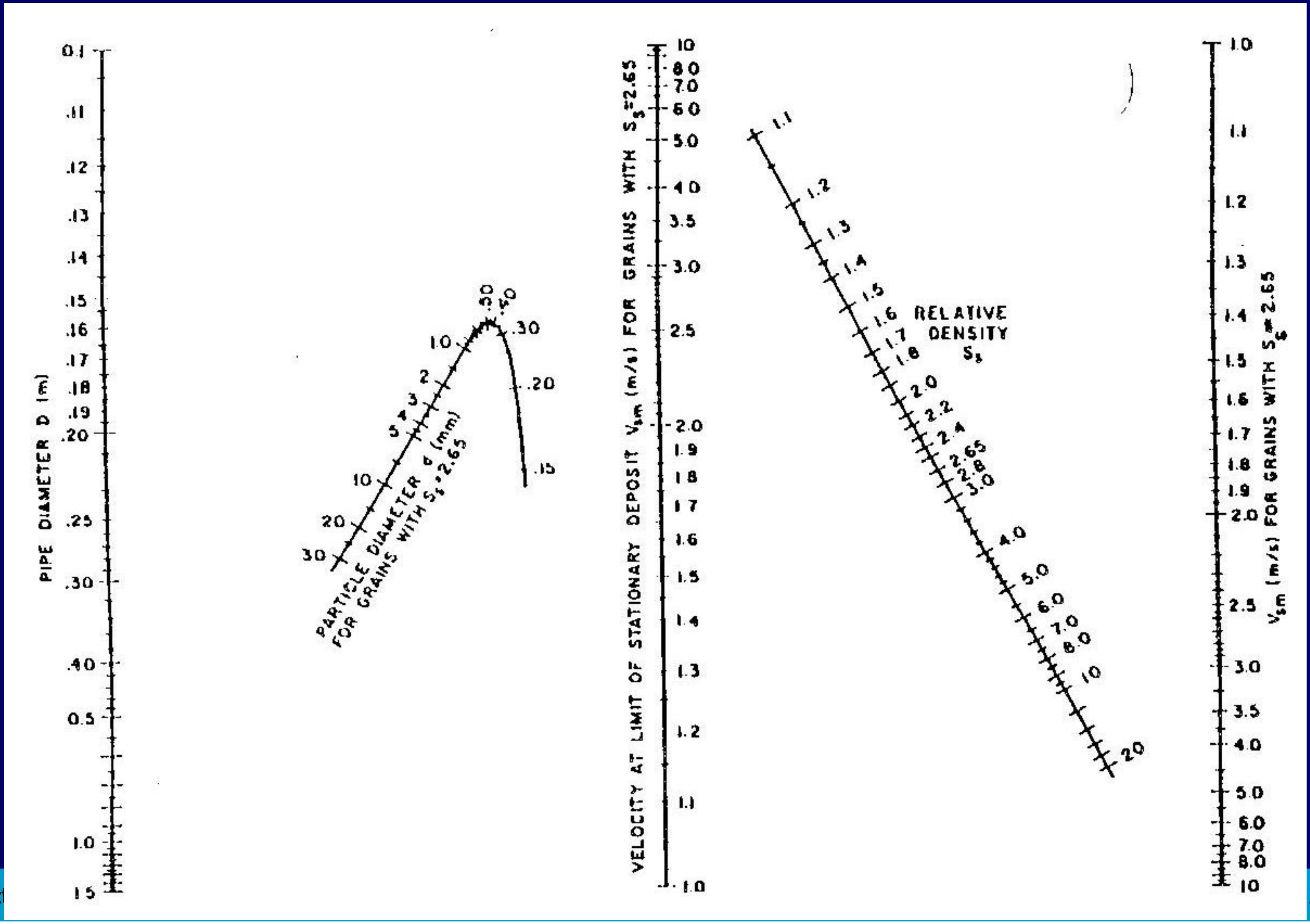
Max velocity at limit of stationary deposit:

$$V_{sm} = fn(d,D)$$



2LM: Applications

$V_{sm} = fn(d, D, S_s)$



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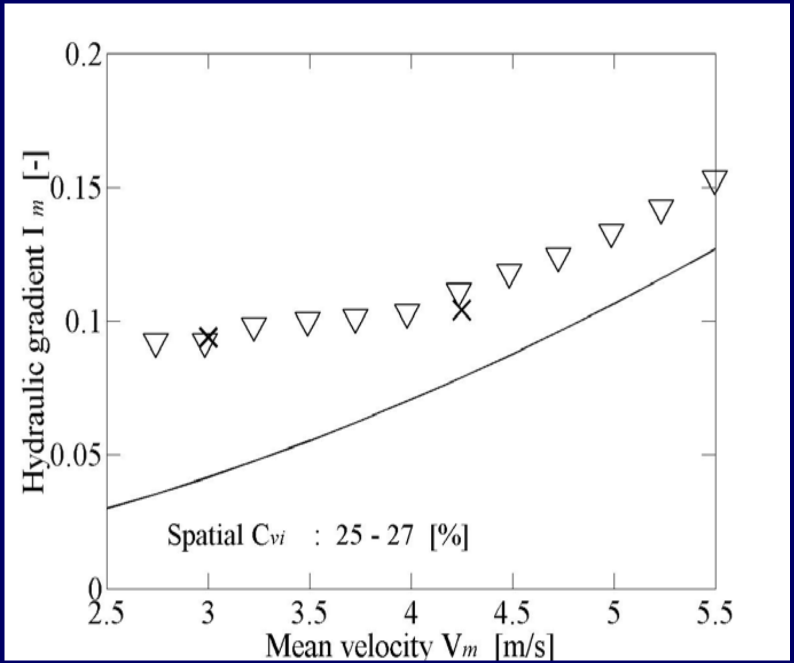
2LM: Applications

Partially Stratified Flows

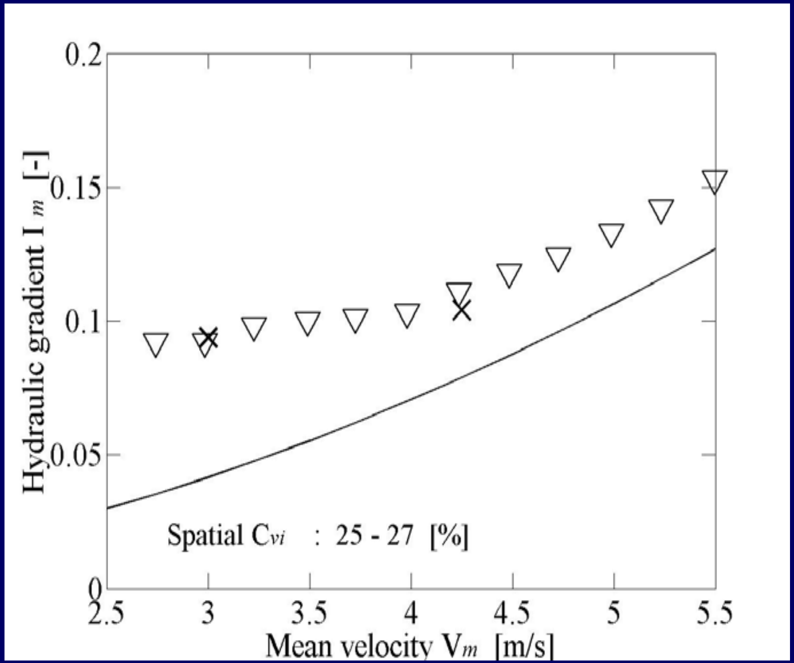
Prediction
of
the deposition-limit velocity
the hydraulic gradient
the thickness of the bed
the velocity of the bed
the slip ratio



2LM: Applications

Partially Stratified Flows

EXAMPLE



Example: Experiments

Measurements in the 150-mm pipe:

Pressure drop

Mean velocity of slurry

Mean concentration of solids

Concentration distribution.

Example: Modeling of suspension

The concentration gradient

the Schmidt-Rouse model with the implemented hindered settling effect

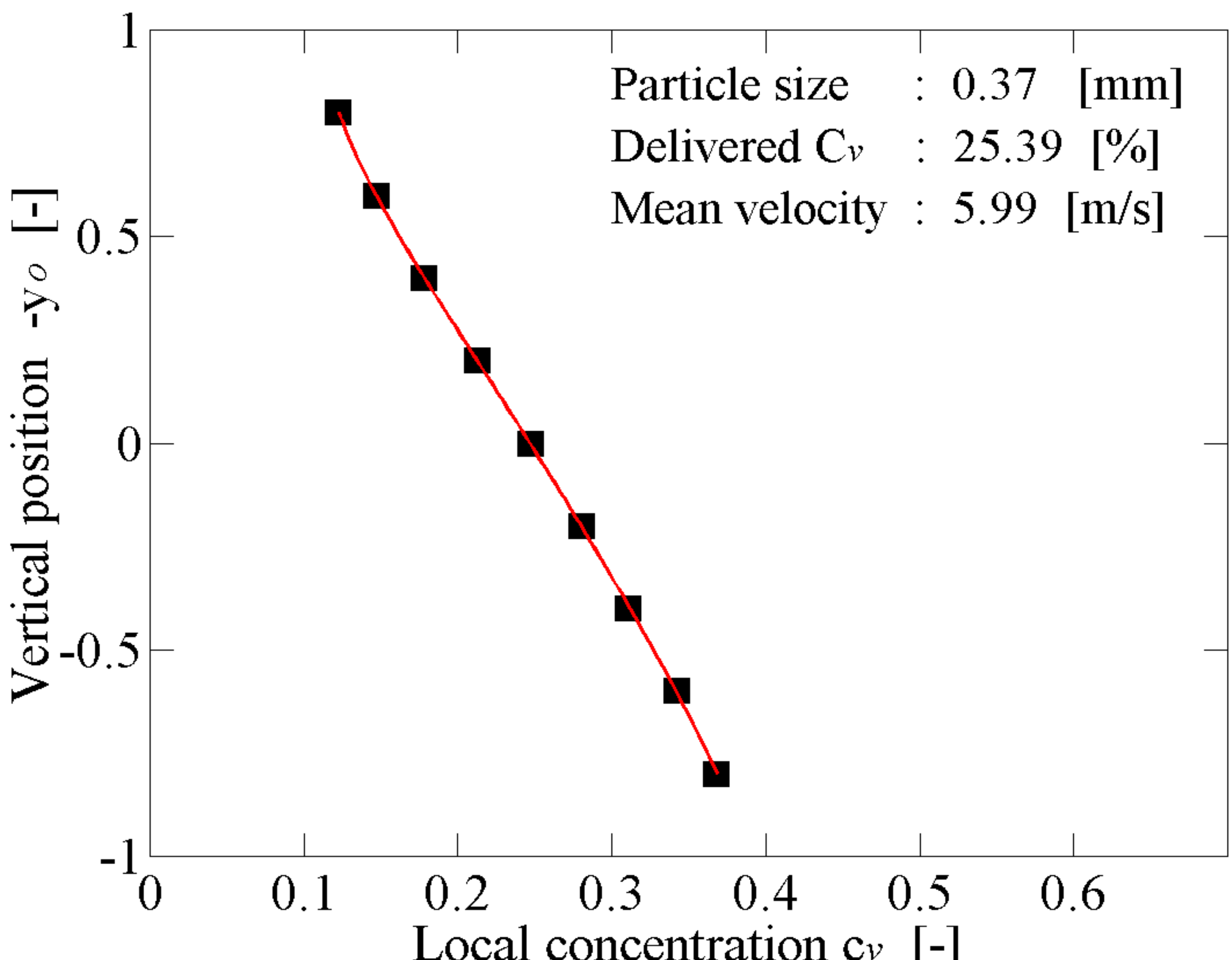
$$\varepsilon_s \frac{dc_v}{dy} = v_t \left(1 - c_v\right)^m c_v$$

$$\varepsilon_s = fn\left(D, u_* = \sqrt{\frac{\lambda}{8}} V_m \right)$$

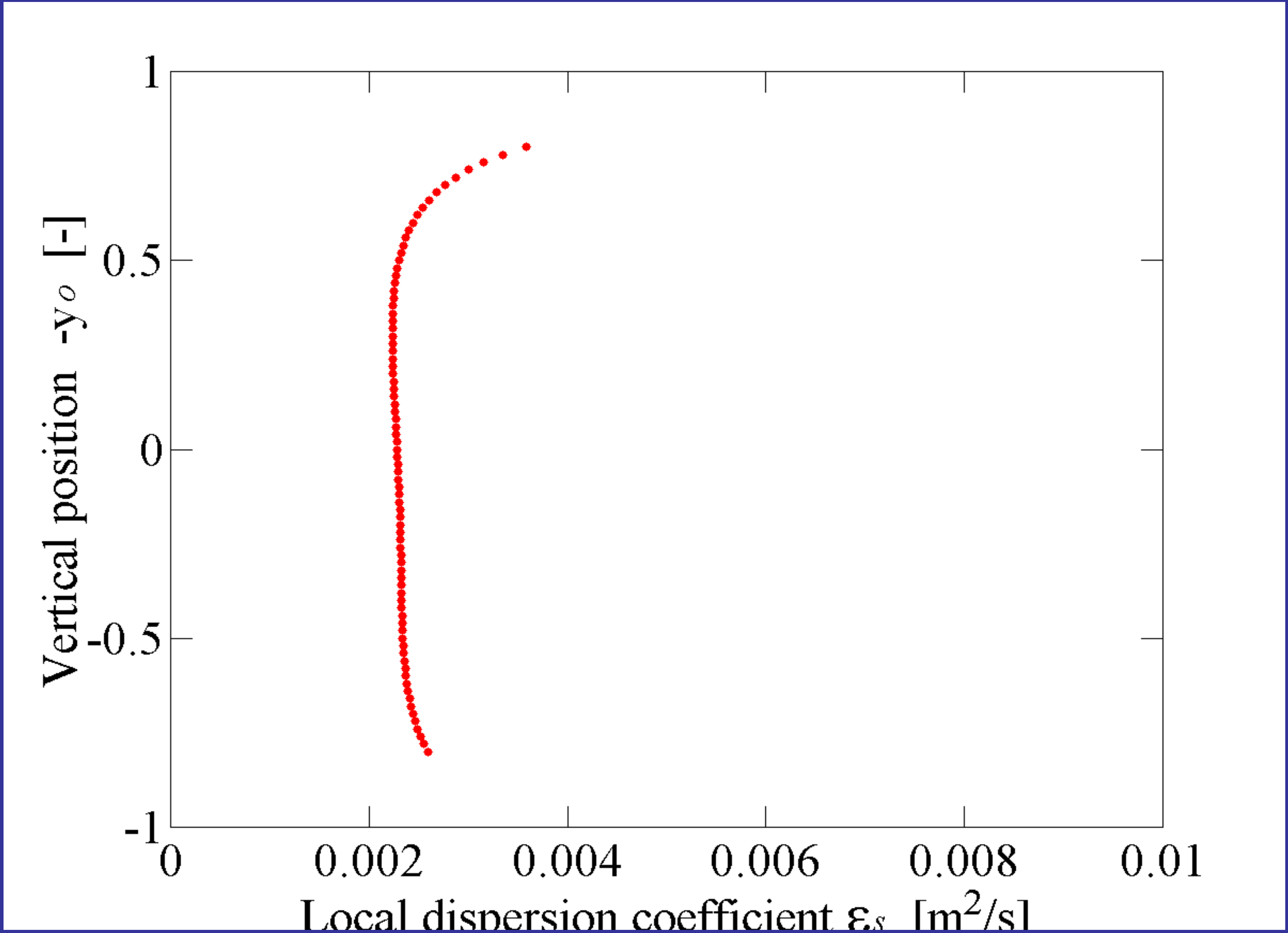
The hydraulic gradient

the two-layer model with the stratification-ratio equation

Example: Measured concentr'n profile



Example: Local solids dispersion coeff.



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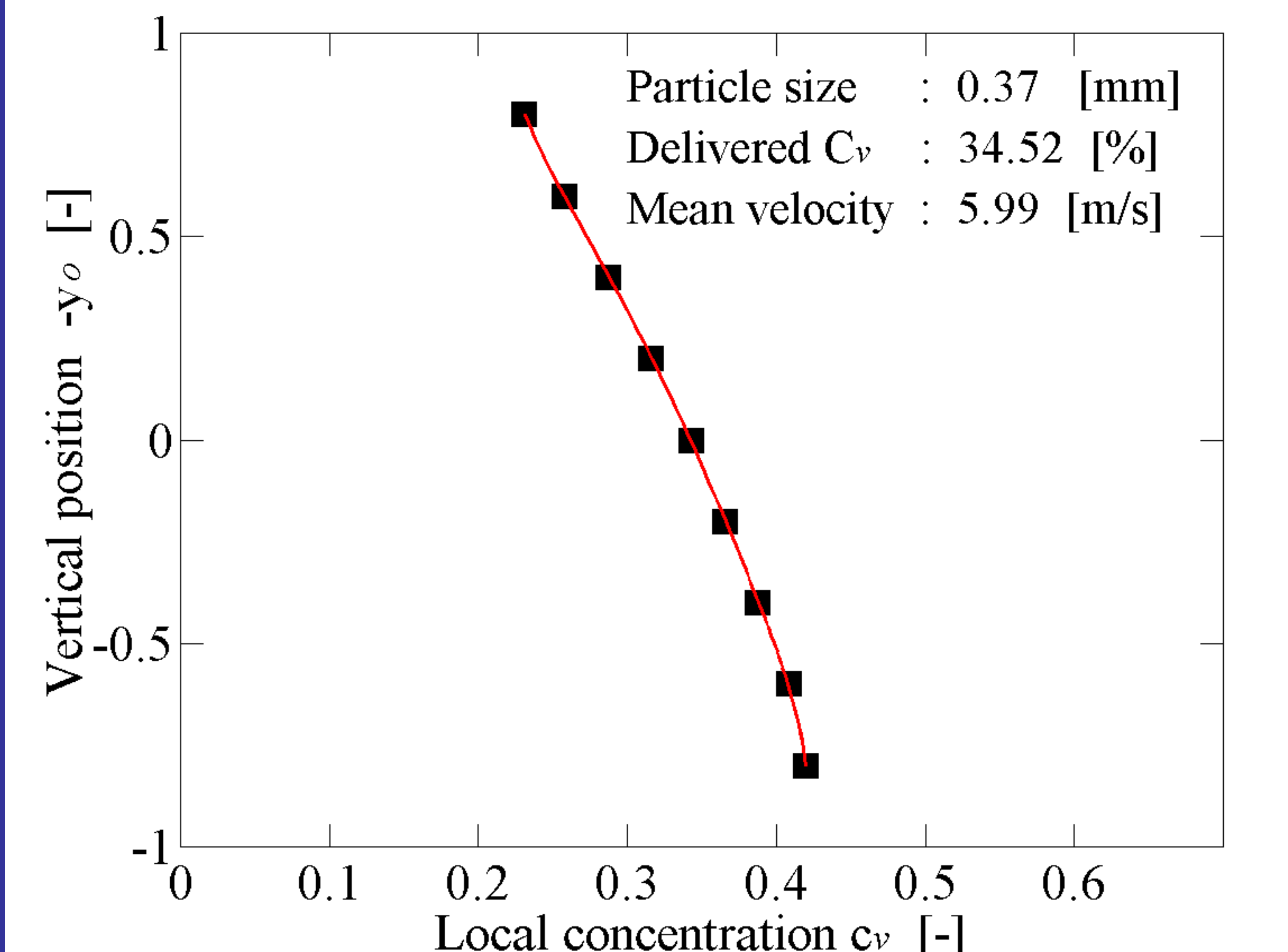

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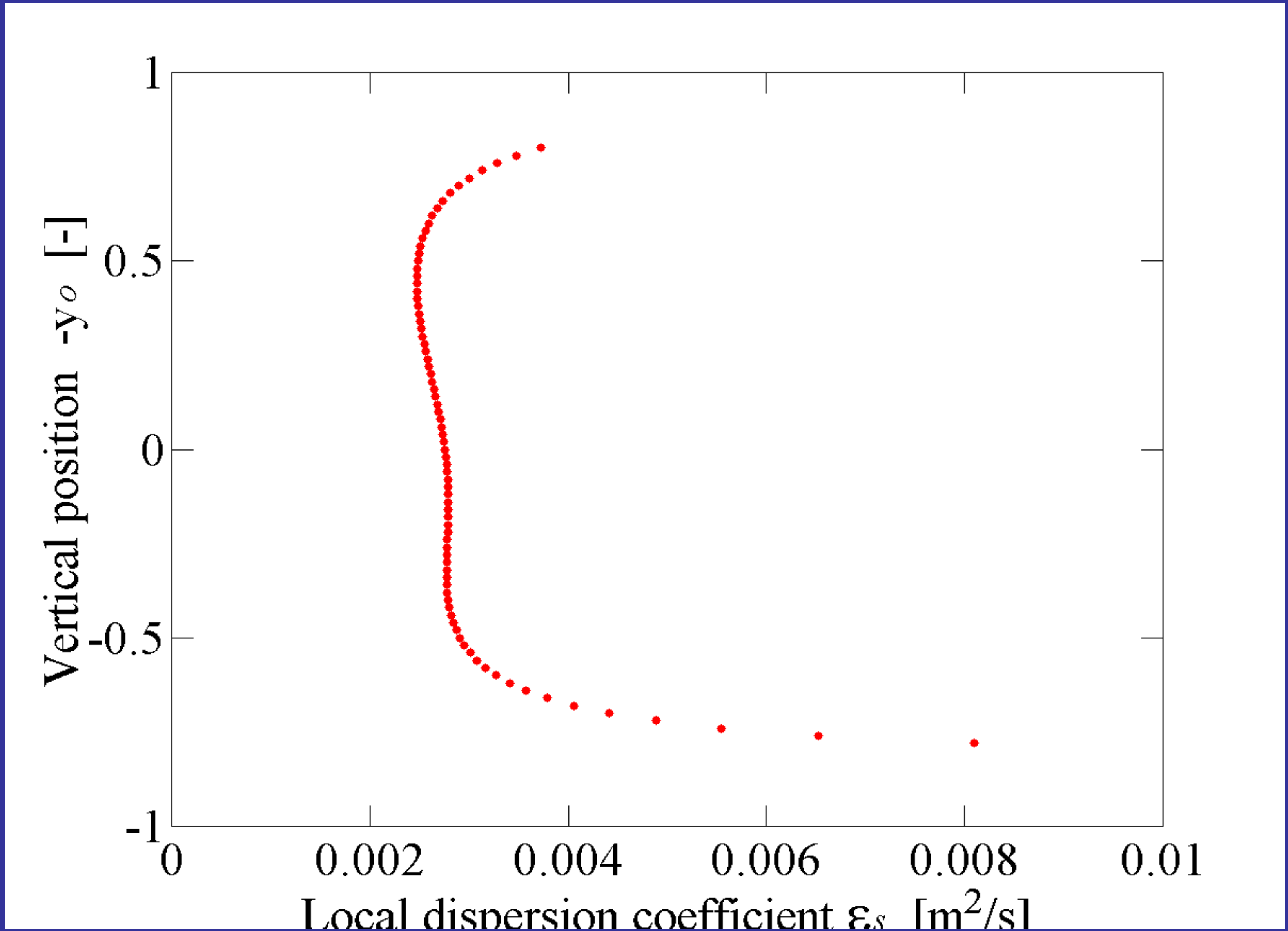
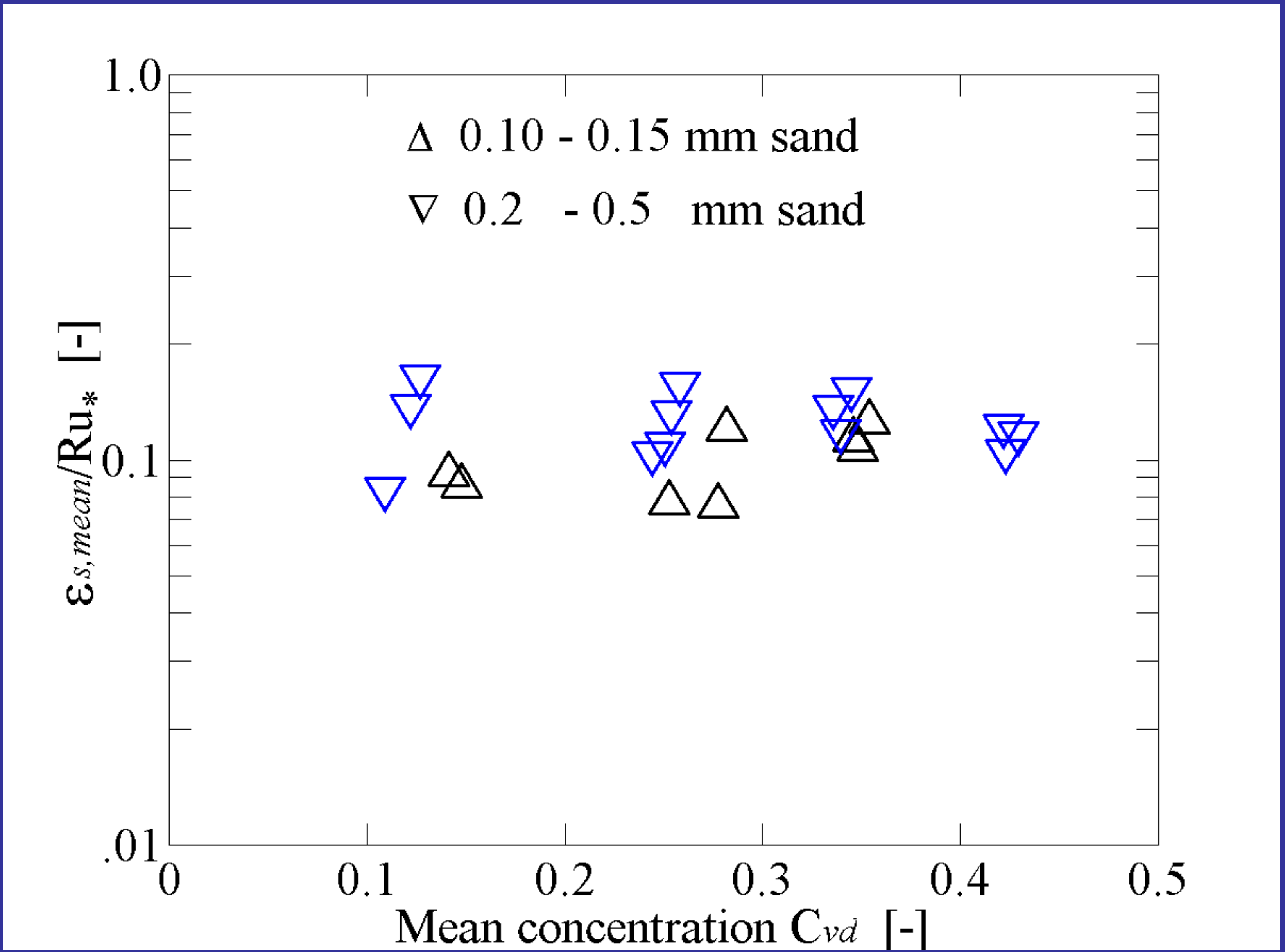
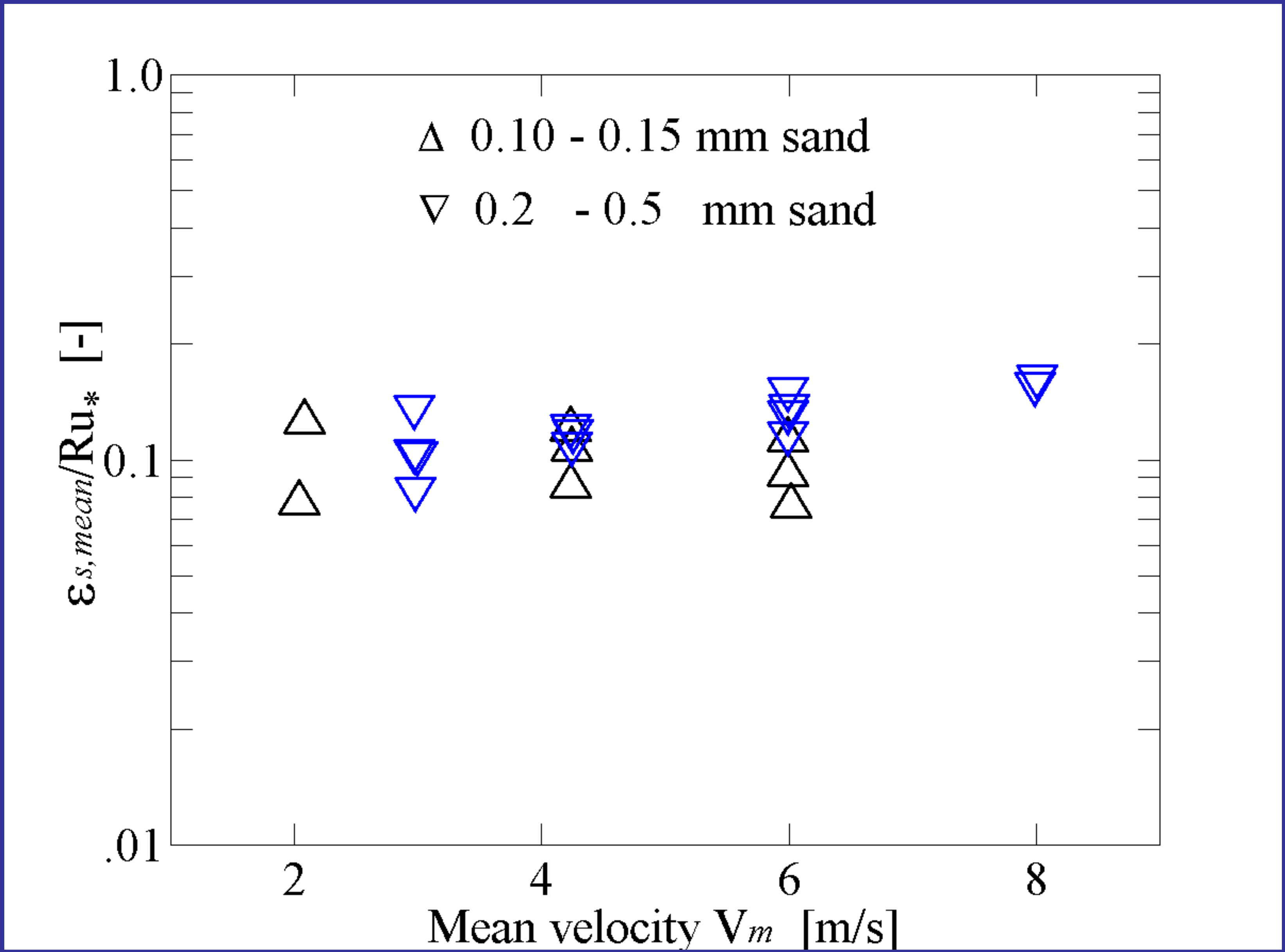


Example: Solids dispersion coefficient



Example: Solids dispersion coefficient



Example: Construction of simplified profile

Inputs:

- Measured concentration profile
- Measured mean concentration C_{vi}.

Outputs:

- The value of the solids dispersion coefficient
- The position of the interface between two layers
- The mean concentration of solids in the bed
- The stratification-ratio value.

Example: Stratification evaluated

Position of the interface:

- The position at which the concentration profile of turbulent suspension is linked to the granular bed.

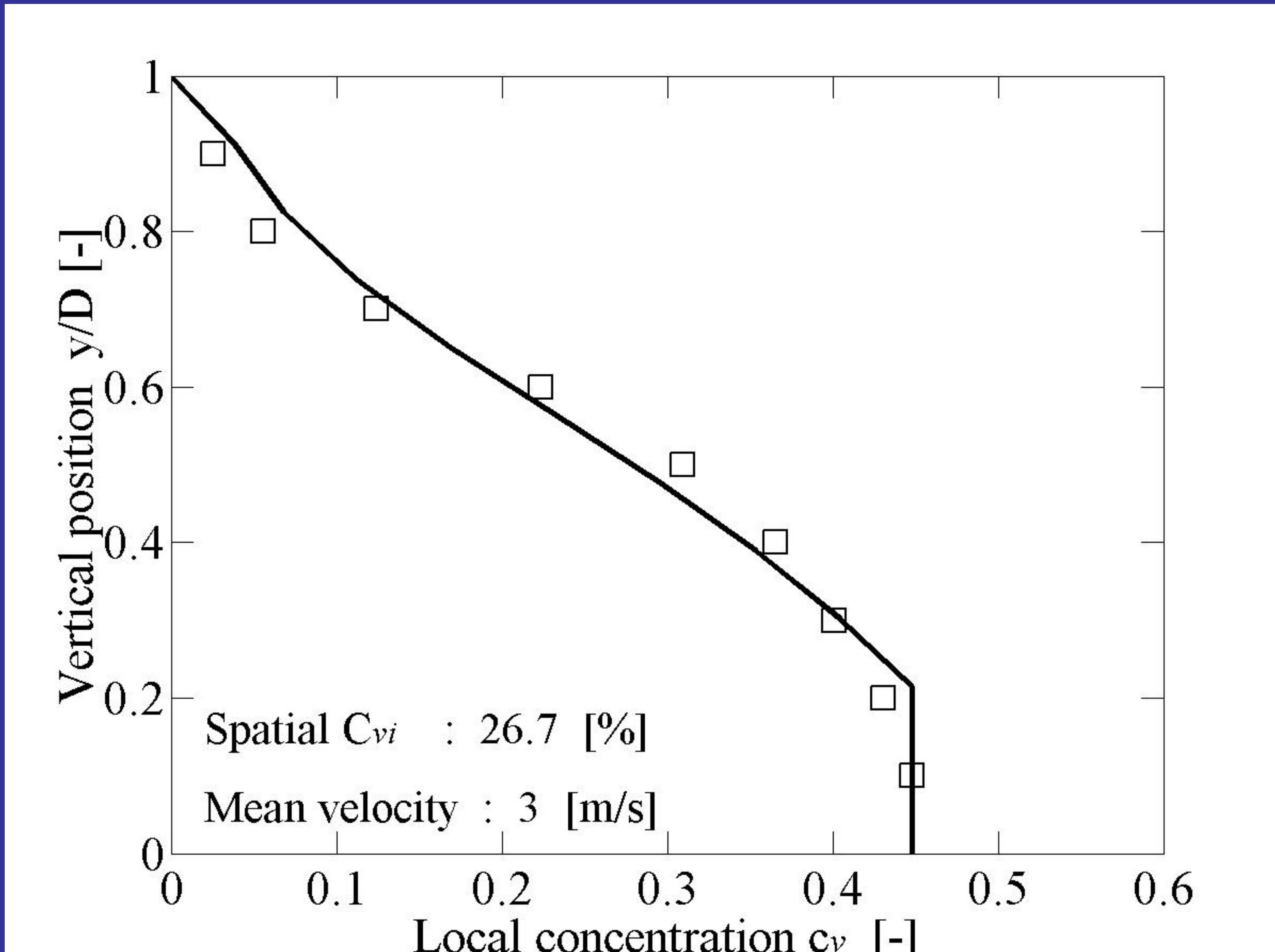
Mean concentration in the bed:

- The mean concentration tend to vary slightly with C_{vi} and V_m.

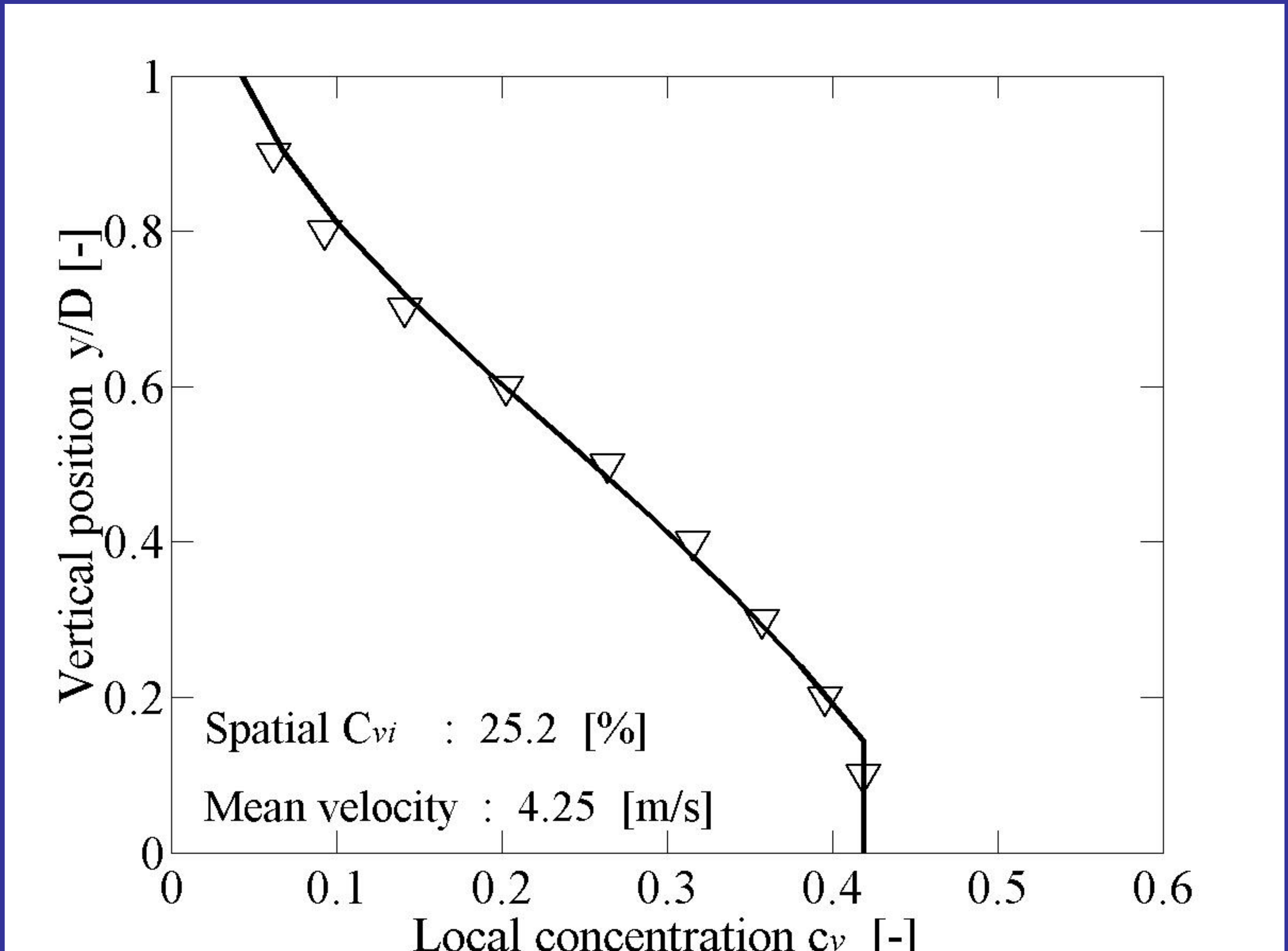
Stratification ratio:

- The portions of solids that contribute to contact or suspended loads.

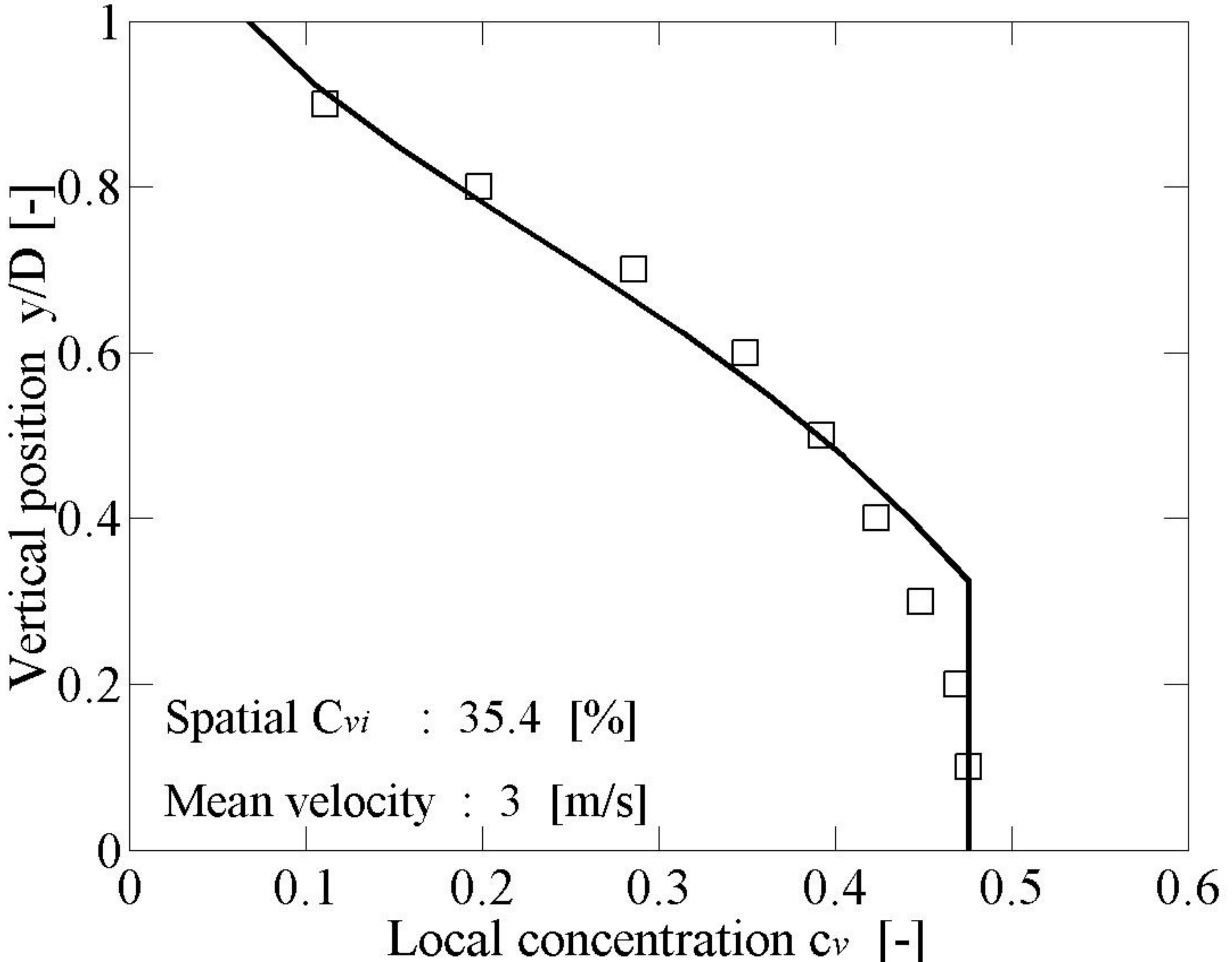
Medium sand: concentr. profile



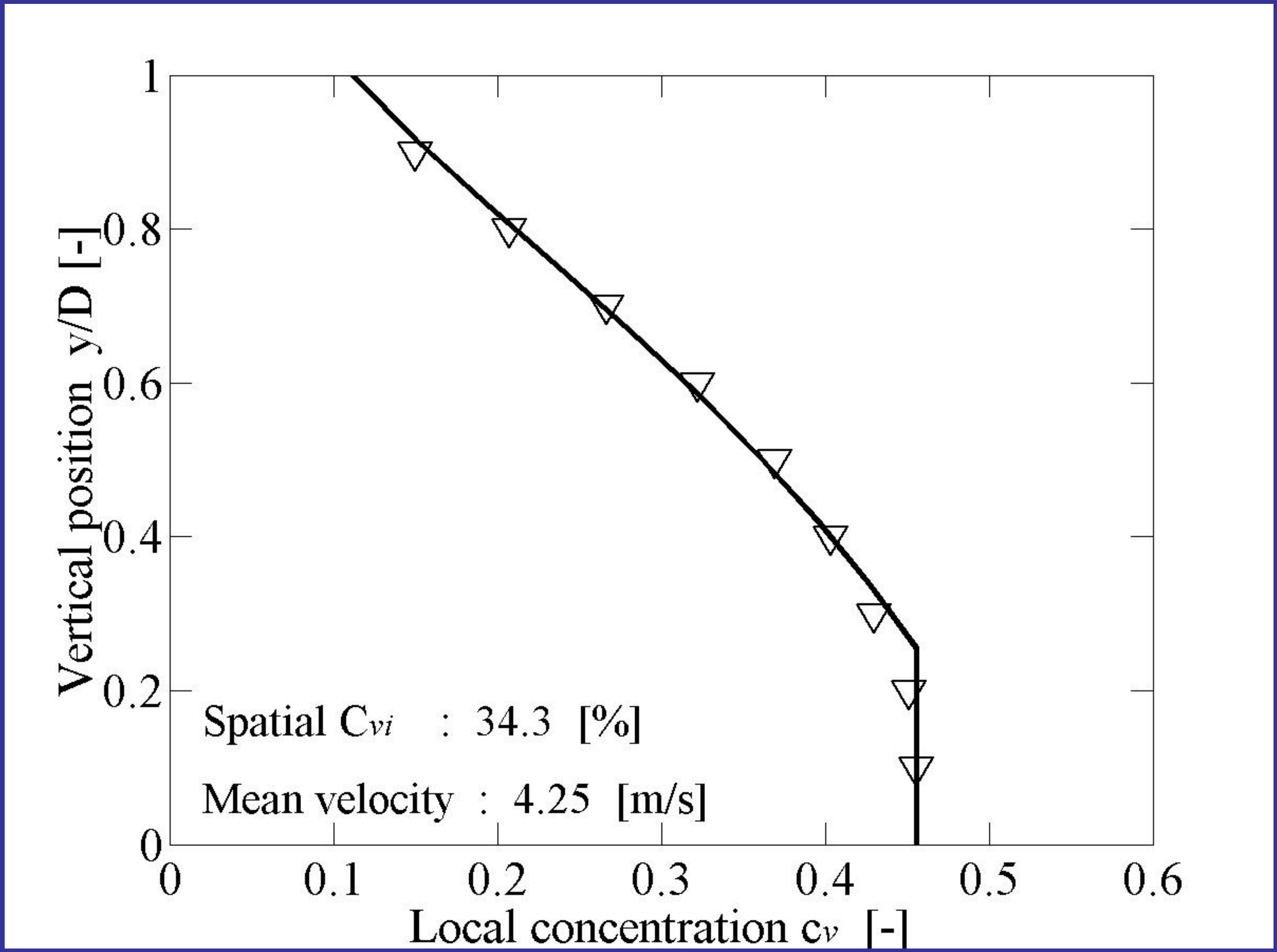
Medium sand: concentr. profile



Medium sand: concentr. profile



Medium sand: concentr. profile



Hydraulic gradient

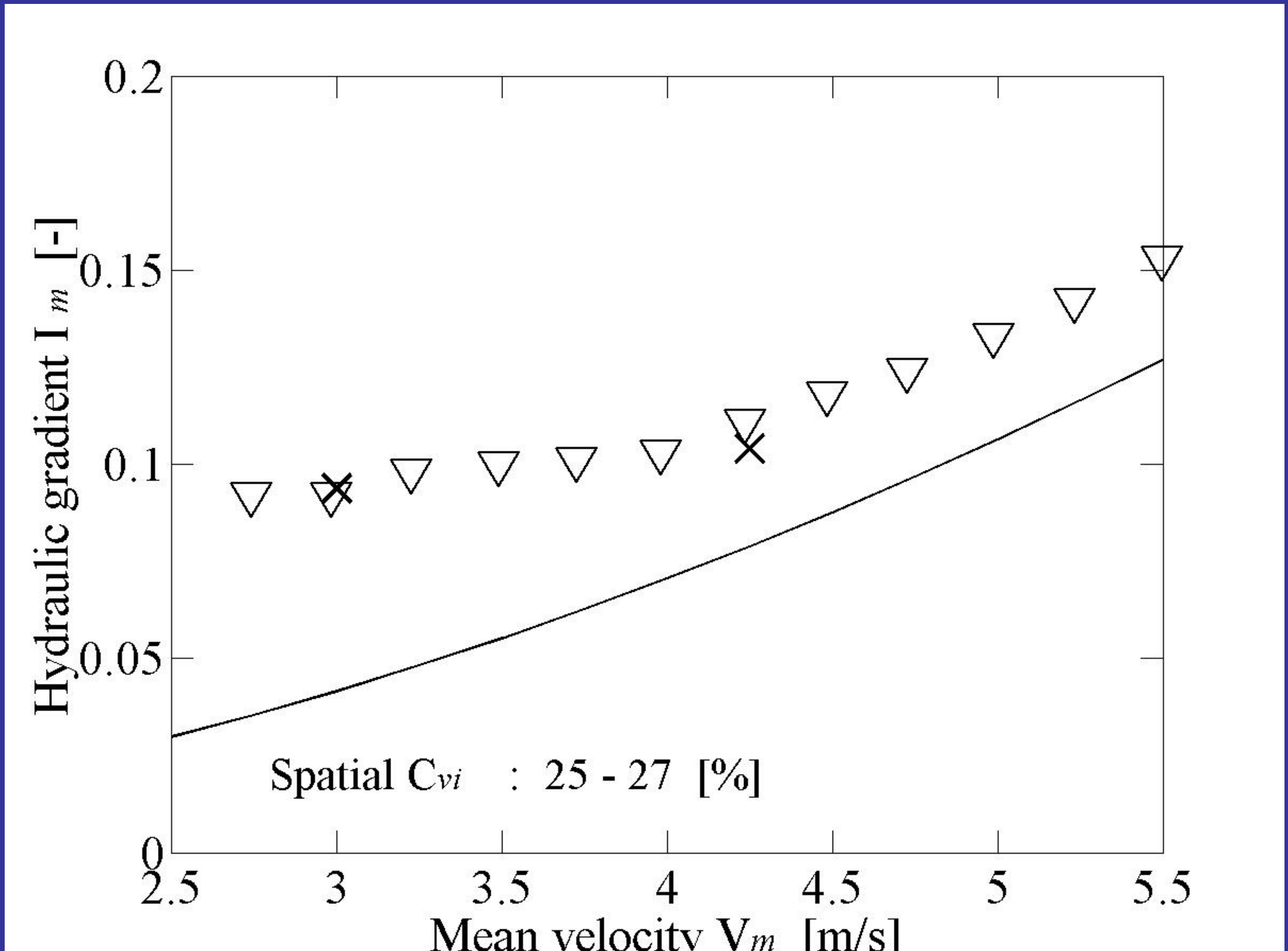
Inputs to the two-layer model:

- Parameters of simplified concentration profile
- Measured mean velocity (V_m)
- Friction coefficients (μ_s, λ_{1f} , λ_{2f} , λ_{12}).

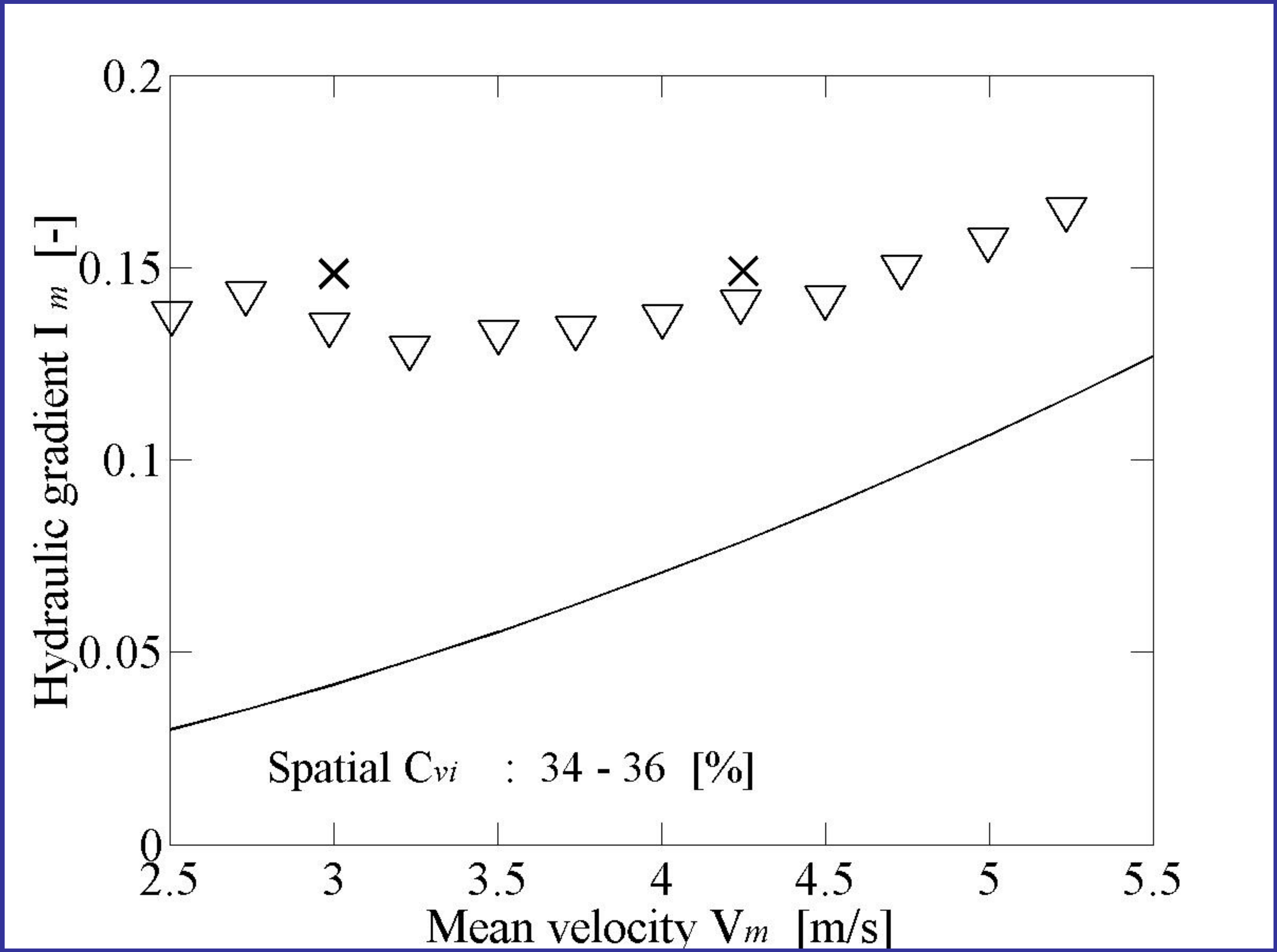
Outputs:

- The value of the hydraulic gradient
- The value of the slip ratio.

Medium sand: hydraulic gradient



Medium sand: hydraulic gradient



Example: Conclusions

- The concentration gradients in slurry flows of fine to medium sands in a 150-mm pipe are due dispersive action of carrier turbulence.
- The concentration gradients can be predicted using the Schmidt-Rouse turbulent diffusion model with the implemented hindered settling effect. The dispersion coefficient can be considered constant across the suspension flow.

Example: Conclusions

- The concentration gradient can be used for the determination of the simplified concentration profile in the two-layer flow pattern.

- The hydraulic gradient determined using the two-layer model fits reasonably the measured value.