oe4625 Dredge Pumps and Slurry Transport

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5. MODELING OF NON-STRATIFIED MIXTURE FLOWS (Pseudo-Homogeneous Flows)

NEWTONIAN SLURRIES

NON-NEWTONIAN SLURRIES

Repetition: Newtonian vs non-Newtonian

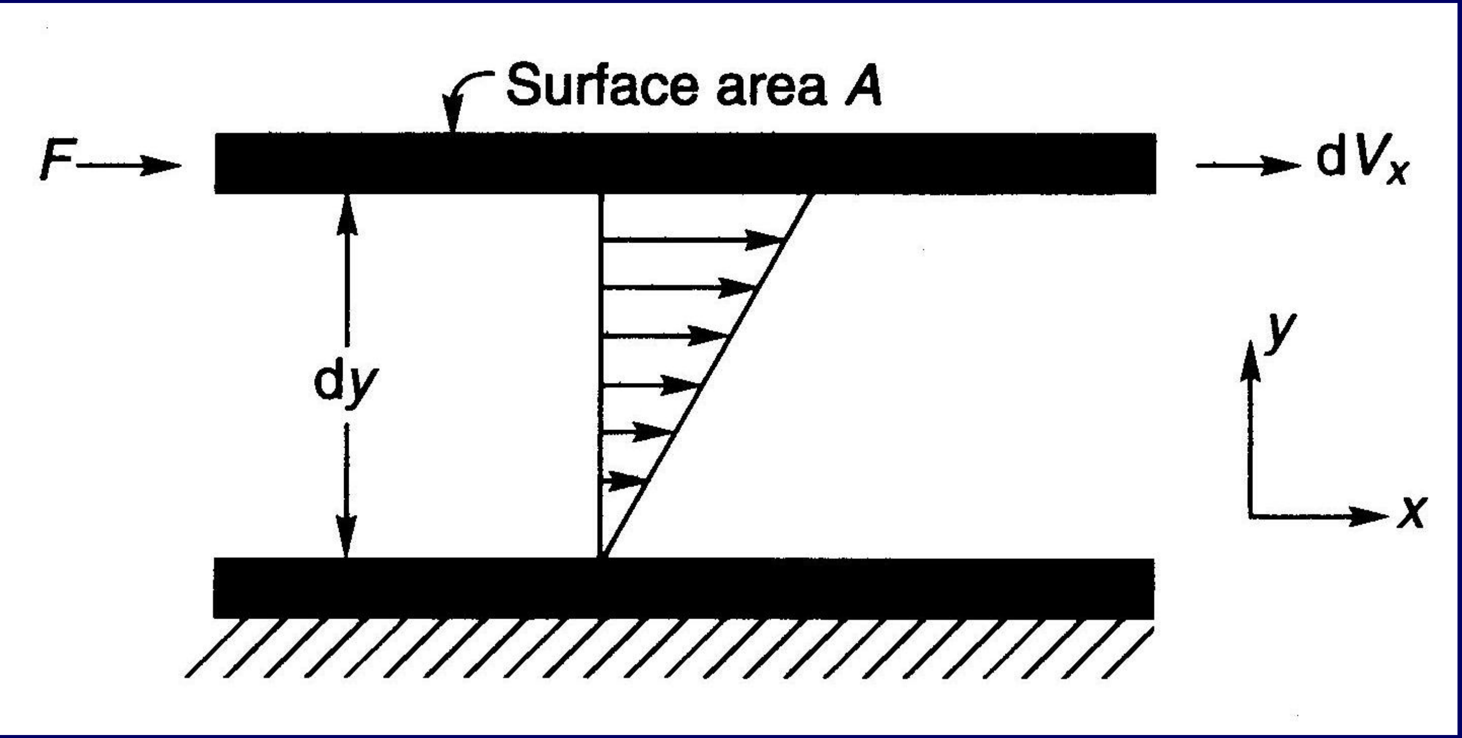
Newton’s law of liquid viscosity (valid for laminar flow)

$$\tau = \mu_f \left(-\frac{dv_x}{dr} \right)$$

Repetition: Newtonian vs non-Newtonian

Newton's law of liquid viscosity (valid for laminar flow):

$$\tau = \frac{F}{A} = \mu_f \left(- \frac{dV_x}{dy} \right)$$



MODELING OF NEWTONIAN SLURRY FLOW

EQUIVALENT-LIQUID MODEL

Equivalent-Liquid Model

A. Physical background

- Slurry flow behaves as a flow of a single-phase liquid having the density of the slurry.

- The "equivalent liquid" has the density of the mixture but other properties (as viscosity) remain the same as in the liquid (water) alone.

Equivalent-Liquid Model

B. Construction of the model for I_m:

- The model suggests that all particles contribute to the increase of the suspension density.
- The increase of the mixture density is responsible for the increase of the liquid-like shear stress resisting the flow at the pipe wall.
- The model equation is obtained in the same way as the Darcy-Weisbach equation with one exception: <u>the density of mixture is considered instead of the liquid density</u>.

Repetition: Conservation of Momentum in 1D-flow

For *additional conditions* :

- incompressible liquid,
- steady and uniform flow in a horizontal straight pipe

$$-\frac{dP}{dx}A = \tau_o O \, , \quad \text{i.e.} \quad -\frac{dP}{dx} = \frac{4\tau_o}{D}$$

for *a pipe of a circular cross section* and internal diameter D.

Repetition: Water Friction in 1D Flow in Pipe

A comparison of the Darcy-Weisbach friction coefficient equation

$$\lambda_f = \frac{8\tau_o}{\rho_f V_f^2}$$

+

with the linear momentum eq. (driving force = resistance force) for pipe flow

$$-\frac{dP}{dx} = \frac{4\tau_o}{D}$$

=

gives the general pressure-drop equation for the pipe flow

(Darcy-Weisbach equation, 1850)

$$-\frac{dP}{dx} = \frac{\lambda_f}{D}\frac{\rho_f V_f^2}{2}$$

Equivalent-Liquid Model

B. Construction of the model for I_m (cont'):

- The shear stress at the pipe wall:

water flow

$$\tau_{o,f} = \frac{\lambda_f}{8} \rho_f V^2$$

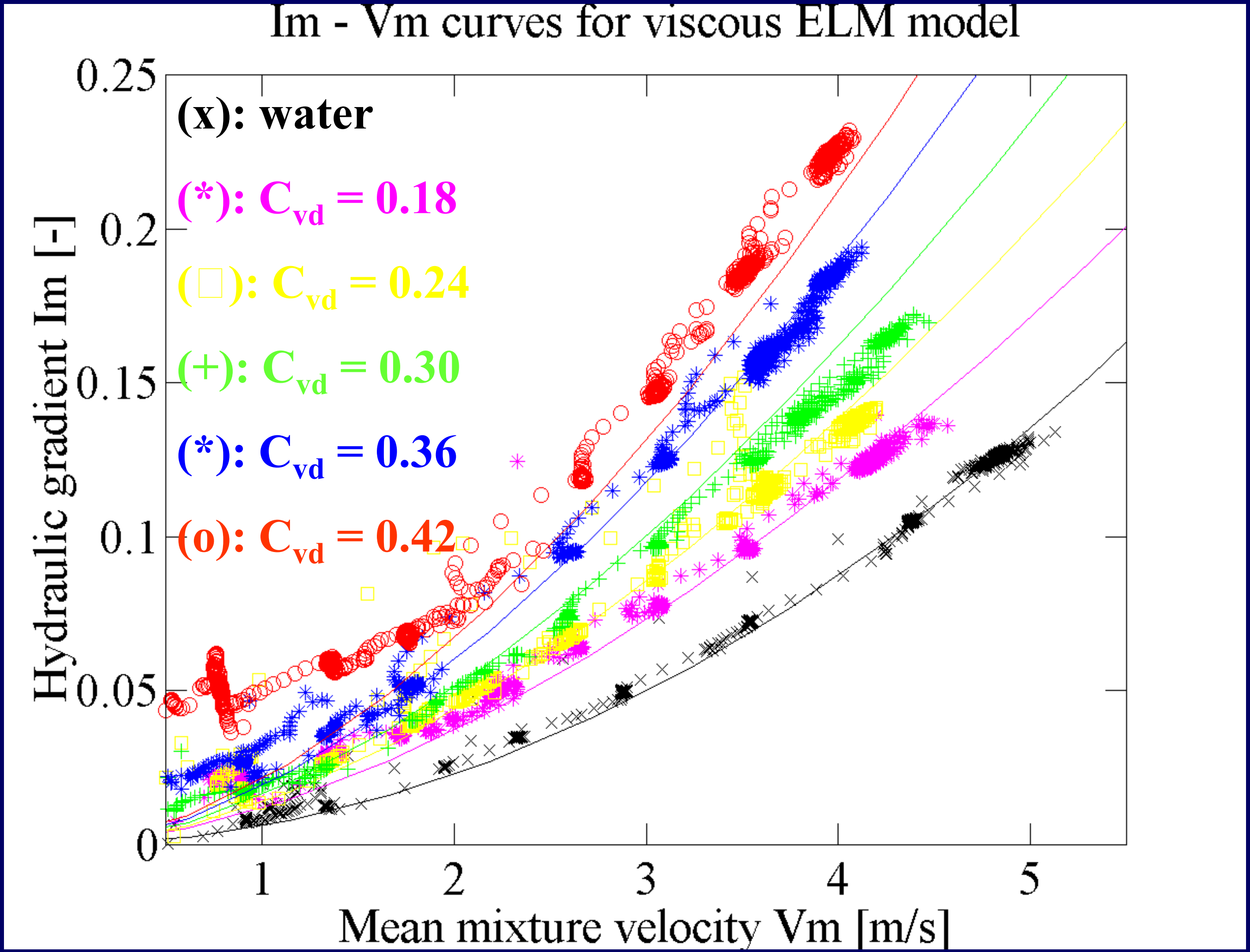
equivalent-liquid flow

$$\tau_{o,m} = \frac{\lambda_f}{8} \rho_m V^2$$

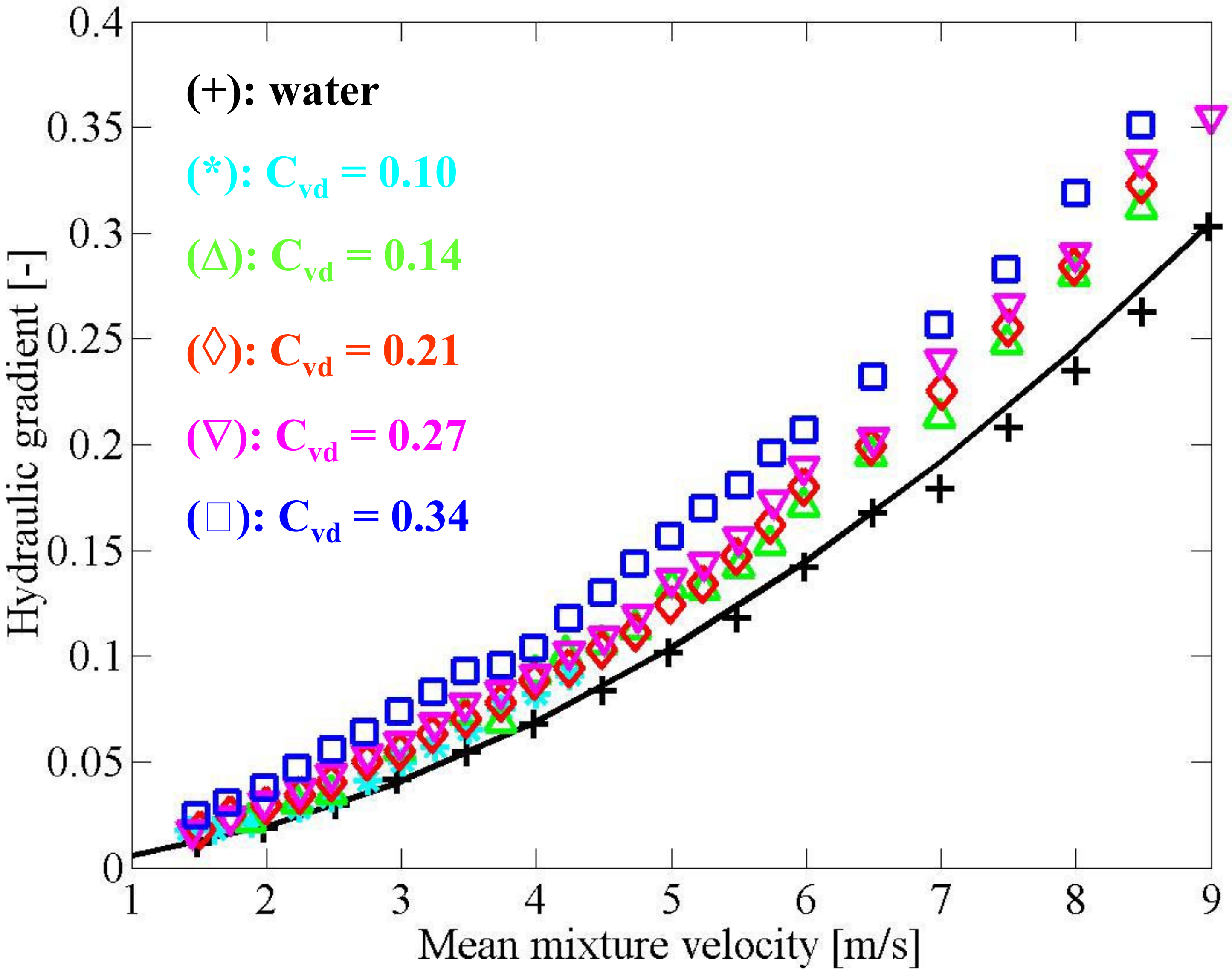
- The hydraulic gradient for slurry (equiv.-liquid) flow, I_m, is

$$I_m = -\frac{dP}{dx \rho_f g} = \frac{\lambda_f}{D} \frac{V_m^2}{2g} \frac{\rho_m}{\rho_f} = I_f \frac{\rho_m}{\rho_f}$$

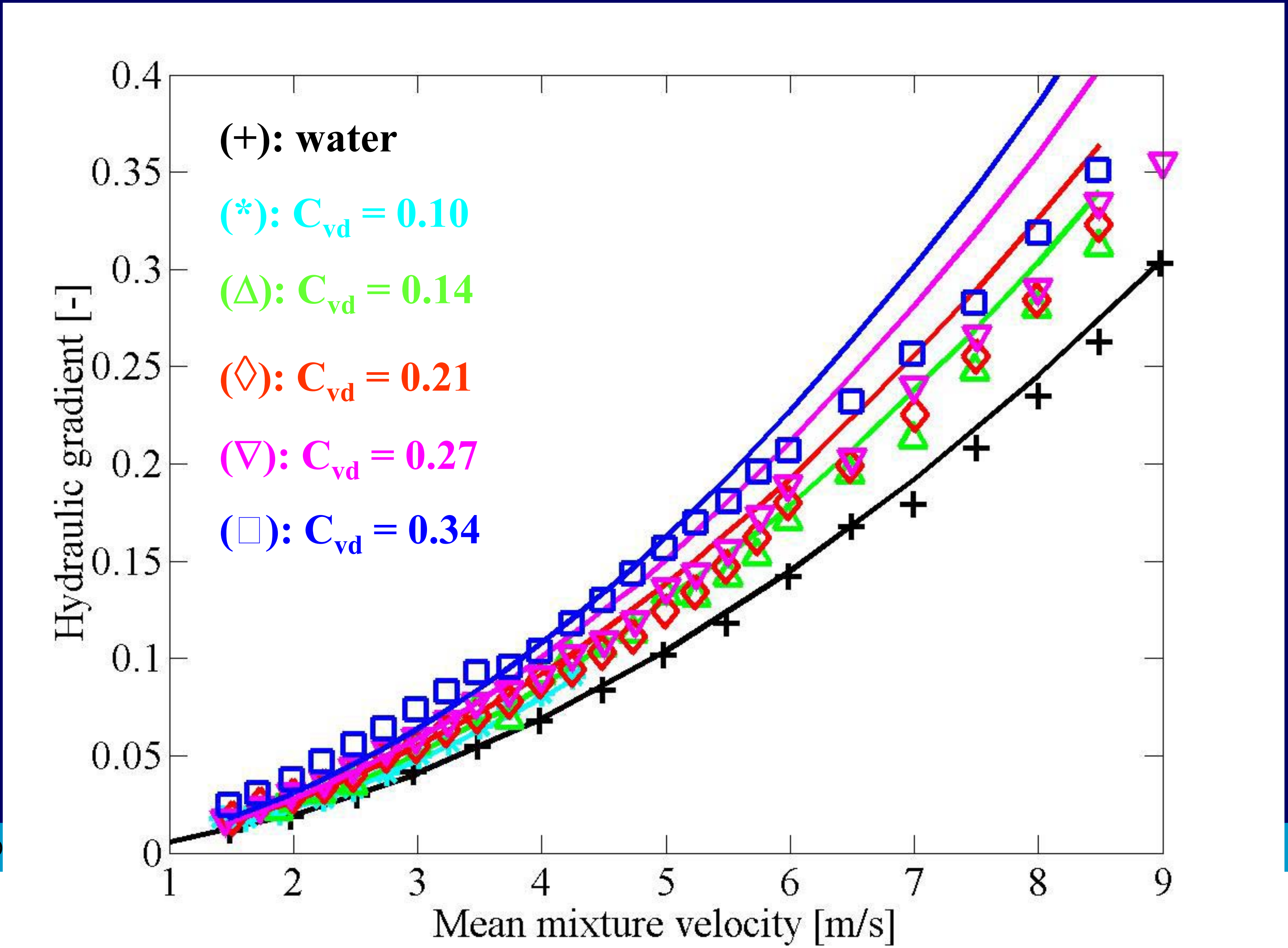
Very fine sand (MTI) in 158-mm horiz pipe



Fine sand (DUT) – vertical flow



Fine sand (DUT) – vertical flow



Generalized Equivalent-Liquid Model

Model for both fine and coarse pseudo-homogeneous flows (e.g. vertical flows):

The hydraulic gradient for slurry flow, I_m, is

$$\frac{I_m - I_f}{C_{vd}\left(S_s - 1\right)} = A' I_f$$

If A′=1, then $I_m = S_m I_f$, i.e. the equivalent-liquid model.

If A′=0, then $I_m = I_f$, i.e. the liquid (water) model.

If $0 < A' < 1$, then I_m according to the generalized model with calibrated A′ value.

MODELING OF NON-NEWTONIAN SLURRY FLOW

MANY DIFFERENT MODELS

Non-Newtonian Slurry Model

A. Physical background

- Very fine particles increase the viscosity of the pseudo-homogeneous mixture.

- The suspension does not obey Newton's law of viscosity and its constitutive rheological equation has to be determined experimentally. <u>Each particular mixture obeys its own law of viscosity</u>.

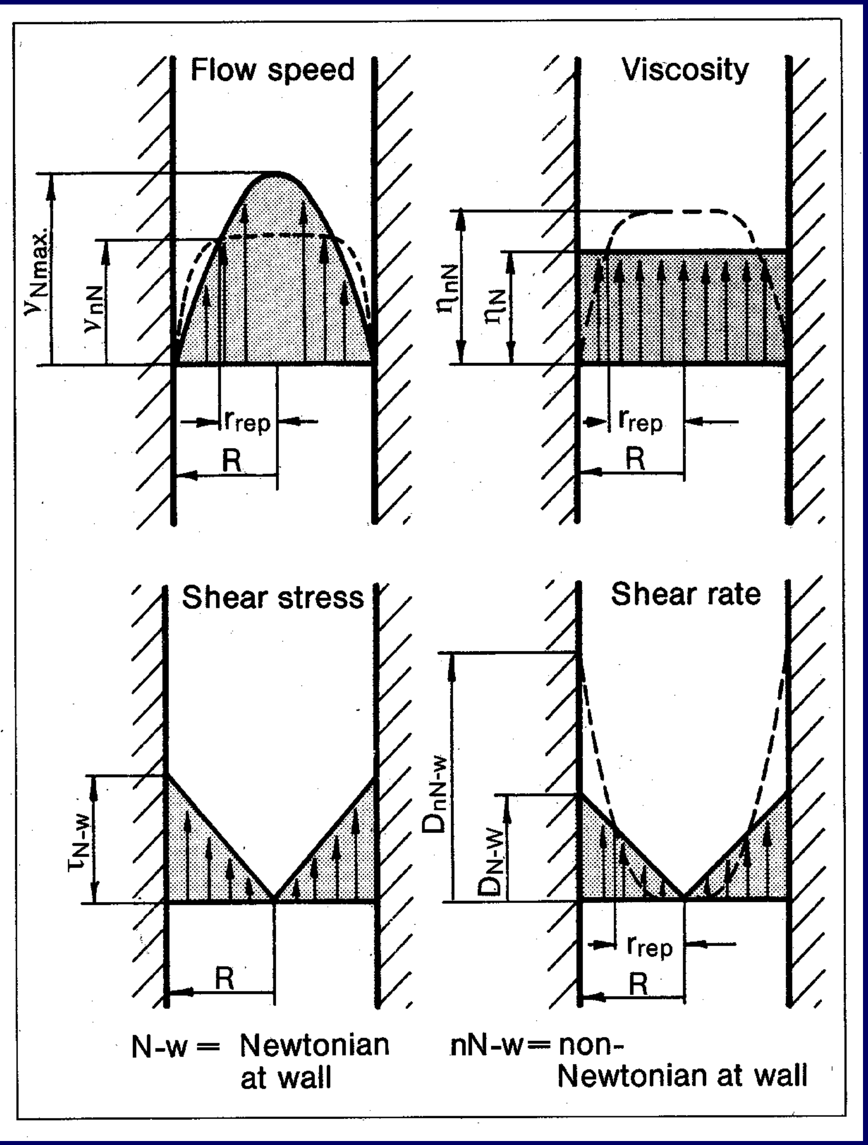
- Modeling of non-Newtonian mixtures is even more complex than the modeling of Newtonian mixtures in pipes.

Non-Newtonian Slurry Model

A. Physical background

- Internal structure of flow compared with Newtonian:

  - different <u>velocity distrib</u>.
  - different <u>viscosity distrib</u>.
  - identical <u>shear-stress</u> distribution



Non-Newtonian Slurry Model

B. Construction of the model for I_m

A general procedure for a determination of the frictional head loss, I_m:

1. the rheological parameters of a slurry

2. the slurry flow regime (laminar or turbulent)

3. the pressure drop using a scale-up method or an appropriate friction model.

Repetition: Newtonian vs non-Newtonian

Newton's law of liquid viscosity (valid for laminar flow)

$$\tau = \mu_f \left(-\frac{dv_x}{dr} \right)$$

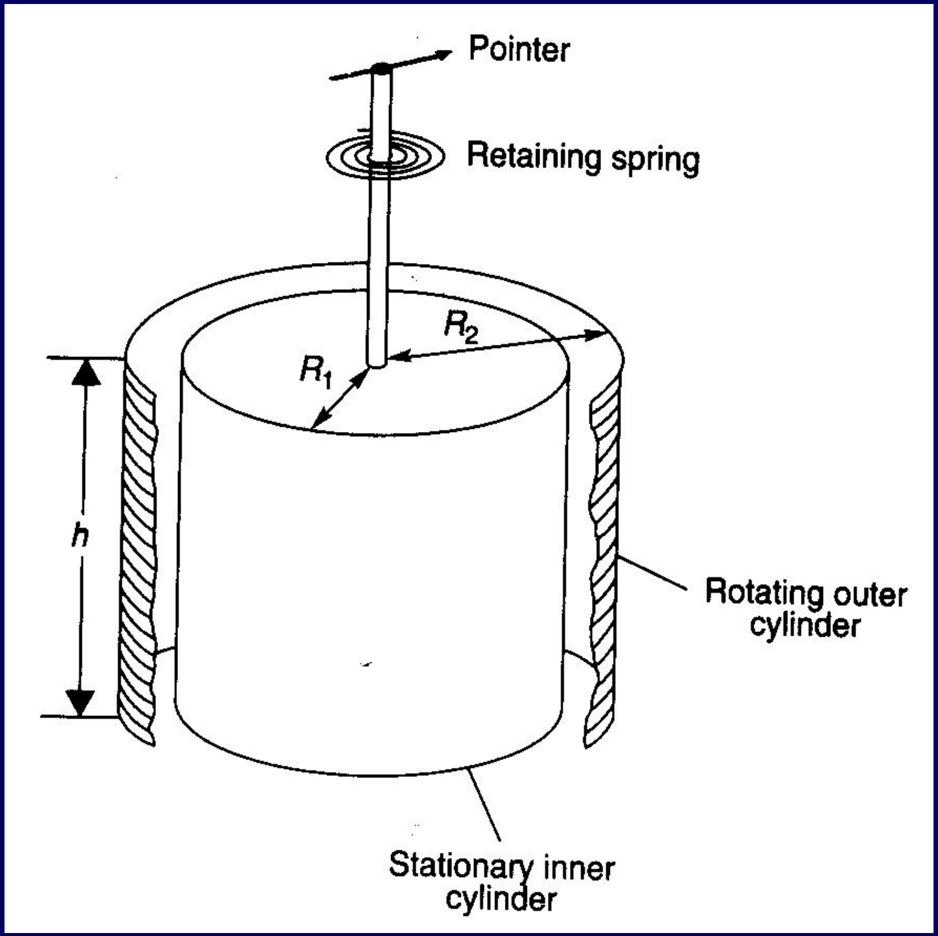
Step 1. Rheological Parameters

Table. Typical values of dynamic viscosity μ at room temperature

Substance	Dyn. Viscosity μ [mPa.s]
Air	10^{-2}
Water	1
Olive oil	10^{2}
Honey	10^{4}
Molten glass	10^{15}

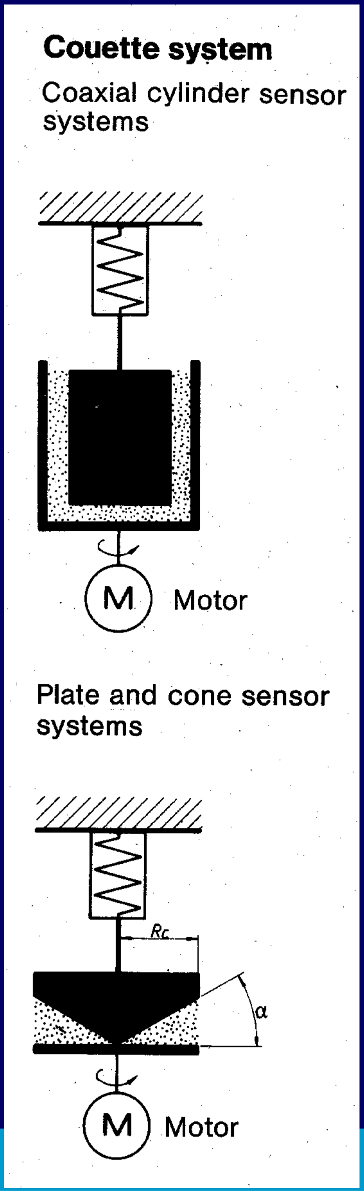
(source: Chhabra & Richardson, 1999)

Step 1. Rheological Parameters



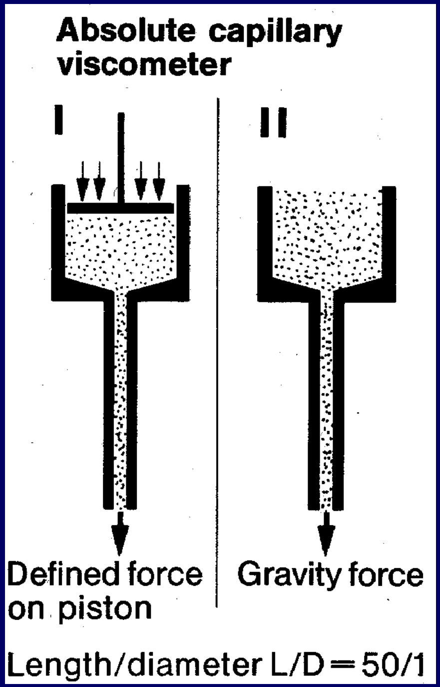
Rotational viscometer

Step 1. Rheological Parameters



Viscometers for determination of rheological parameters:

Rotational **Capillary**



Step 1. Rheological Parameters

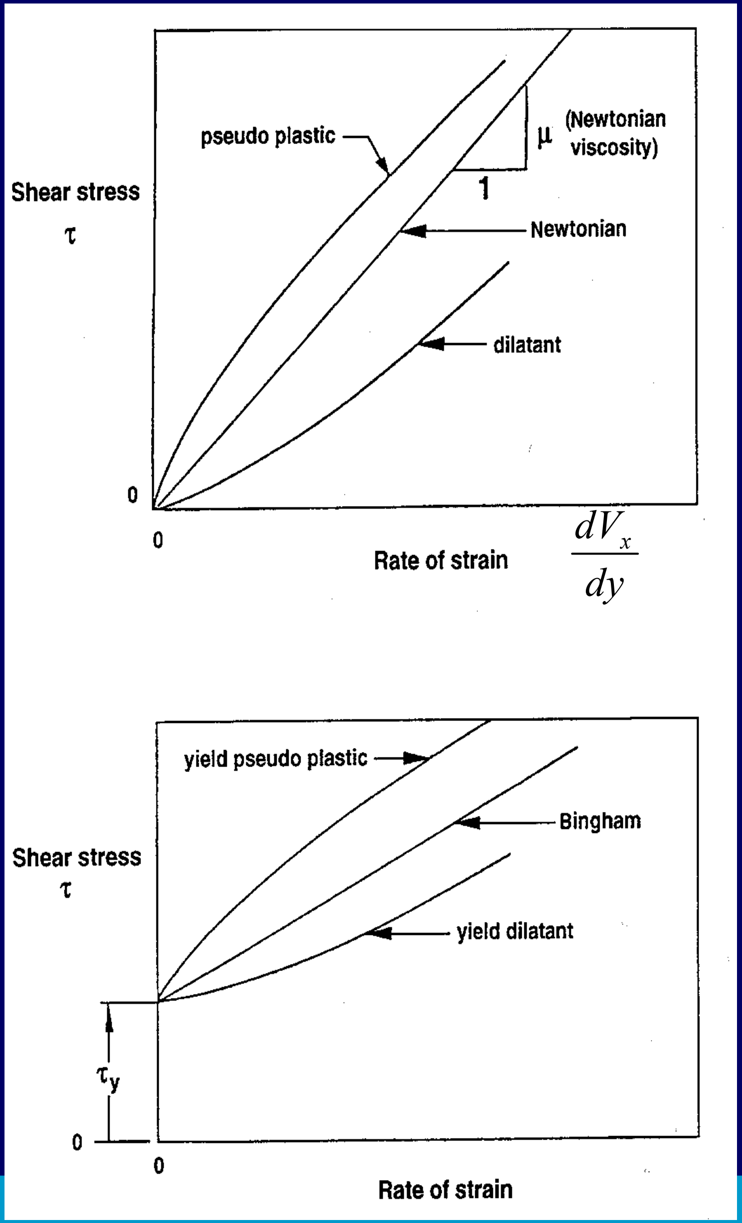
Rheograms

from rotational viscometers:

$$\tau = fn\left(-\frac{dv_x}{dr}\right)$$

from tube (capillary) viscometers:

$$\tau = fn\left(-\frac{dv_x}{dy}\right)$$



Step 1. Rheological Parameters: Tube Viscometer

Conditions :

- incompressible and homogeneous slurry,
- steady and uniform flow in horizontal straight tube,
- laminar flow (will be required in next slide)

$$-\frac{dP}{dx} = \frac{4\tau_o}{D} \Rightarrow \tau_o = \frac{\Delta P}{L}\frac{D}{4}$$

The wall shear stress τ_0 obtained from $-dP/dx = \Delta P/L$ measured in a tube of the internal diameter D over the length L.

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Step 1. Rheological Parameters: Tube Viscometer

Conditions :

- incompressible and homogeneous slurry,
- steady and uniform flow in horizontal straight tube,
- laminar flow $\lambda_m = 64/Re$!!!

$$\tau_o = \frac{\lambda_m}{8}\rho_m V_m^2 \; and \; \lambda_m = \frac{64}{\mathrm{Re}} = \frac{64\mu_m}{V_m D \rho_m}$$

Combination of these two equations gives the relationship representing the pseudo-rheogram $\tau_0 = fn(V_m/D)$ obtained from the tube viscometer.

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Step 1. Rheological Parameters: Tube Viscometer

Conditions :

- incompressible and homogeneous slurry,
- steady and uniform flow in horizontal straight tube,
- laminar flow

$$\tau_o = \mu_m \frac{8V_m}{D} \ (pseudo)equivalent\ to\ \tau = \mu_m \left(-\frac{dv_x}{dy} \right)$$

The method must be found that transforms a pseudo-rheogram to a real rheogram.

Intermezzo. Rabinowitsch-Mooney Transformation

Principle

The Rabinowitsch-Mooney transformation transforms:

- **the pseudo-rheogram [T_o, $8V_m/D$] to**
- **the normal rheogram [T, dv_x/dy] and vice versa**

<u>for a laminar flow of non-Newtonians</u>.

Intermezzo. Rabinowitsch-Mooney Transformation

Procedure:

1. The <u>pseudo-rheogram</u>: determined experimentally in a tube viscometer: points [T_o, $8V_m/D$] plotted in the ln-ln co-ordinates give $\tau_o = K\left(\frac{8V_m}{D}\right)^{n'}$, where $n' = K\frac{d(\ln \tau_o)}{d\left(\ln \frac{8V_m}{D}\right)}$

2. <u>Transformation</u> rules: $T=T_o$ and $\frac{dv_x}{dy} = \left(\frac{3n'+1}{4n'}\right)\frac{8V_m}{D}$ (for Newtonians n'=1)

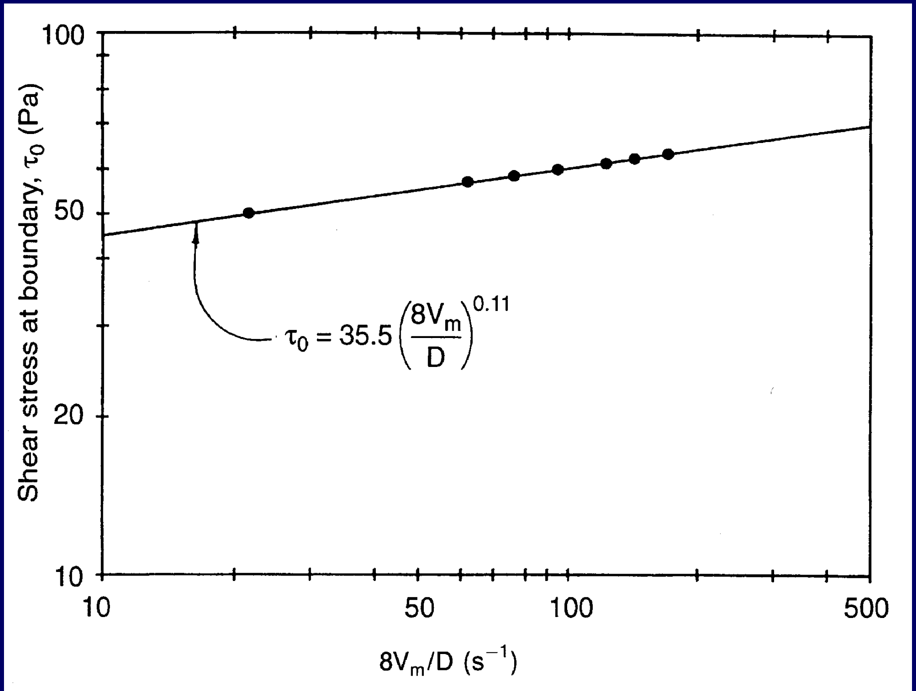
3. The <u>normal rheogram</u>: the [T, dv_x/dy] points are fitted by a suitable rheological model to determine the rheol. parameters

Step 1. Rheological Parameters

Pseudo-rheograms by capillary or tube viscometers:

wall shear stress, T_o

mean velocity, V_m



Step 1. Rheological Parameters

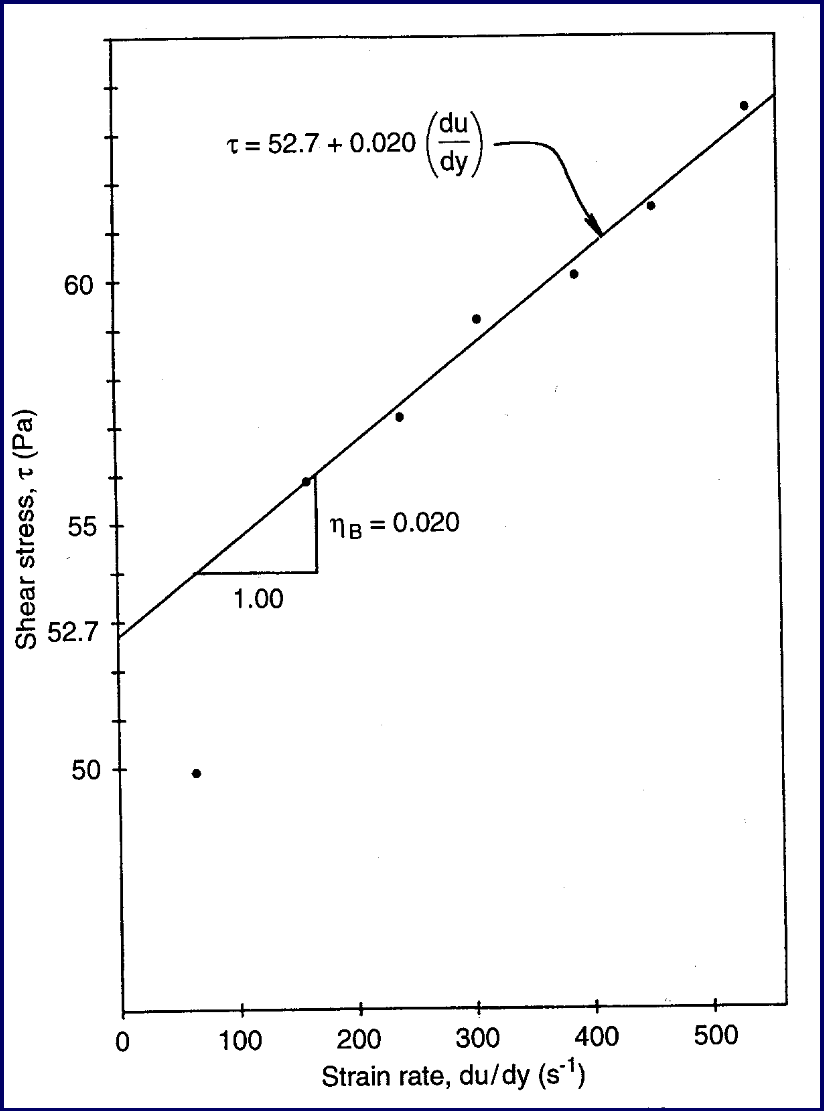
Transformed to the **normal rheograms** using the

Rabinowitsch-Mooney transformation:

shear stress, T

shear rate, dv/dy

Result: the slurry behaves as a Bingham plastic liquid, its internal friction is described by two rheological parameters, T_y, n_B.



Step 1. Rheological Parameters

Rheograms of various

non-Newtonian slurries:

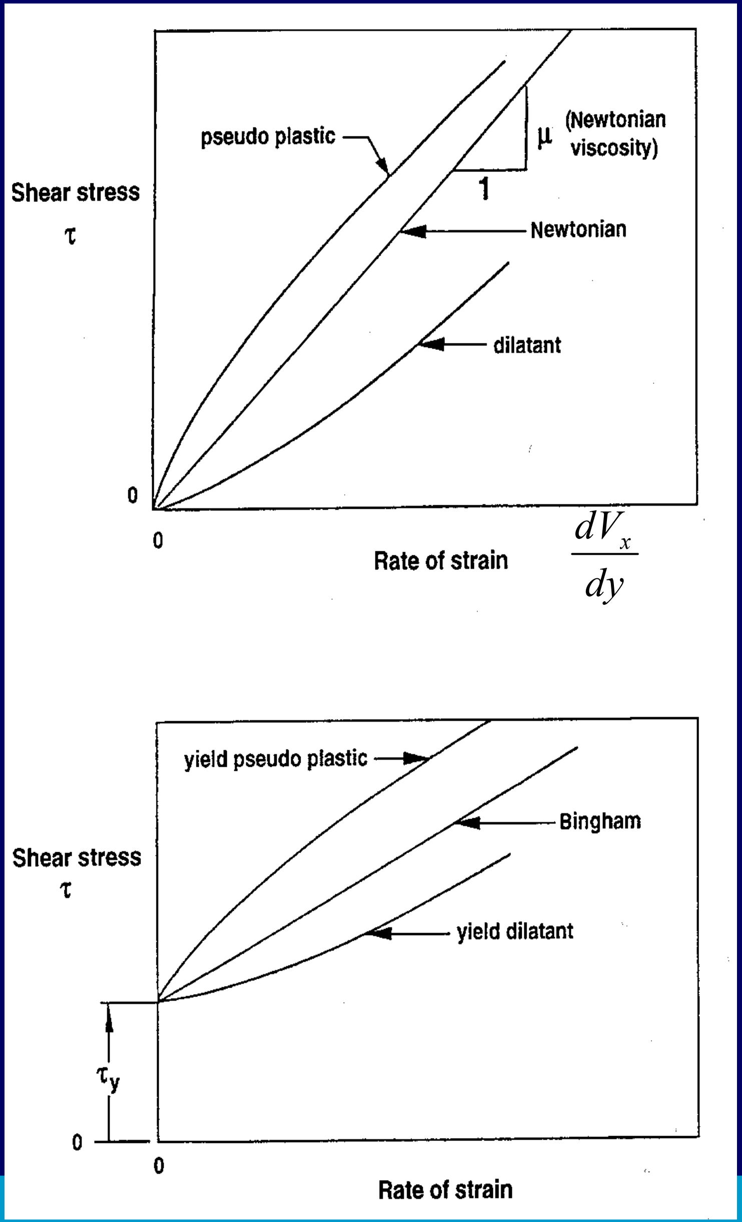
Dilatant (2 parameters)

Pseudo-plastic (2 par.)

Bingham (2 par.)

Yield dilatant (3 parameters)

Yield pseudo-plastic (3 par.)



Step 1. Rheological Parameters

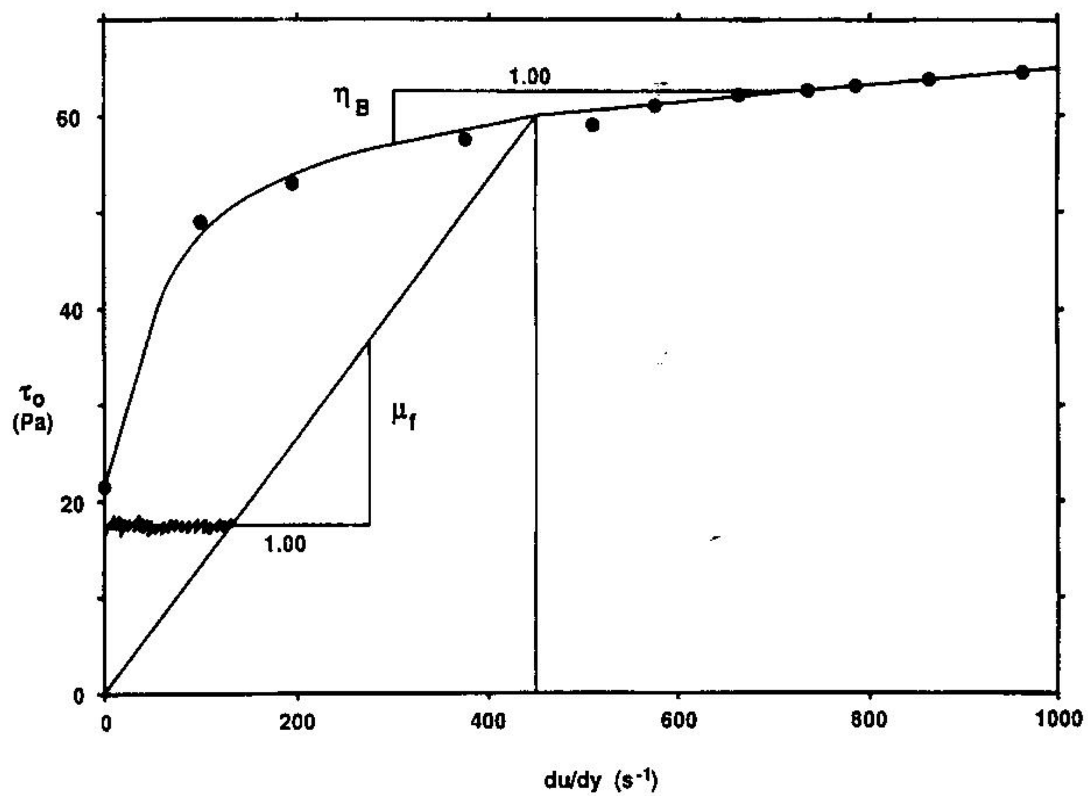


Fig. 7.1 Rheogram for clay slurry.

Example of rheogram obtained from viscometer test

Step 1. Rheological Parameters

Rheograms of various non-Newtonian slurries:

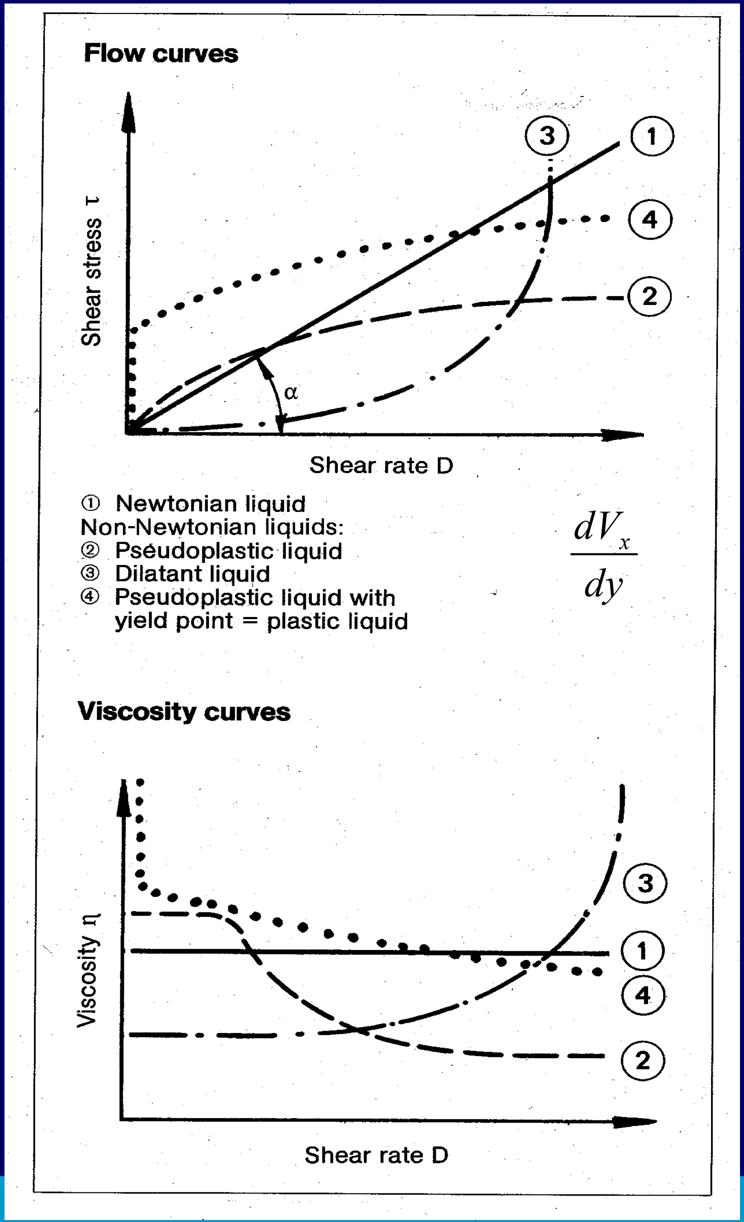
Dilatant (2 parameters)

Pseudo-plastic (2 par.)

Bingham (2 par.)

Yield dilatant (3 parameters)

Yield pseudo-plastic (3 par.)



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Step 1. Rheological Parameters

Rheological model with two-parameters

The Bingham-plastic model contains rheological parameters T_y, n_B. The rheogram is a straight line with the slope n_B (called plastic viscosity or tangent viscosity). T_y is called the yield stress.

$$\tau = \tau_y + \eta_B \frac{dv_x}{dy}$$

This model satisfies the flow behaviour of the majority of fine homogeneous high-concentrated slurries that are dredged (aqueous mixtures of non-cohesive clay etc.).

Step 1. Rheological Parameters

Rheological model with two-parameters

The Power-law model contains rheological parameters K, n. The rheogram is a curve. n is called the flow index (it is not the viscosity). K is called the flow coefficient.

$$\tau = K\left(\frac{dv_x}{dy}\right)^n$$

Different types of fluid behavior occur:

0 < n < 1: pseudo plastic slurry (becomes *thinner* under the increasing shear rate)

n > 1: dilatant slurry (becomes *thicker* under the increasing dv/dy).

Step 1. Rheological Parameters

Rheological model with three-parameters

<u>The Yield Power-law model (Buckley-Herschel model)</u> **contains rheological parameters T_y, K, n. The rheogram is a curve which does not pass through origin.**

$$\tau = \tau_y + K\left(\frac{dv_x}{dy}\right)^n$$

Different types of fluid behavior occur:

$0 < n < 1$: <u>pseudo plastic slurry</u> (becomes *thinner* under the increasing shear rate)

$n > 1$: <u>dilatant slurry</u> (becomes *thicker* under the increasing dv/dy).

Step 2. Flow Regime (Lam/Turb)

Threshold for Bingham plastic slurries_
is given by the value of the modified Bingham Reynolds number

$$\mathrm{Re}_B = \frac{\rho_m V_m D}{\eta_B \left(1 + \frac{\tau_y D}{6 \eta_B V_m} \right)} = 2100$$

in which T_y and n_B are the rheological parameters of the Bingham plastic slurry.

Step 3. Pressure drop due to friction

Two Approaches:

Scale Up
or
Friction model.

There are different scale-up methods for laminar and turbulent flows.
There are different friction models for laminar and turbulent flows.

Step 3. Pressure drop: Scale Up

Principle

The I_m-V_m measurement results from tube viscometers can be scaled up to prototype pipes without an intermediary of a rheological model (for the flow of identical slurry).

The principle of the scaling-up technique is that in non-Newtonian flows the wall shear stress is unchanged in pipes of different pipe sizes D.

Step 3. Pressure drop: Scale Up

Laminar flow

using the *Rabinowitsch-Mooney transformation*:

$$\tau_o = \frac{D.\Delta P}{4L} = idem \qquad \frac{8V_m}{D} = idem$$

This says that if a T_o versus $8V_m/D$ relationship is determined experimentally for a laminar flow of certain slurry in one pipe it is valid also for pipes of all different sizes.

Scaling up between two different pipeline sizes [e.g. between a tube viscometer (index 1) and a prototype pipeline (index 2)]:

$$I_{m2} = I_{m1}\frac{D_1}{D_2} \qquad V_{m2} = V_{m1}\frac{D_2}{D_1}$$

Step 3. Pressure drop: Scale Up

Turbulent flow

The *Rabinowitsch-Mooney transformation* not applicable:

$$\tau_o = \frac{D.\Delta P}{4L} = idem \qquad \frac{8V_m}{D} \neq idem$$

because the near-wall velocity gradient is not described by $8V_m/D$, instead the wall-friction law uses an empirical friction coefficient to relate the wall shear stress with the mean velocity.

Scaling up between two different pipeline sizes [e.g. between a tube viscometer (index 1) and a prototype pipeline (index 2)]:

$$I_{m2} = I_{m1}\frac{D_1}{D_2} \qquad V_{m2} = V_{m1}\left[1 + 2.5\sqrt{\frac{\lambda_f}{8}}\ln\left(\frac{D_2}{D_1}\right)\right]$$

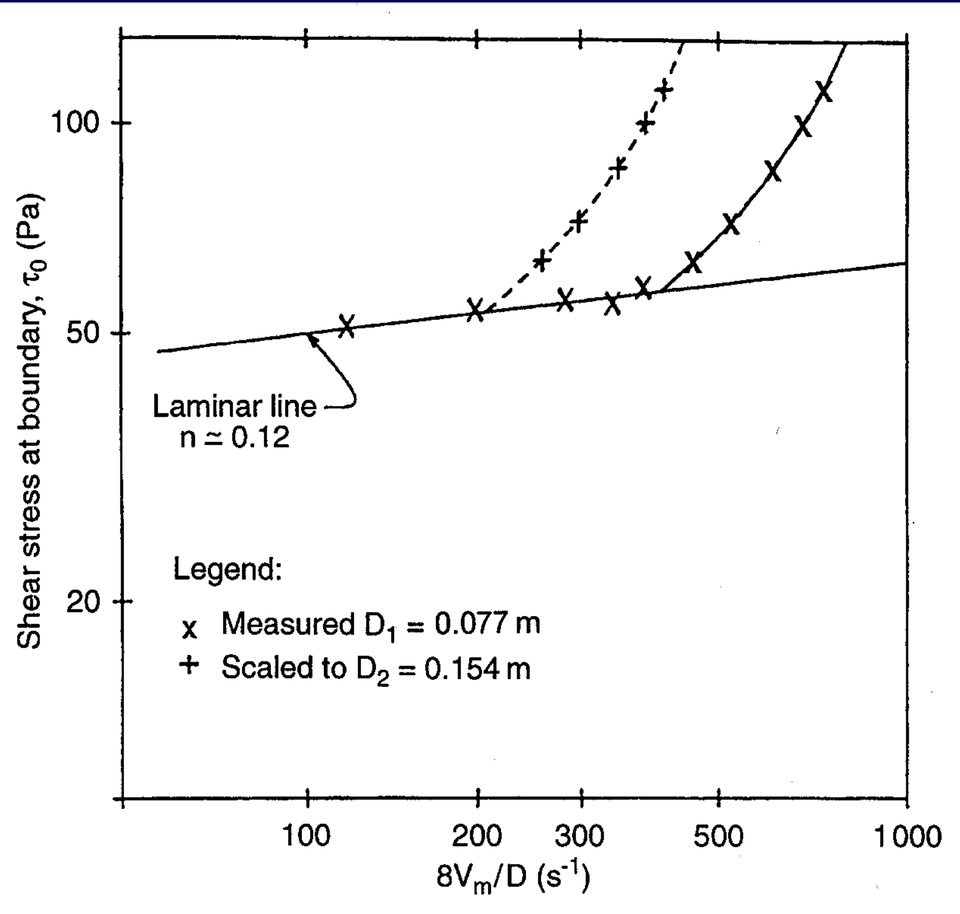
Step 3. Pressure drop: Scale Up

Turbulent flow

The example of the scale up of the turbulent flow of slurry:

data *measured* in the 77-mm pipe

and *predicted* for the 154-mm pipe.



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Step 3. Pressure drop: Friction Model

Laminar flow

Integrating of a rheological model over a pipe cross section gives the mean velocity and the relation between V_m and the pressure drop (the same procedure as for the water flow).

The obtained equation can be rearranged to the standard eq.

$$I_m = \frac{\Delta P}{\rho_f g L} = \frac{\lambda_{nN}}{D} \frac{V_m^2}{2g}$$

in which the wall-friction law for the laminar flow: and thus all effect on the frictional pressure drop due to slurry rheology are expressed in the modified Reynolds number.

$$\lambda_{nN} = \frac{64}{\text{Re}_{nN}}$$

Step 3. Pressure drop: Friction Model

Laminar flow – EXAMPLE: Bingham plastic slurry flow

The rheological model

$$\tau = \tau_y + \eta_B \frac{dv_x}{dy}$$

after integrating

$$\frac{8V_m}{D} = \frac{\tau_o}{\eta_B}\left(1 - \frac{4\tau_y}{3\tau_o} + \frac{\tau_y^4}{3\tau_o^4}\right)$$

compared with the basic force-balance and Darcy-W. equations

$$\frac{\Delta P}{L} = \frac{4\tau_o}{D}$$

and

$$I_m = \frac{\Delta P}{\rho_f g L} = \frac{\lambda_{nN}}{D}\frac{V_m^2}{2g}$$

leads to the wall-friction law for the Bingham-slurry flow =>

Step 3. Pressure drop: Friction Model

Laminar flow – EXAMPLE: Bingham plastic slurry flow

leads to the wall-friction law for the Bingham-slurry flow

$$\lambda_{nN} = \frac{64\eta_B}{DV_m\rho_m}\left(1 - \frac{4\tau_y}{3\tau_o} + \frac{\tau_y^4}{3\tau_o^4}\right)^{-1} = \frac{64}{\mathrm{Re}_B}$$

The modified Bingham Reynolds number is

$$\mathrm{Re}_B = \frac{\rho_m V_m D}{\eta_B\left(1 + \frac{\tau_y D}{6\eta_B V_m}\right)}$$

(the fourth-power term in the above eq. is neglected)

Step 3. Pressure drop: Friction Model

Turbulent flow

In turbulent flow, a velocity profile differs from that determined by a rheological model (analogy with a Newtonian flow).

The standard Darcy-Weisbach equation is valid

$$I_m = \frac{\Delta P}{\rho_f g L} = \frac{\lambda_{nN}}{D} \frac{V_m^2}{2g}$$

but the wall-friction law is an empirical function

$$\lambda_{nN} = fn(\mathrm{Re}_{nN}, k, \ldots)$$

Step 3. Pressure drop: Friction Model

Turbulent flow - EXAMPLE

The most successful models for turbulent flows of non-Newtonian slurries are:

Wilson – Thomas model

Slatter model.