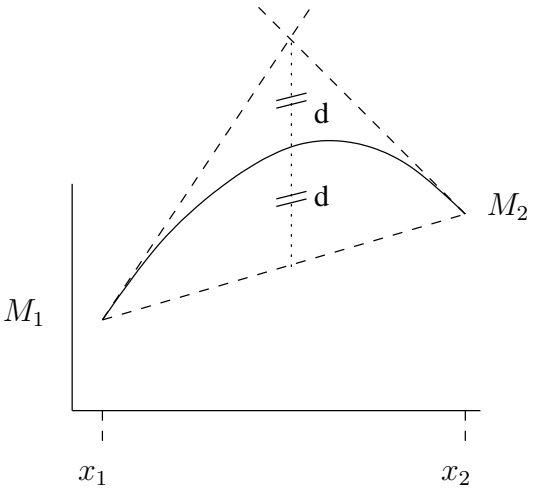
Properties of a parabola



Consider a parabola passing through the points (x_1, M_1) and (x_2, M_2). The parabola represents a bending moment diagram as a result of a constant distributed load q. The bending moment formula as a function of x is given by

$$M(x) = -\frac{1}{2}qx^2 + C_1x + C_2 \tag{1}$$

Since we have depicted a concave parabola, we introduce a minus sine in front of the square term to be consistent.

Using the information that the parabola passes through the two given points, derive that the formula can be written as

$$M(x) = -\frac{1}{2}qx^2 + \left(\frac{M_1 - M_2}{x_1 - x_2} + \frac{1}{2}q(x_1 + x_2)\right)x - \frac{1}{2}qx_1x_2 + M_1 - \frac{M_1 - M_2}{x_1 - x_2}x_1 \tag{2}$$

The tangent line to the bending moment diagram in $x = x_1$ is given by

$$f(x) = \left(\frac{dM}{dx}(x = x_1)\right)x + C_3 \tag{3}$$

Use the information $M(x = x_1) = M_1$ and the preceding formula for $M(x)$ to derive that this results in

$$f(x) = \left(\frac{M_1 - M_2}{x_1 - x_2} - \frac{1}{2} q(x_1 - x_2) \right) x + M_1 - \left(\frac{M_1 - M_2}{x_1 - x_2} - \frac{1}{2} q(x_1 - x_2) \right) x_1 \tag{4}$$

We already know that the tangent lines at both ends will intersect at $x = \frac{1}{2}(x_1 + x_2)$, since the distributed load is constant. From (2) and (4) we then find the values of $f(\frac{1}{2}(x_1 + x_2))$ and $M(\frac{1}{2}(x_1 + x_2))$ as

$$M(\frac{1}{2}(x_1 + x_2)) = \frac{1}{2}(M_1 + M_2) + \frac{1}{8} q(x_1 - x_2)^2 \tag{5}$$

and

$$f(\frac{1}{2}(x_1 + x_2)) = \frac{1}{2}(M_1 + M_2) + \frac{1}{4} q(x_1 - x_2)^2 \tag{6}$$

Now note that the connecting line between the points (x_1, M_1) and (x_2, M_2) passes through the point $(\frac{1}{2}(x_1 + x_2), \frac{1}{2}(M_1 + M_2))$. We can now conclude that at the position $x = \frac{1}{2}(x_1 + x_2)$ the distance between the connecting line and the bending moment diagram and the distance between the the bending moment diagram and the tangent line $f(x)$ are both equal to

$$d = \frac{1}{8} q(x_1 - x_2)^2 \tag{7}$$

This can be used to generate another point for the bending moment diagram in order to draw the diagram more accurately.