Problem set 1: Quantum states and transformations

Due 21 Sept 2007

1) Are the following states separable? If so, factor them.

$$\frac{|00\rangle + i|01\rangle - |10\rangle + i|11\rangle}{2}, \qquad \frac{|00\rangle + i|01\rangle - |10\rangle - i|11\rangle}{2}$$
$$\frac{|000\rangle - |010\rangle}{\sqrt{2}}, \qquad \frac{|100\rangle + |010\rangle}{\sqrt{2}}.$$

2) What is the Bloch sphere representation (x, y, z coordinates) of

$$\frac{|0\rangle + \frac{1-i}{\sqrt{2}}|1\rangle}{\sqrt{2}} \qquad \text{and} \qquad \sqrt{\frac{1}{4}}|0\rangle - \sqrt{\frac{3}{4}}|1\rangle \ ?$$

3) Write down the 4×4 unitary transformation which flips qubit 1 and does nothing to qubit 2.

4) Write down the 4×4 unitary transformation which flips the phase of qubit 1 if and only if qubit 2 is in $|1\rangle$. Demonstrate by example that this quantum gate can entangle two initially unentangled qubits.