

Problem set 2: Hamiltonian and unitary evolution

Due 28 Oct 2007

1) Show analytically that

$$\exp\left(-i\frac{\theta}{2}\sigma_x\right) = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2}\sigma_x$$

2) Analytically determine the rotation axis and angle of the Hadamard gate and the gate U :

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

Check your result by examining to what final states H and U transform the initial states $|0\rangle$, $|1\rangle$ and $(|0\rangle + |1\rangle)/\sqrt{2}$.

3) How would you realize a 90° rotation about $+\hat{y}$ if you can implement directly only rotations about \hat{x} or \hat{z} ? (i.e. give a decomposition in \hat{x} and \hat{z} rotations)

Same question for a 45° rotation about $+\hat{y}$ and 180° rotation about $+\hat{y}$.