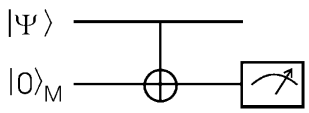
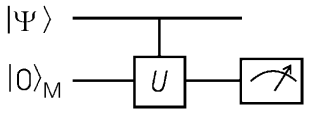
Problem set 6: Quantum measurement
Due 2 Nov 2007

1) Consider a quantum bit in a pure state $|\psi\rangle = a|0\rangle + b|1\rangle$, and a meter initially in the state $|0\rangle_M$. A CNOT is performed from the qubit onto the meter, and subsequently the meter is read out, as in the figure. In effect, we are thereby (indirectly) measuring the qubit state.



- What is the joint state of qubit and meter after the CNOT?
- What are the probabilities for the measurement to give outcome 0 or 1?
- What is the post-measurement state of the qubit?
- What are the POVM measurement operators E_m (2×2 matrices) that describe this indirect measurement of the qubit? Verify that $\langle\psi|E_m|\psi\rangle$ gives the probability for outcome m.
- *Optional, but highly recommended:* Find the measurement operators M_m, such that the probabilities for obtaining m are given by $\langle\psi|M_m^\dagger M_m|\psi\rangle$, and such that $M_m|\psi\rangle/\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}$ gives the post-measurement state.
- Is the measurement of the qubit projective?

2) Same questions as under 1), but now with the following measurement circuit,



$$\text{where} \quad U = \sqrt{\text{NOT}} = \sqrt{\frac{-i}{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$