Problem set 8: Grover's algorithm
Due 23 Nov 2007

1) Consider a search space of 4 elements, indexed 0 through 3, for an oracle that marks the 3rd element ($f(2) = 1$ and $f(x) = 0\ \forall x \neq 2$). For each step in Grover's algorithm, draw the 'stick diagrams' like we did in class, and indicate the amplitude of each element in the diagram (corresponding to the amplitude of the corresponding term in the superposition - including normalization factors). What is the optimal number of iterations, k? What is the probability of success (i.e. of identifying that $x = 2$ is the marked element) after each iteration? Classically, how often would you have to evaluate $f(x)$ *on average* in order to find the marked element?

2) Now use the other graphical method (rotations in the space spanned by $|\alpha\rangle$ and $|\beta\rangle$) to determine the optimal number of iterations, k, for a search space of 8 elements. The elements are indexed 0 through 7, and again the oracle marks the 3rd element ($f(2) = 1$ and $f(x) = 0\ \forall x \neq 2$). Compute the probability of success after each step. Do this *without* making any approximations such as $\sin\theta \approx \theta$. Again, how often would you have to evaluate $f(x)$ *on average* in order to find the marked element classically?

3) Since in Grover's algorithm, we have to implement the function that marks one or more elements, it would seem that we need to know the answer (the marked elements) in advance. Or not? Please comment.