Problem set 9: Quantum error correction
Due 30 Nov 2007

1) Consider the three-qubit code for correcting bit flips, where a qubit in the state $|\psi\rangle = a|0\rangle + b|1\rangle$ is encoded as $|\psi_C\rangle = a|000\rangle + b|111\rangle$. Suppose the encoded state is distorted by a rotation about $+\hat{x}$ over 60° of the third qubit. Use the error correction circuit discussed in class and show that the state $|\psi_C\rangle$ is recovered after error correction.

We know that any quantum process can be expressed in the form $\sum_i E_i \rho E_i^\dagger$, with $E_i = e_{i0}I + e_{ix}X + e_{iy}Y + e_{iz}Z$. Give the discretization of the "error process" explicitly for this 60° rotation.

2) Give a quantum circuit that implements the X_1X_2 syndrome measurement in the 3-qubit phase flip code.

3) Show that the operator $X_1X_2X_3X_4X_5X_6$ represents a syndrome measurement that compares the sign of the first and second blocks in the 9-qubit Shor code. (no need to give a quantum circuit that implements it)