10

Losses and Efficiency Limits

In the previous chapters we have learned the basic physical principles of solar cells. In this chapter we will bring the different building blocks together and analyse, how efficient a solar cell theoretically can be. After discussing different efficiency limits and the major loss mechanisms, we will finalise this chapter with the formulation of *three design rules* that always should be kept in mind when designing solar cells.

It is very important to understand, why a solar cell cannot convert 100% of the incident light into electricity. Different efficiency limits can be formulated, each taking different effects into account.

10.1 The thermodynamic limit

The most general efficiency limit is the *thermodynamic efficiency limit*. In this limit, the photovoltaic device is seen as a thermodynamic *heat engine*, as illustrated in Fig. 10.1. Such a heat engine operates between two heat reservoirs; a hot one with temperature T_H and a cold one temperature T_C. For the heat engine, three energy flows are relevant. First, the heat flow $\dot{Q}_H$ from the hot reservoir to the engine. Secondly, the *work* $\dot{W}$ that is performed by the engine and thirdly, heat flowing from the engine to the cold reservoir that serves as a *heat sink*, $\dot{Q}_C$. Clearly, the third energy flow is a loss and consequently, the efficiency of the heat engine is given by

$$\eta = \frac{\dot{W}}{\dot{Q}_h}.$$ (10.1)

The *second law of thermodynamics* teaches us that the entropy of an independent system never decreases. It only increases or stays the same. While the heat flows $\dot{Q}_H$ and $\dot{Q}_C$ carry entropy, the performed work W is an entropy-free form of energy. Thermodynamics
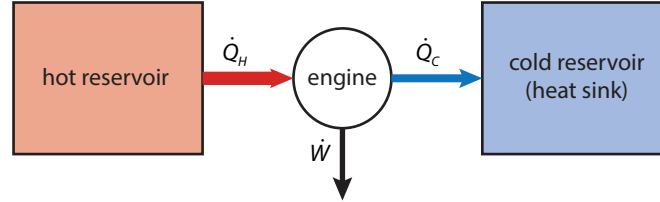
Figure 10.1: Illustrating the major heat flows in a generic heat engine.

teaches us that there is an efficiency limit for the transformation of heat into entropy-free energy. An (ideal) engine that has this maximal efficiency is called a *Carnot engine* and its efficiency is given by

$$\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H}. \tag{10.2}$$

For a Carnot engine, the entropy does not increase. Note that all the temperatures must be given in a temperature scale where the absolute zero takes the value 0. For example, the Kelvin scale is such a scale. From Eq. (10.2) we can already see two important trends that are basically true for every heat engine, *i.e.* also steam engines of combustion engines. The efficiency increases, if the higher temperature T_H is increased and/or the lower temperature T_C is decreased.

Let us now look at a solar cell that we imagine as a heat engine operating between an *absorber* of temperature T_A (this is our hot reservoir) and a cold reservoir, which is given by the surroundings and that we assume to be of temperature $T_C = 300$ K. What this heat engine actually does is that it converts the energy stored in the heat of the absorber into entropy-less chemical energy that is stored in the electron-hole pairs. Here, we assume that the transformation of chemical energy into electrical energy happens lossless, *i.e.* with an efficiency of 1. Hence, the efficiency of this thermodynamic heat engine is given by

$$\eta_{\text{TD}} = 1 - \frac{T_C}{T_A}. \tag{10.3}$$

The absorber will be heated as it absorbs sunlight. As we look at the ideal situation, we assume the absorber to be a black body that absorbs all incident radiation. Further, we assume the sun to be a black body of temperature $T_S = 6000$ K. As we have seen in Chapter 5, the solar irradiance incident onto the absorber is given by

$$I_e^S = \sigma T_S^4 \Omega_{\text{inc}}, \tag{10.4}$$

where Ω_{inc} is the solid angle covered by the incident sunlight. As the absorber is a black body of temperature T_A it also will emit radiation. The emittance of the absorber is given by

$$E_e^A = \sigma T_A^4 \Omega_{\text{emit}}. \tag{10.5}$$

Ω_{emit} is the solid angle into that the absorber can emit.

The efficiency of the absorption process is given by

$$\eta_A = \frac{I_e^S - E_e^A}{I_e^S} = 1 - \frac{E_e^A}{I_e^S} = 1 - \frac{\Omega_{\text{emit}}}{\Omega_{\text{inc}}} \frac{T_A^4}{T_S^4}. \tag{10.6}$$
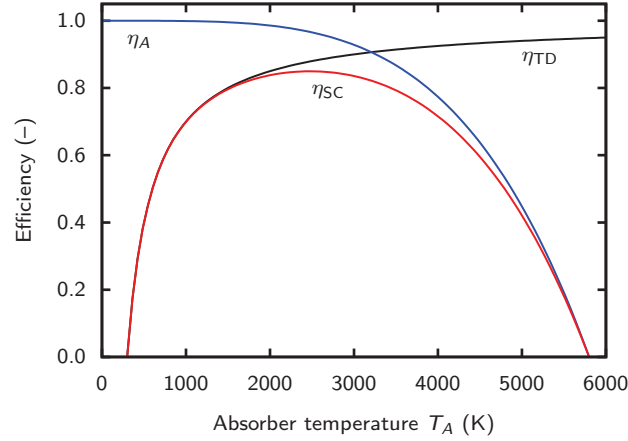
Figure 10.2: The absorber efficiency η_A, the thermodynamic efficiency η_{TD} and the combined solar cell efficiency η_{SC} under full concentration for a solar temperature of 5800 K and an ambient temperature of 300 K.

The absorber efficiency can be increased by increasing Ω_{inc}, which can be achieved by *concentrating* the incident sunlight. Under *maximal concentration* sunlight will be incident onto the absorber from all angles of the hemisphere, *i.e.* $\Omega_{inc}^{max} = 2\pi$. We assume the absorber to be open towards the surroundings and hence the sun on the top side. Its bottom side is connected to the heat engine such that radiative loss only can happen via the top side. Therefore, also $\Omega_{emit} = 2\pi$. Hence, the maximal absorber efficiency is achieved under maximal concentration and it is given by

$$\eta_A^{max} = 1 - \frac{T_A^4}{T_S^4}. \tag{10.7}$$

Note that η_A is the larger the lower T_A while the efficiency of the heat engine η_{TD} is the larger the higher T_A.

For the total efficiency of the ideal solar cell we combine Eq. (10.3) with Eq. (10.7) and obtain

$$\eta_{SC} = \left(1 - \frac{T_A^4}{T_S^4}\right)\left(1 - \frac{T_C}{T_A}\right). \tag{10.8}$$

Figure 10.2 shows the absorber efficiency, the thermodynamic efficiency and the solar cell efficiency. We see that the solar cell efficiency reaches its maximum of about 85% for an absorber temperature of 2480 K. Please note that the solar cell model presented in this section does not resemble a real solar cell but is only intended to discuss the physical limit of converting solar radiation into electricity. Several much more detailed studies on the thermodynamic limit have been performed. We want to refer the interested reader to works by Würfel [25] and Markvart *et al.* [36–38].

10.2   The Shockley-Queisser limit

We now will take a look at the the theoretical limit for single-junction solar cells. This limit is usually referred to as the *Shockley-Queisser* (SQ) limit, as they where the first ones to formulate this limit based purely on physical assumptions and without using empirically determined constants [27]. We will derive the SQ limit in a two-step approach. First, we will discuss the losses due to *spectral mismatch*. Secondly, we also will take into account that the solar cell will have a temperature different from 0 K which means that it emits electromagnetic radiation according to Planck's law. Just like William B. Shockley (1910-1989) and Hans-Joachim Queisser (*1931), we will do this with the *detailed balance* approach.

10.2.1   Spectral mismatch

There are two principal losses that strongly reduce the energy conversion efficiency of single-junction solar cells. As discussed in Chapter 8, an important part of a solar cell is the absorber layer, in which the photons of the incident radiation are efficiently absorbed resulting in a creation of electron-hole pairs. In most cases, the absorber layer is formed by a semiconductor material, which we characterise by its bandgap energy E_G. In principle, only photons with energy higher than the band gap energy of the absorber can generate electron-hole pairs. Since the electrons and holes tend to occupy energy levels at the bottom of the conduction band and the top of the valence band, respectively, the extra energy that the electron-hole pairs receive from the photons is released as heat into the semiconductor lattice in the *thermalisation* process. Photons with energy lower than the band gap energy of the absorber are in principle not absorbed and cannot generate electron-hole pairs. Therefore these photons are not involved in the energy conversion process. The *non-absorption* of photons carrying less energy than the semiconductor band gap and the *excess energy* of photons, larger than the band gap, are the two main losses in the energy conversion process using solar cells. Both of these losses are thus related to the spectral mismatch between the energy distribution of photons in the solar spectrum and the band gap of a semiconductor material.

Shockley and Queisser call the efficiency that is obtained when taking the spectral mismatch losses into account the *ultimate efficiency*, that is given according to the hypothesis that '*each photon with energy greater than $h\nu_G$ produces one electronic charge q at a voltage of $V_G = h\nu_G/e$*' [27].

Let us now determine the fraction of energy of the incident radiation spectrum that is absorbed by a single-junction solar cell. When we denote λ_G as the wavelength of photons that corresponds to the band gap energy of the absorber of the solar cell, only the photons with $\lambda \leq \lambda_G$ are absorbed. The fraction p_{abs} of the incident power that is absorbed by a solar cell and used for energy conversion can be expressed as

$$p_{abs} = \frac{\int_0^{\lambda_G} \frac{hc}{\lambda} \Phi_{ph,\lambda} \, d\lambda}{\int_0^{\infty} \frac{hc}{\lambda} \Phi_{ph,\lambda} \, d\lambda}, \qquad (10.9)$$

where $\Phi_{ph,\lambda}$ is the spectral photon flux of the incident light as defined in Chapter 5. The fraction of the absorbed photon energy exceeding the bandgap energy is lost because of thermalisation. The fraction of the absorbed energy that the solar can deliver as useful
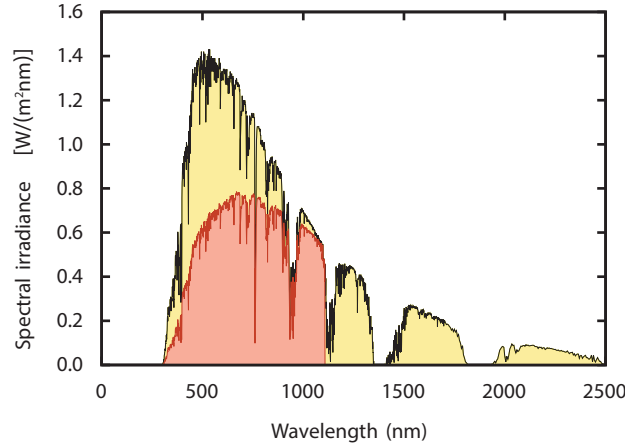
Figure 10.3: The fraction of the AM1.5 spectrum that can be converted into a usable energy by a crystalline silicon solar cell with $E_G = 1.12$ eV.

energy is then given by

$$p_{\text{use}} = \frac{E_G \int_0^{\lambda_G} \Phi_{\text{ph}, \lambda} \, d\lambda}{\int_0^{\lambda_G} \frac{hc}{\lambda} \Phi_{\text{ph}, \lambda} \, d\lambda}. \tag{10.10}$$

By combining Eqs. (10.9) and (10.10), we can determine the *ultimate conversion efficiency*,

$$\eta_{\text{ult}} = p_{\text{abs}} p_{\text{use}} = \frac{E_G \int_0^{\lambda_G} \Phi_{\text{ph}, \lambda} \, d\lambda}{\int_0^{\infty} \frac{hc}{\lambda} \Phi_{\text{ph}, \lambda} \, d\lambda}. \tag{10.11}$$

Figure 10.3 illustrates the fraction of the AM1.5 spectrum that can be converted into a usable energy by a crystalline silicon solar cell. Figure 10.4 shows the ultimate conversion efficiency in dependence of the the absorber band gap for three different radiation spectra, black-body radiation at 6000 K, AM0 and AM1.5 solar radiation spectra. The figure demonstrates that in case of a crystalline silicon solar cell ($E_G = 1.12$ eV) the losses due to spectral mismatch account for almost 50%. It also shows that an absorber material for a single junction solar cell has an optimal band gap of 1.1 eV and 1.0 eV for the AM0 and AM1.5 spectra, respectively. Note that the maximum conversion efficiency for the AM1.5 spectrum is higher than that for AM0, while the AM0 spectrum has a higher overall power density. This is because of the fact that the AM1.5 spectrum has a lower power density in parts of the spectrum that are not contributing to the energy conversion process as can be seen in Fig. 10.3. The dips in the AM1.5 spectrum also result in the irregular shape of the conversion efficiency as function of the band gap.

10.2.2 Detail balance limit of the efficiency

Similar to *Shockley and Queisser* we now will formulate the *detail balance limit of the efficiency*. But before we start we briefly will discuss the reason that the ultimate efficiency formulated above is not physical for solar cells with temperatures higher than 0 K.
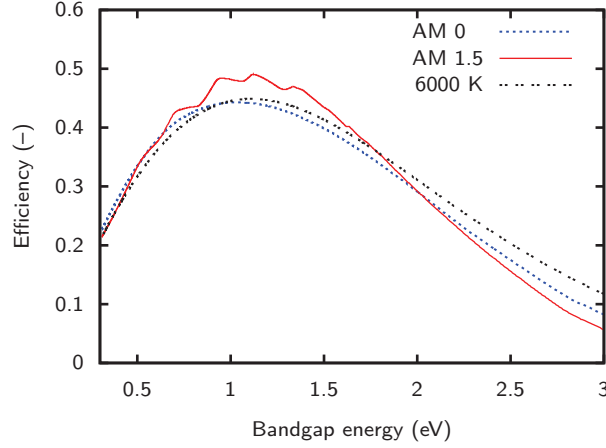
Figure 10.4: The ultimate conversion efficiency for the black body spectrum at 6000 K, the AM0 and AM1.5 solar radiation spectra, limited only by the spectral mismatch as a function of the band gap of a semiconductor absorber in single junction solar cells.

Let us estimate that the solar cell is embedded in an environment of ambient temperature of 300 K and that the solar cell temperature also is 300 K. As the solar cell will be in thermal equilibrium with its surroundings, it will absorb thermal radiation according to the ambient temperature *and* it will also emit the same amount of radiation. Therefore recombination of electron-hole pairs will be present in the semiconductor leading to a recombination current density different from zero. As we have seen in Eq. (9.1), the open circuit voltage will be reduced with increasing recombination current, which is an efficiency loss.

For deriving the detailed balance limit we first recall the definition of the efficiency from Eq. (9.5),

$$\eta = \frac{J_{\mathrm{ph}} V_{\mathrm{oc}} \, FF}{P_{\mathrm{in}}}. \tag{10.12}$$

For calculating η_{ult} we made the assumption that *'each photon with energy greater than $h\nu_G$ produces one electronic charge q at a voltage of $V_G = h\nu_G/q$'*. Under the same assumption, we obtain for the *short circuit current density*

$$J_{\mathrm{ph}}(E_G) = -q \int_0^{\lambda_G} \Phi_{\mathrm{ph}, \lambda} \, \mathrm{d}\lambda \tag{10.13}$$

with $\lambda_G = hc/E_G$. Note that we here implicitly assumed that the photo-generated current density J_{ph} is equivalent to the short circuit current density. This approximation is valid as the recombination current originating from thermal emission is orders of magnitude lower than the photo-generated current. By combining Eqs. (10.11) and (10.13) we find

$$J_{\mathrm{ph}} = -\frac{q}{E_G} P_{\mathrm{in}} \eta_{\mathrm{ult}} = -\frac{P_{\mathrm{in}} \eta_{\mathrm{ult}}}{V_G}. \tag{10.14}$$

Let us now define the *bandgap utilisation efficiency* η_V that is given by

$$\eta_V = \frac{V_{oc}}{V_G} \tag{10.15}$$

and tells us the fraction of the bandgap that can be used as open-circuit voltage (Shockley and Queisser use the letter v for this efficiency). We now combine Eqs. (10.12), (10.14) and (10.15) and find

$$\eta = \eta_{\text{ult}}\eta_V FF. \tag{10.16}$$

For determining the efficiency in the detailed balance limit, we therefore must determine the bandgap utilisation efficiency and the fill factor. Let us start with η_V.

According to Eq. (9.1), the open circuit voltage will be reduced with increasing recombination current density, which is an efficiency loss. It is given as

$$V_{oc} = \frac{k_B T}{q} \ln \left(\frac{J_{\text{ph}}}{J_0} + 1 \right). \tag{10.17}$$

The only unknown in this equation is the dark current density J_0. We assume the solar cell to be in *thermal equilibrium* with its surroundings at an ambient temperature of $T_a = 300$ K. Further, we assume that the solar cell absorbs and emits as a black body for wavelengths shorter than the bandgap wavelength of the solar cell absorber. For wavelengths longer than the bandgap we assume the solar cell to be completely transparent thus to neither absorb nor emit. This is the same assumption that we already used for the absorption of sunlight.

Using the equation for the *blackbody radiance* $L_{e\lambda}^{BB}$ as given in Eq. (5.18a) we find for the radiative recombination current density

$$\begin{aligned} J_0(E_G) &= -2q \int_0^{\lambda_G} \int_{2\pi} L_{e\lambda}^{BB}(\lambda; T_a) \cos\theta \, \mathrm{d}\Omega \, \mathrm{d}\lambda \\ &= -2q\pi \int_0^{\lambda_G} \frac{2hc^2}{\lambda^5} \left[\exp\left(\frac{hc}{\lambda k_B T_a} \right) - 1 \right]^{-1} \mathrm{d}\lambda, \end{aligned} \tag{10.18}$$

where the factor 2 arises from the fact that we assume the solar cell to emit thermal radiation both at its front and back sides.

Combining Eqs. (10.15) with (10.17) we find

$$\eta_V(E_G) = k_B T / E_G \ln \left[\frac{J_{\text{ph}}(E_G)}{J_0(E_G)} + 1 \right]. \tag{10.19}$$

Figure 10.5 shows the bandgap utilisation efficiency for three different spectra of the incident sunlight. For a bandgap of 1.12 eV this efficiency is about $\eta_V \approx 77\%$.

For the fill factor we take the empirical but very accurate approximation

$$FF = \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1} \tag{10.20}$$

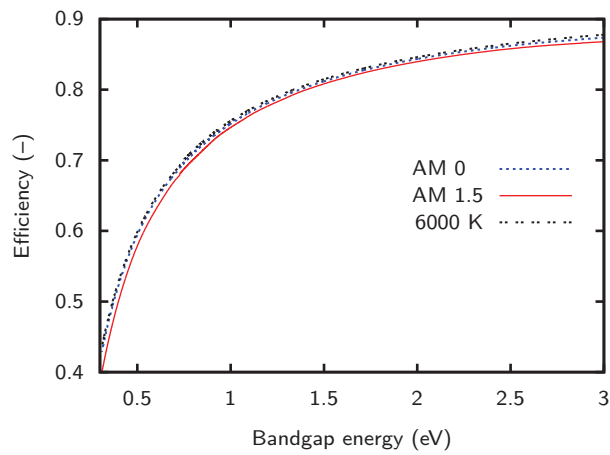with $v_{oc} = qV_{oc}/k_B T$. We already discussed this approximation in Eq. (9.3).

Figure 10.5: The bandgap utilisation efficiency η_V for the black body spectrum at 6000 K, and the AM0 and AM1.5 solar spectra.
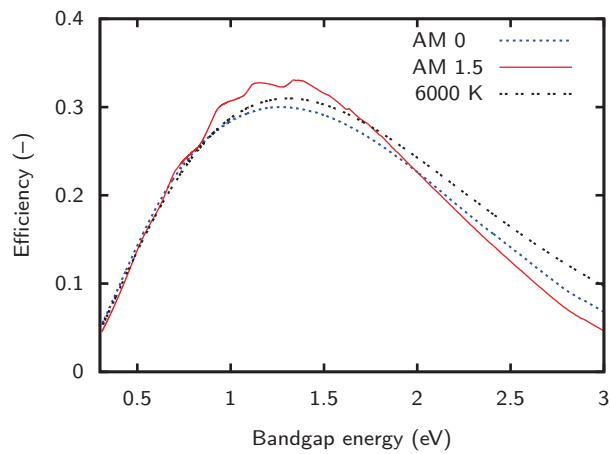

Figure 10.6: The Shockley-Queisser efficiency limit for the black body spectrum at 6000 K, and the AM0 and AM1.5 solar spectra.
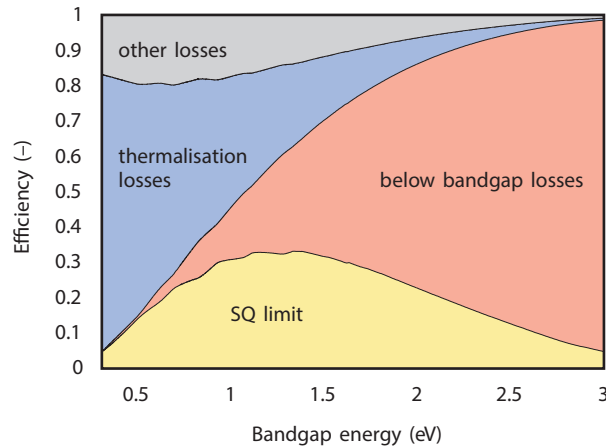
Figure 10.7: The major loss mechanisms in the Shockley Queisser limit. For this calculation the AM1.5 spectrum was used as incident light.

Figure 10.6 finally shows the Shockley-Queisser efficiency limit for three different spectra of the incident light. For the AM1.5 spectrum the limit is about 33.1% at 1.34 eV. For AM0 it is 30.1% at 1.26 eV.

The major loss mechanisms that are taken into account in the Shockley-Queisser limit are illustrated in Fig. 10.7. The major losses are non-absorbed photons below the bandgap and thermalised energy of photons above the bandgap. The other losses are due to the voltage loss because of thermal radiation and the fill factor being different from 100%.

10.2.3   Efficiency limit for silicon solar cells

It is very important to note that the Shockley-Queisser (SQ) limit is not directly applicable to solar cells made from crystalline silicon. The reason for this is that silicon is a so-called *indirect bandgap* semiconductor as we will discuss in detail in Chapter 12. This means that Auger recombination, which is a non-radiative recombination mechanism, is dominant. For the derivation of the SQ limit we assumed that only radiative recombination is present. Clearly, this assumption cannot be valid for crystalline silicon solar cells. Several attempts to calculate the efficiency limit while taking radiative recombination mechanisms into account were performed in the past. A study from 2013 by Richter *et al.* derives an efficiency limits of 29.43% for silicon solar cells .

As the Shockley-Queisser limit only considers radiative recombination, it is most valid for direct band gap materials such as GaAs. Because of its direct band gap, radiative recombination is the limiting recombination mechanism for GaAs.

10.3   Additional losses

The Shockley-Queisser limit is a very idealised model. For example all optical losses are neglected. Now we will discuss several loss mechanisms that have to be taken into account