18
Location Issues

In Chapter 5 we discussed solar radiation on Earth. There, we introduced the AM1.5 spectrum, which is used to evaluate the performance of solar cells and modules in laboratories and industry. The AM1.5 spectrum represents the solar irradiance if the centre of the solar disc is at an angle of 41.8° above the horizon.

It is clear that the Sun not always is at this position, but the position is dependent on the time of the day and the year, and also on the location on Earth. In this chapter we will discuss how to calculate the position of the Sun at every location on Earth at an arbitrary time and date. Furthermore we will discuss scattering of Sunlight when it traverses the atmosphere and how this influences the direct and diffuse spectrum. We also will discuss the influence of the mounting angle and position of a PV module on the irradiance at the module.

18.1 The position of the Sun

When planning a PV system it is crucial to know the position of the Sun in the sky at the location of the solar system at a given time. In this section we explain how this position can be calculated.

Since celestial objects like the Sun, the moon and the stars are very far away from the Earth it is convenient to describe their motion projected on a sphere with arbitrary radius and concentric to the Earth. This sphere is called the *celestial sphere*. The position of every celestial object thus can be parameterised by two angles. For photovoltaic applications it is most convenient to use the *horizontal coordinate system*, where the horizon of the observer constitutes the *fundamental plane*. In this coordinate system, the position of the Sun is expressed by two angles that are illustrated in Fig. 18.1: The *altitude* a is the angular elevation of the centre of the solar disc above the horizontal plane. Its angular range is $a \in [-90°, 90°]$, where negative angles correspond to the object being below the horizon and thus not visible. The *azimuth* A is the angle between the line of sight projected on the horizontal plane and due North. It is counted eastward, such that $A = 0°, 90°, 180°, 270°$ correspond to due *North*, *East*, *South* and *West*, respectively. Its angular range is $A \in [0°, 360°]$. In a different convention also used by the PV community, due South corresponds to 0° and is counted

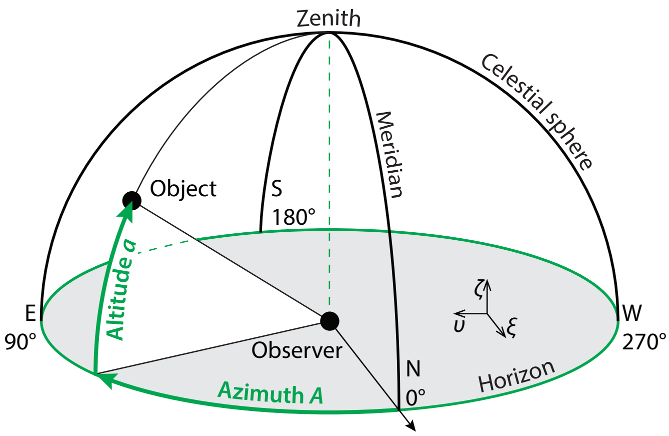

Figure 18.1: Illustrating the definition of the *altitude* a and the *azimuth* A in the horizontal coordinate system. Note that North is at the bottom of the figure.

westward, the angles then are in between $-180°$ and $180°$. In this book we will use the convention where $A = 0°$ corresponds to due North. Figure 18.1 also shows the *meridian*, which is great circle on the celestial sphere passing through the celestial North and South poles as well as the zenith.

Instead of using the spherical coordinates a and A, we also could use Cartesian coordinates, that we here call ξ (xi), υ (upsilon) and ζ (zeta) and that are also depicted in Fig. 18.1. The principal direction is parallel to ξ. The Cartesian coordinates are connected to the spherical coordinates via

$$\begin{pmatrix} \xi \\ \upsilon \\ \zeta \end{pmatrix} = \begin{pmatrix} \cos a \cos A \\ \cos a \sin A \\ \sin a \end{pmatrix}. \tag{18.1}$$

Note that on the celestial sphere $\xi^2 + \upsilon^2 + \zeta^2 = 1$ for all points.

In the horizontal coordinate system, the position of the sun is given by the *solar altitude* a_S and the *solar azimuth* A_S. As their calculation is quite complex, it is elaborated in Appendix E. This appendix also contains an example and a discussion on the *equation of time*.

Now we take a look at the solar paths throughout the year at four locations, shown in Figures 18.2-18.5: Delft, the Netherlands ($\phi_0 = 52.01°$ N), the North Cape, Norway ($\phi_0 = 71.17°$ N), Cali, Colombia ($\phi_0 = 3.42°$ N), and Sydney, Australia ($\phi_0 = 33.86°$ S). Note that all the times are given in the apparent solar time (AST, see Section E.2). While in Delft and on the North Cape, the Sun at noon always is South of the zenith, in Sydney it is always North. In Cali, close to the Equator, the Sun is either South or North, depending on the time of the year. Since the North Cape is north of the Arctic Circle, the Sun does not set around 21 June. This phenomenon is called the *midnight Sun*. On the other hand, the Sun always stays below the horizon around 21 December - this is called the *polar night*.

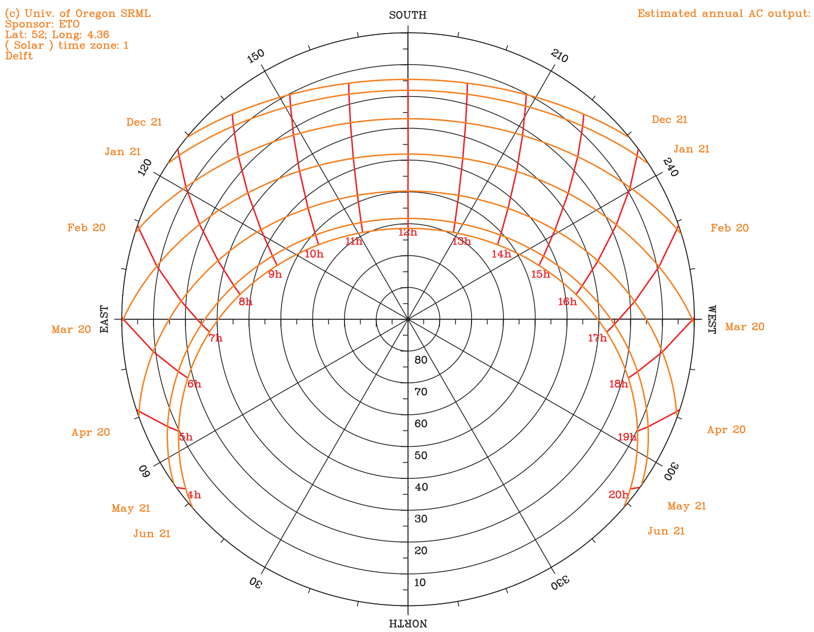

Figure 18.2: The Sun path in apparent solar time in Delft, the Netherlands ($\phi_0 = 52.01°$ N). The Sun path was calculated with the Sun path chart program by the *Solar Radiation Monitoring Lab.* of the *University of Oregon* (with kind permission from F. Vignola, University of Oregon) [137].

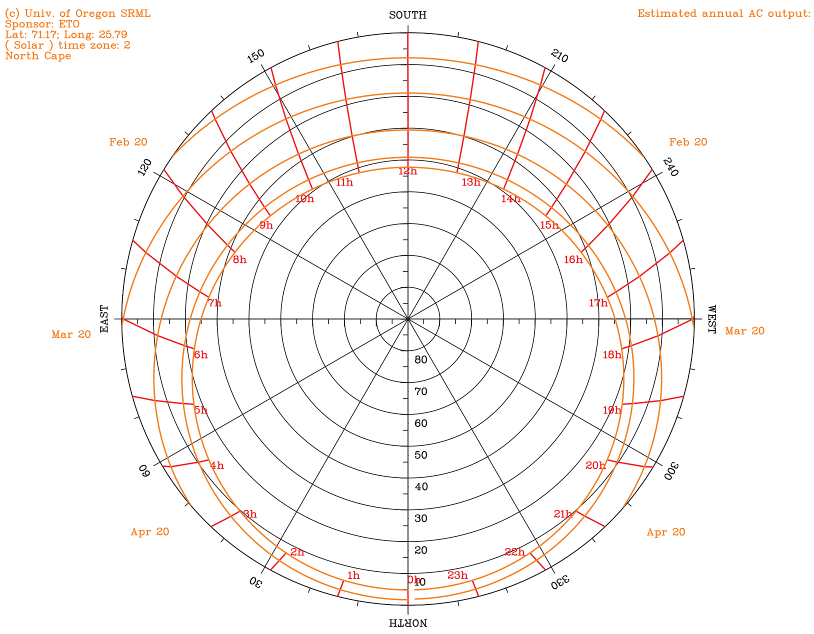

Figure 18.3: The Sun path in apparent solar time on the North Cape, Norway ($\phi_0 = 71.17°$ N). The Sun path was calculated with the Sun path chart program by the *Solar Radiation Monitoring Lab.* of the *University of Oregon* (with kind permission from F. Vignola, University of Oregon) [137].

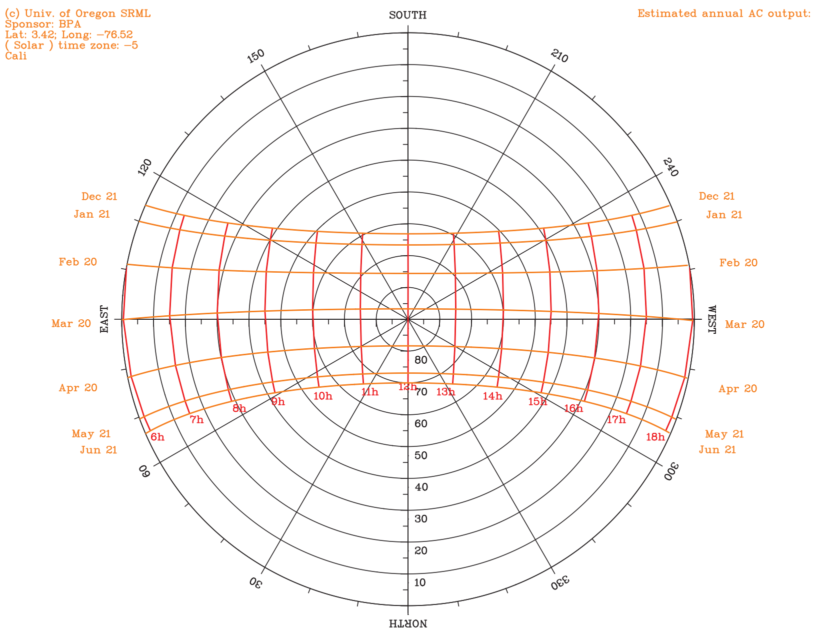

Figure 18.4: The Sun path in apparent solar time in Cali, Colombia ($\phi_0 = 3.42°$ N). The Sun path was calculated with the Sun path chart program by the *Solar Radiation Monitoring Lab.* of the *University of Oregon* (with kind permission from F. Vignola, University of Oregon) [137].

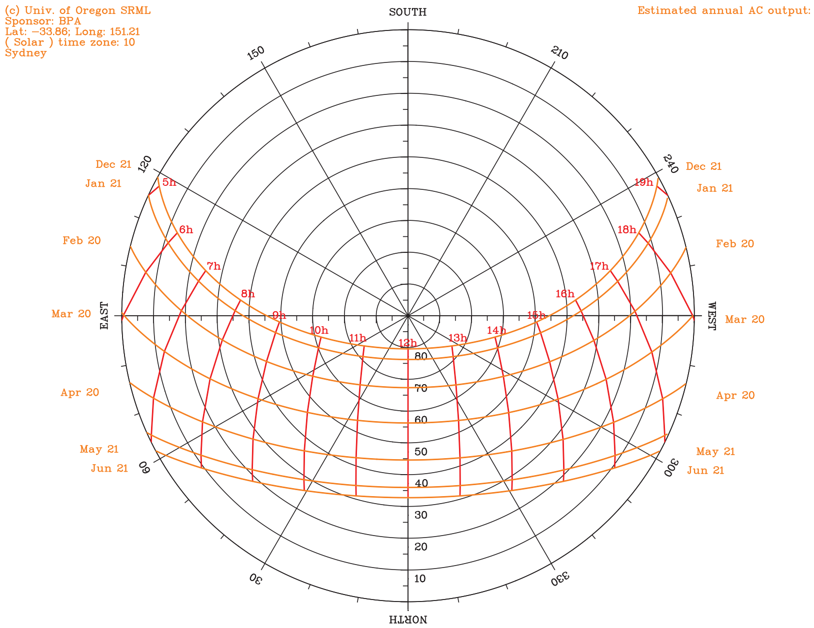

Figure 18.5: The Sun path in apparent solar time in Sydney, Australia ($\phi_0 = 33.86°$ S). The Sun path was calculated with the Sun path chart program by the *Solar Radiation Monitoring Lab.* of the *University of Oregon* (with kind permission from F. Vignola, University of Oregon) [137].

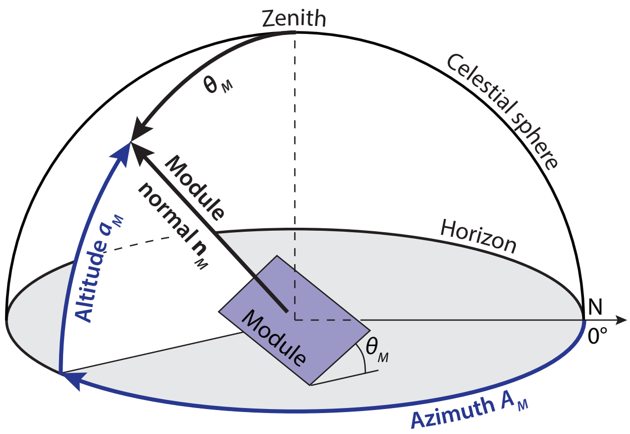

Figure 18.6: Illustrating the angles used to describe the orientation of a PV module installed on a horizontal plane.

18.2 Irradiance on a PV module

18.2.1 The angle of incidence (AOI)

In this section we discuss the implications of the changing position of the Sun on the irradiance present on solar modules. For this discussion we assume that the solar module is mounted on a horizontal plane and that it is tilted under an angle θ_M, as illustrated in Fig. 18.6. The angle between the projection of the normal of the module onto the horizontal plane and due north is A_M. We then can describe the position of the module by the direction of the module normal in horizontal coordinates (A_M, a_M), where the altitude is given by $a_M = 90° - \theta_M$. Let now the Sun be at the position (A_S, a_S). Then the direct irradiance on the module G_M^{dir} is given by the equation

$$G_M^{\text{dir}} = I_e^{\text{dir}} \cos\gamma, \tag{18.2}$$

where I_e^{dir} is the *direct normal irradiance* (DNI); $\gamma = \sphericalangle(A_M, a_M)(A_S, a_S)$ is the angle between the surface normal and the incident direction of the sunlight or — in other words — the *angle of incidence* (AOI).

As derived in Appendix E.3, $\cos\gamma$ is given by

$$\cos\gamma = \cos a_M \cos a_S \cos(A_M - A_S) + \sin a_M \sin a_S. \tag{18.3}$$

Thus we obtain for the irradiance

$$\begin{aligned} G_M^{\text{dir}} &= I_e^{\text{dir}} \left[\cos a_M \cos a_S \cos(A_M - A_S) + \sin a_M \sin a_S\right] \\ &= I_e^{\text{dir}} \left[\sin\theta \cos a_S \cos(A_M - A_S) + \cos\theta \sin a_S\right]. \end{aligned} \tag{18.4}$$

Note that this equation only holds when the Sun is above the horizon $(a_S > 0)$ and the azimuth of the Sun is within $\pm 90°$ of A_M, $A_S \in [A_S - 90°, A_S + 90°]$. Otherwise, $G_M^{\text{dir}} = 0$.

If the azimuth of the solar position is the same as the azimuth of the module normal $A_M = A_S$, Eq. (18.4) becomes

$$\begin{aligned} G_M^{\text{dir}} &= I_e^{\text{dir}} \left[\cos a_M \cos a_S + \sin a_M \sin a_S\right] \\ &= I_e^{\text{dir}} \cos(a_M - a_S). \end{aligned} \tag{18.5}$$

When using the tilt angle $\theta = 90° - a_M$ we find

$$G_M^{\mathrm{dir}} = I_e^{\mathrm{dir}} \sin(\theta + a_S)\,. \tag{18.6}$$

Very often, PV modules are not installed on a horizontal plane, but on a tilted roof, which makes the calculation of $\cos\gamma$ more complex. The derivations for modules on a tilted roof as well as some examples are shown in Appendix E.4.

18.2.2 Shading

As we have seen in Section 15.3 shading can have a detrimental effect on the power output of a PV module. Therefore, mutual *shading* between different rows of PV modules must be kept in mind, when designing a PV system.

As shown in Appendix E.5 the length of the shadow of a module of length l is given by

$$d = l\left[\cos\theta_M + \sin\theta_M \cot a_S \cos(A_M - A_S)\right], \tag{18.7}$$

where θ_M and A_M are the tilt and azimuth of the PV module, respectively, as illustrated in Fig. 18.6. a_S and A_S are the altitude and azimuth of the Sun, respectively.

For a first estimation, one can use the rule of thumb that the distance between two rows of modules should be at least three times the length l of the module.

Example

A PV system should be installed on a flat roof in Naples (Italy). The area of the roof that can be utilized for installing the PV system is 10×10 m^2. *The roof is oriented such that the sides are parallel to the East-West and North-South directions, respectively.*

The owner of the roof decides to use Yingli PANDA 60 *modules with dimensions of* $1650\times990\times40$ mm^3. *The modules are installed facing south with a tilt of* $30°$.

He wants to install ***as many modules as possible*** *under the condition that on the shortest day of the year* ***no mutual shading must occur for the duration of 6 hours****.*

Should the modules be mounted with the long or short side touching the ground? How many modules can be mounted in this case?

Answer: *The shortest day of course is 21 December. The solar position on this day at 9:00 h and 15:00 h is*

Time	*Altitude* (°)	*Azimuth* (°)
9:00	13.59	138.55
15:00	13.13	222.17

Because of the Equation of Time, *the Sun is not at its highest point at exactly 12:00 noon. We see, that the solar altitude at 15:00 is just slightly lower than at 9:00. Thus, when using 9:00 for calculating the length of the shadow, the duration without mutual shading will be slightly shorter than 6 hours. Thus, we use the position at 15:00 for the calculation.*

The length of the shadow can be calculated with equation (18.7)

$$d = l\left[\cos\theta_M + \sin\theta_M \cot a_S \cos(A_M - A_S)\right].$$

We have θ_M=30°, A_M=180°, a_S=13.13°, *and* A_S=222.17°.

If the module is mounted to the ground on the short side, we have l=1650 mm. *Hence, we find* d=4050 mm. *The area directly beneath the module at the last row is*

$$d' = l \cos\theta_M = 1429 \text{ mm}.$$

Thus, we can mount ***three rows*** *behind each other, because*

$$2d + d' = 2 \cdot 4050 + 1429 = 9529 \text{ mm},$$

which is less than 10 m. *In one row fit* 10 *modules because* $10 \cdot 990 = 9900$ mm. *Thus, we can place* 30 *modules.*

If the module is mounted to the ground on the long side, we have l=990 mm. *Hence, we find* d=2430 mm. *The area directly beneath the module at the last row is*

$$d' = l \cos\theta_M = 857 \text{ mm}.$$

Thus, we can mount ***four rows*** *behind each other, because*

$$3d + d' = 3 \cdot 2430 + 857 = 8147 \text{ mm},$$

which is lower than 10 m. *In one row fit 6 modules because* $6 \cdot 1650 = 9900$ mm. *Thus, we can place 24 modules.*

18.3 Direct and diffuse irradiance

As Sunlight traverses the atmosphere, it is partially scattered, leading to attenuation of the *direct beam* component. On the other hand, the scattered light also partially will arrive at on the Earth's surface as *diffuse light*. For PV applications it is important to be able to estimate the strength of the direct and diffuse components.

First, we discuss a simple model that allows to estimate the irradiance on a *cloudless sky* independent of the *air mass* and hence the altitude of the Sun. As we have seen in section 5.5, the air mass is defined as

$$\text{AM} := \frac{1}{\cos\theta} = \frac{1}{\sin a_S}, \tag{18.8}$$

where we used that the angle between the Sun and the zenith θ is connected to the solar altitude via $\theta = 90° - a_S$. This equation, however, does not take the curvature of the Earth into account. If the curvature is taken into account, we find [138]

$$\text{AM}(a_S) = \frac{1}{\sin a_S + 0.50572(6.07995 + a_S)^{-1.6364}}. \tag{18.9}$$

To estimate the direct normal irradiance at a certain solar altitude a_S and altitude of the observer h, we can use the following empirical equation [139]

$$I_e^{\text{dir}} = I_e^0 \left[(1 - ch) \cdot 0.7^{\left(\text{AM}^{0.678}\right)} + ch\right], \tag{18.10}$$

with the constant $c = 0.14$. The solar constant is given as $I_e^0 = 1361$ Wm^{-2}. Even during clear sky conditions the diffuse irradiance is about 10% of the direct irradiance. Thus the global irradiance on a module perpendicular to the sun can be approximated as [140]

$$I_e^{\text{global}} \approx 1.1 \cdot I_e^{\text{dir}}. \tag{18.11}$$

For a high diffusion percentage this approach does not work well any more. A more accurate model was developed in the framework of the *European Solar Radiation Atlas* [141]. In that model the direct irradiance for clear sky is given by

$$I_e^{\mathrm{dir}} = I_e^0 \varepsilon \exp\left[-0.8662\, T_L(\mathrm{AM2})\, m\, \delta_R(m)\right]. \tag{18.12}$$

I_0 is the *solar constant* that takes a value of 1361 Wm^{-2}. The factor ε allows to correct for deviations of the Sun-Earth distance from its mean value. a_S is the solar altitude angle. $T_L(\mathrm{AM2})$ is the *Linke turbidity factor* with which the haziness of the atmosphere is taken into account. In this equation its value at an Air Mass 2 is used. m is the relative optical air mass, and finally $\delta_R(m)$ is the integral Rayleigh optical thickness. The different components can be evaluated as follows:

The *correction factor* ε is given by

$$\varepsilon = \frac{I_e(R)}{I_e^0} = \frac{R^2}{\mathrm{AU}^2}. \tag{18.13}$$

The distance between Earth and Sun as a multiple of astronomic units (AU) is given in Eq. (E.6). Hence,

$$\varepsilon = \left(1.00014 - 0.01671 \cos g - 0.00014 \cos 2g\right)^2, \tag{18.14}$$

which leads to annual variations of about $\pm 3.3\%$.

The *Linke Turbidity factor* approximates absorption and scattering in the atmosphere and takes both absorption by water vapour and scattering by aerosol particles into account. It is a unit-less number and typically takes on values between 2 for very clear skies and 7 for heavily polluted skies.

The relative optical air mass m expresses the ratio of the optical path length of the solar beam through the atmosphere to the optical path through a standard atmosphere at sea level with the Sun at zenith. In can be approximated as a function of the solar altitude a_S by

$$m(a_S) = \frac{\exp(-z/z_h)}{\sin a_S + 0.50572(a_S + 6.07995)^{-1.6364}}. \tag{18.15}$$

Here, z is the site elevation and z_h is the scale height of the Rayleigh atmosphere near the Earth surface, given by 8434.5 m.

Finally, the Rayleigh optical thickness $\delta_R(m)$ is given by

$$\begin{aligned} \frac{1}{\delta_R(m)} =& 6.62960 + 1.75130\, m - 0.12020\, m^2 \\ & + 0.00650\, m^3 - 0.00013\, m^4. \end{aligned} \tag{18.16}$$

In their paper, Rigollier *et al.* also take the effect of refraction at very low altitudes into account [141]. This however, is not relevant for our application.

They also present an expression for the *diffuse horizontal irradiance* (DHI) of the light, which is given by

$$I_e^{\mathrm{dif}} = I_e^0 \varepsilon T_{rd}[T_L(\mathrm{AM2})] F_d[a_S, T_L(\mathrm{AM2})], \tag{18.17}$$

where T_{rd} is the diffuse transmission function at zenith, which is a second-order polynomial of $T_L(\mathrm{AM2})$. T_{rd} has typical values in between 0.05 for very clear skies and 0.22 for a very turbid atmosphere. F_d is a diffuse angular function, given as a second-order polynomial of $\sin a_S$. For more details we refer to the paper [141].

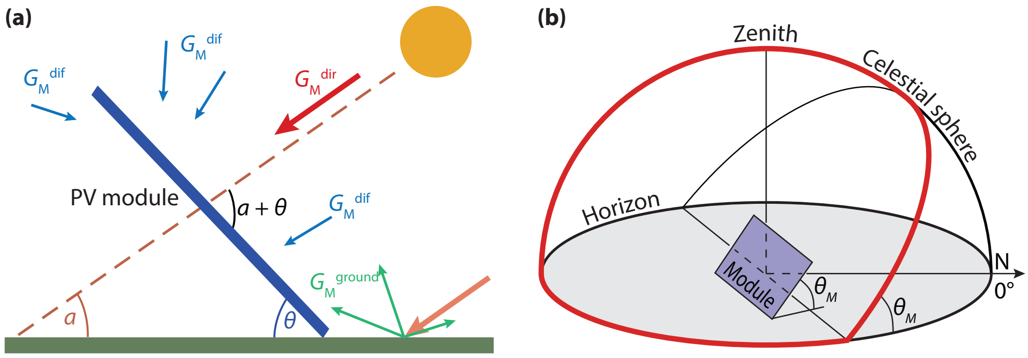

Figure 18.7: (a) The three contributions to the irradiance on a PV module G_M. (b) Illustrating the definition of the *sky view factor* (SVF), which is the fraction of the celestial hemisphere enclosed by the thick red line.

18.3.1 Computation of the irradiance on the module

In Section 18.2 we found that the direct irradiance on a module G_M is given by

$$G_M^{\mathrm{dir}} = I_e^{\mathrm{dir}} \cos\gamma, \tag{18.18}$$

i.e. it is given as the *direct normal irradiance* I_e^{dir} (DNI) times the cosine of the angle of incidence γ (AOI).

For PV modules installed on Earth also two other factors contribute to G_M, as illustrated in Fig. 18.7 (a): first, the diffuse component from the sky, G_M^{dif}. It is proportional to the *sky view factor* (SVF), *i.e.* the portion of the sky from which the module can receive diffuse radiation, as illustrated in Fig. 18.7 (b). It is given by

$$\mathrm{SVF} = \frac{1 + \cos\theta_M}{2}. \tag{18.19}$$

For the detailed calculation of G_M^{dif}, different *sky models* can be used [142–146].

Secondly, the module receives radiation that is reflected from the ground, which can be approximated by

$$G_M^{\mathrm{ground}} = \mathrm{GHI} \cdot \alpha \cdot (1 - \mathrm{SVF}). \tag{18.20}$$

GHI is the *global horizontal irradiance*, which is given by

$$\mathrm{GHI} = \mathrm{DNI} \cdot \cos(a_S) + \mathrm{DHI}, \tag{18.21}$$

where a_S is the altitude of the Sun. DNI and DHI can be measured in meteorological stations with a pyrheliometer and a pyranometer, respectively. The factor α is the *albedo* of the ground, *i.e.* the reflection coefficient of the ground. The lower the albedo, the more light is absorbed by the ground. For example, the albedo of forest is between 0.05 and 0.10, that of snow is 0.6 and the albedo of urban areas is between 0.05 and 0.20 [147]. More example can be found in Ref. [148].

In summary, the irradiance on a PV module is given by

$$G_M = G_M^{\mathrm{dir}} + G_M^{\mathrm{dif}} + G_M^{\mathrm{ground}}. \tag{18.22}$$

If only the measured GHI is available, sky models in combination with Eq. (18.21) allows to retrieve the irradiance components that are necessary for the evaluation of G_M.