5 Solar Radiation

In this chapter we discuss the aspects of solar radiation, which are important for solar energy. After defining the most important radiometric properties in Section 5.2, we discuss blackbody radiation in Section 5.3 and the wave-particle duality in Section 5.4. Equipped with these instruments, we than investigate the different solar spectra in Section 5.5. However, prior to these discussions we give a short introduction about *the Sun*.

5.1 The Sun

The Sun is the central star of our solar system. It consists mainly of hydrogen and helium. Some basic facts are summarised in Table 5.1 and its structure is sketched in Fig. 5.1. The mass of the Sun is so large that it contributes 99.68% of the total mass of the solar system. In the center of the Sun the pressure-temperature conditions are such that *nuclear fusion* can

Table 5.1: Some facts on the Sun

Mean distance from the Earth	149 600 000 km (the astronomic unit, AU)
Diameter	1 392 000 km (109 $\times$ that of the Earth)
Volume	1 300 000 $\times$ that of the Earth
Mass	1.993×10^{27} kg (332 000 times that of the Earth)
Density (at its center)	$>10^5$ kg m^{-3} (over 100 times that of water)
Pressure (at its center)	over 1 billion atmospheres
Temperature (at its center)	about 15 000 000 K
Temperature (at the surface)	6 000 K
Energy radiation	3.8×10^{26} W
The Earth receives	1.7×10^{18} W

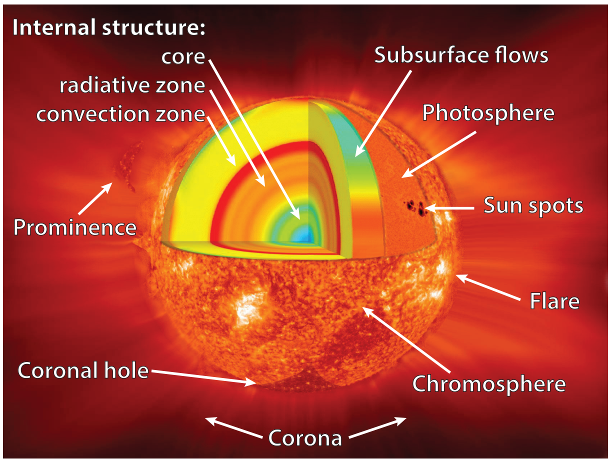

Figure 5.1: The layer structure of the Sun (adapted from a figure obtained from NASA [28]).

take place. In the major nuclear reaction, the *proton-proton reaction*, via a number of steps four protons react to produce

- 1 helium core (two protons and two neutrons),
- 2 positrons (the anti-particles of electrons),
- 2 neutrinos,
- electromagnetic radiation.

The positrons annihilate with electrons leading to additional radiation. The mass of the helium core is 0.635% less than that of four protons, the energy difference is converted into energy according to Einstein's equation

$$E = mc_0{}^2. \tag{5.1}$$

The total power is about 3.8×10^{26} W. Therefore, every second approx. 4 million tons of mass are converted into energy. However, the power density at the centre of the Sun is estimated by theoretical assumptions only to be about 275 W/m^3. As we have seen in Chapter 1, the average power of an adult male is 115.7 W. If we assume his average volume to be 70 L, *i.e.* 0.07 m^3, the average power density of the human body is 1650 W, hence much higher than the fusion power in the centre of the sun!

The neutrinos hardly interact with matter and thus can leave the solar core without any hindrance. Every second, about 6.5×10^{10} cm^{-2} pass through the Earth and hence also through our bodies. Neutrinos carry about 2% of the total energy radiated by the Sun.

The remainder of the radiation is released as electromagnetic radiation. The core of the Sun is so dense that radiation cannot travel freely but is continuously absorbed and re-emitted, such that it takes the radiation between 10 000 and 170 000 years to travel to the solar surface. The surface of the Sun is called the photosphere. It has a temperature of about 6000 K. It behaves very closely to a blackbody (see section 5.3) and is the source of the solar radiation that hits the Earth. The total irradiance of the solar radiation at the mean

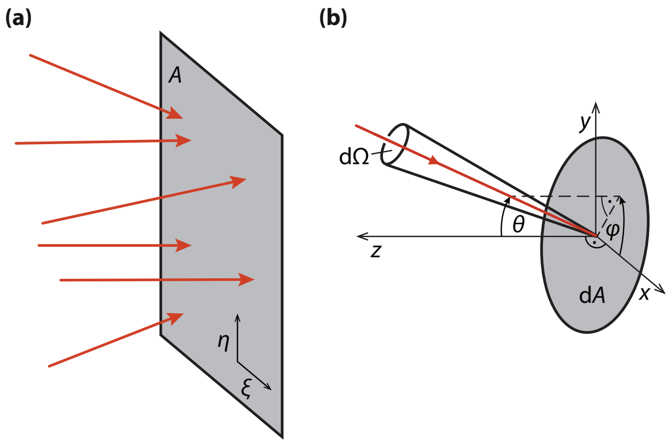

Figure 5.2: (a) Illustrating a surface A irradiated by light from various directions and (b) a surface element $\mathrm{d}A$ that receives radiation from a solid angle element $\mathrm{d}\Omega$ under an angle θ with respect to the surface normal.

Earth-sun distance on a plane perpendicular to the direction of the Sun, outside the Earth's atmosphere, is referred to as the *solar constant*. Its value is approximately 1361 $\mathrm{W/m^2}$.

5.2 Radiometric properties

Radiometry is the branch of optics concerned with the *measurement of light*. Since photovoltaics deals with sunlight that is converted into electricity it is very important to discuss how the "amount of energy" of the light can be expressed physically and mathematically.

In solar science, it is not the total amount of the energy that is important, but the amount of energy per unit time, *i.e.* the *power* P. For our discussion we assume a surface A that is irradiated by light, as illustrated in Fig. 5.2 (a). For obtaining the total power that is incident on the surface, we have to integrate over the whole surface. Further we have to take into account that light is incident from all the different directions, which we parameterise with the spherical coordinates (θ, ϕ). The *polar angle* θ is defined with respect to the normal of the surface element $\mathrm{d}A$ and ϕ is the *azimuth*, as sketched in Fig. 5.2 (b). Thus, we have to integrate over the hemisphere, from that light can be incident on the surface element $\mathrm{d}A$, as well. We therefore obtain

$$P = \int_A \int_{2\pi} L_e \cos\theta \,\mathrm{d}\Omega\,\mathrm{d}A, \tag{5.2}$$

where

$$\mathrm{d}\Omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \tag{5.3}$$

is a *solid angle* element. The quantity L_e is called the *radiance* and it is one of the most fundamental radiometric properties. The subscript e for L_e and all the other radiometric properties indicates that these are *energetic* properties, *i.e.* they are related to energies or powers. The physical dimension of L_e is

$$[L_e] = \mathrm{Wm^{-2}sr^{-1}}.$$

The factor $\cos\theta$ expresses the fact that not the surface element $\mathrm{d}A$ itself is the relevant property but the projection of $\mathrm{d}A$ to the normal of the direction (θ, ϕ). This is also known as the *Lambert cosine law*.

We can express Eq. (5.2) as integrals of the surface coordinates (ξ, η) and the direction coordinates (θ, ϕ), which reads as

$$P = \int_A \int_{2\pi} L_e(\xi, \eta; \theta, \phi) \cos\theta \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi \,\mathrm{d}\xi \,\mathrm{d}\eta. \tag{5.4}$$

Since sunlight consists of of a spectrum of different frequencies (or wavelengths), it is useful to use *spectral properties*. These are given by

$$P_\nu = \frac{\mathrm{d}P}{\mathrm{d}\nu}, \qquad P_\lambda = \frac{\mathrm{d}P}{\mathrm{d}\lambda}, \tag{5.5}$$

$$L_{e\nu} = \frac{\mathrm{d}L_e}{\mathrm{d}\nu}, \qquad L_{e\lambda} = \frac{\mathrm{d}L_e}{\mathrm{d}\lambda}, \tag{5.6}$$

etc. Their physical dimensions are

$$[P_\nu] = \mathrm{W\,Hz^{-1}} = \mathrm{Ws}, \qquad [P_\lambda] = \mathrm{Wm^{-1}},$$

$$[L_{e\nu}] = \mathrm{Wm^{-2}sr^{-1}s}, \qquad [L_{e\lambda}] = \mathrm{Wm^{-2}sr^{-1}m^{-1}},$$

Since wavelength and frequency are connected to each other via $\nu\lambda = c$, P_ν and P_λ are related via

$$P_\nu = \frac{\mathrm{d}P}{\mathrm{d}\nu} = \frac{\mathrm{d}P}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}\nu} = P_\lambda \left(-\frac{c}{\nu^2}\right), \tag{5.7}$$

and similarly for $L_{e\nu}$ and $L_{e\lambda}$. The $-$ sign is because of the changing direction of integration when switching between ν and λ and usually is omitted.

The spectral power in wavelength thus can be obtained via

$$P_\lambda = \int_A \int_{2\pi} L_{e\lambda} \cos\theta \,\mathrm{d}\Omega \,\mathrm{d}A, \tag{5.8}$$

and analogously for P_ν. The radiance is given by

$$L_e = \frac{1}{\cos\theta}\frac{\partial^4 P}{\partial A\,\partial\Omega}, \tag{5.9}$$

and similarly for $L_{e\nu}$ and $L_{e\lambda}$.

Another very important radiometric property is the *irradiance* I_e that tells us the power density at a certain point (ξ, η) of the surface. It often also is called the *(spectral) intensity* of the light. It is given as the integral of the radiance over the solid angle,

$$I_e = \int_{2\pi} L_e \cos\theta \,\mathrm{d}\Omega = \int_{2\pi} L_e(\xi, \eta; \theta, \phi) \cos\theta \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi. \tag{5.10}$$

The *spectral irradiance* $I_{e\nu}$ or $I_{e\lambda}$ is calculated similarly. The physical dimensions are

$$[I_e] = \mathrm{Wm^{-2}}, \qquad [I_{e\nu}] = \mathrm{Wm^{-2}s}, \qquad [I_{e\lambda}] = \mathrm{Wm^{-2}m^{-1}}.$$

The irradiance also is given as

$$I_e = \frac{\partial^2 P}{\partial A}, \tag{5.11}$$

and similarly for $I_{e\nu}$ and $I_{e\lambda}$. Irradiance refers to radiation, that is received by the surface. For radiation emitted by the surface, we instead speak of *radiant emittance*, M_e, $M_{e\nu}$, and $M_{e\lambda}$.

As we discussed earlier, the energy of a photon is proportional to its frequency, $E_{ph} = h\nu = hc/\lambda$. Thus, the spectral power P_λ is proportional to the *spectral photon flow* $\Psi_{\mathrm{ph},\lambda}$,

$$P_\lambda = \Psi_{\mathrm{ph},\lambda} \frac{hc}{\lambda}, \tag{5.12}$$

and similarly for P_ν and $\Psi_{\mathrm{ph},\nu}$. The total photon flow Ψ_{ph} is related to the spectral photon flow via

$$\Psi_{\mathrm{ph}} = \int_0^\infty \Psi_{\mathrm{ph},\nu}\, \mathrm{d}\nu = \int_0^\infty \Psi_{\mathrm{ph},\lambda}\, \mathrm{d}\lambda. \tag{5.13}$$

The physical dimensions of the (spectral) photon flow are

$$[\Psi_{\mathrm{ph}}] = \mathrm{s}^{-1}, \qquad [\Psi_{\mathrm{ph},\nu}] = 1, \qquad [\Psi_{\mathrm{ph},\lambda}] = \mathrm{s}^{-1}\mathrm{m}^{-1}.$$

The *(spectral) photon flux* Φ_{ph} is defined as the photon flow per area,

$$\Phi_{\mathrm{ph}} = \frac{\partial^2 \Psi_{\mathrm{ph}}}{\partial A}, \tag{5.14}$$

and similarly for $\Phi_{\mathrm{ph},\nu}$ and $\Phi_{\mathrm{ph},\lambda}$. The physical dimensions are

$$[\Phi_{\mathrm{ph}}] = \mathrm{s}^{-1}\mathrm{m}^{-2}, \qquad [\Phi_{\mathrm{ph},\nu}] = \mathrm{m}^{-2}, \qquad [\Phi_{\mathrm{ph},\lambda}] = \mathrm{s}^{-1}\mathrm{m}^{-2}\mathrm{m}^{-1}.$$

By comparing Eqs. (5.11) and (5.14) and looking at Eq. (5.12), we find

$$I_{e\lambda} = \Phi_{\mathrm{ph},\lambda} \frac{hc}{\lambda}, \tag{5.15}$$

and analogously for $I_{e\nu}$ and $\Phi_{\mathrm{ph},\nu}$.

5.3 Blackbody radiation

If we take a piece of *e. g.* metal and start heating it up, it will start to glow, first in a reddish colour, then getting more and more yellowish as we increase the temperature even further. It thus emits electromagnetic radiation that we call *thermal radiation*. Understanding this phenomenon theoretically and correctly predicting the emitted spectrum was one of the most important topics of physics in the late nineteenth century.

For discussing thermal radiation, the concept of the *blackbody* is very useful. A blackbody, which does not exist in nature, absorbs all the radiation that is incident on it, regardless of wavelength and angle of incidence. Its reflectivity therefore is 0. Of course, since it also will emit light according to its equilibrium temperature, it does not need to appear black to the eye.

Two approximations for the blackbody spectrum were presented around the turn of the century: First, in 1896, Wilhelm Wien empirically derived the following expression for the spectral blackbody radiance:

$$L_{e\lambda}^{W}(\lambda; T) = \frac{C_1}{\lambda^5} \exp\left(-\frac{C_2}{\lambda T}\right), \tag{5.16}$$

where λ and T are the wavelength and the temperature, respectively. While this approximation gives good results for short wavelengths, it fails to predict the emitted spectrum at long wavelengths, thus in the infrared.

Secondly, in 1900 and in a more complete version in 1905, Lord Rayleigh and James Jeans, derived the equation

$$L_{e\lambda}^{RJ}(\lambda; T) = \frac{2ck_BT}{\lambda^4}, \tag{5.17}$$

where $k_B \approx 1.381 \times 10^{-23}$ J/K is the Boltzmann constant. The derivation of this equation was based on electrodynamic arguments. While $L_{e\lambda}^{RJ}$ is in good agreement to measured values at long wavelengths, it diverges to infinity for short wavelength. Further, the radiant emittance, which is obtained via integration over all wavelength, diverges towards infinity. This so called *ultraviolet catastrophe* demonstrates that Rayleigh and Jeans did not succeed in developing a model that can adequately describe thermal radiation.

In 1900, Max Planck found an equation, that interpolates between the Wien approximation and the Rayleigh-Jeans law,

$$L_{e\lambda}^{BB}(\lambda; T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}, \tag{5.18a}$$

where $c \approx 2.998 \times 10^8$ m/s is the speed of light *in vacuo* and $h \approx 6.626 \times 10^{-34}$ m^{2}kg/s is the nowadays called *Planck constant*. Via Eq. (5.7) we find the *Planck law* expressed as a function of the frequency ν,

$$L_{e\nu}^{BB}(\nu; T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}, \tag{5.18b}$$

It is remarkable to see that the Planck law contains three fundamental constants, c, k_B, and h, which are amongst the most important constants in physics.

Figure 5.3 shows the spectrum of a blackbody of 6000 K temperature and the Wien approximation and the Rayleigh-Jeans law. We indeed see that the Wien approximation fits well at short wavelengths, while the Rayleigh-Jeans law matches well at long wavelengths but completely fails at short wavelengths.

Both the Wien approximation [Eq. (5.16)] and the Rayleigh-Jeans law [Eq. (5.17)] can be directly derived from the Planck law:

For short wavelength,

$$\exp\left(\frac{hc}{\lambda k_B T}\right) \gg 1,$$

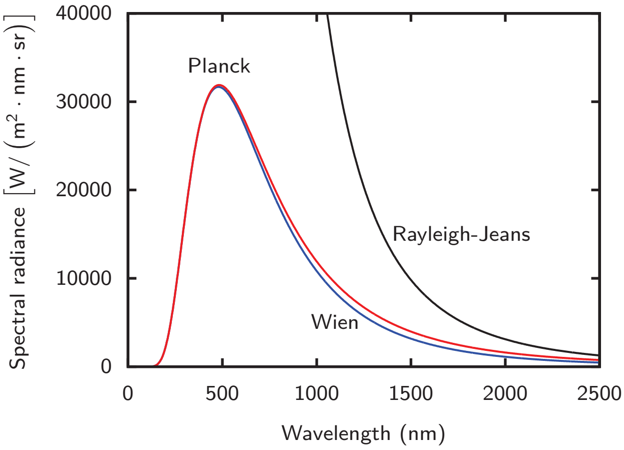

Figure 5.3: The blackbody spectrum at 6000 K as calculated with the Wien approximation, the Rayleigh-Jeans law and the Planck law.

such that the -1 can be ignored and we arrive at the Wien approximation with $C_1 = 2hc^2$ and $C_2 = hc/k_B$.

For long wavelength we can use the approximation

$$\exp\left(\frac{hc}{\lambda k_B T}\right) - 1 \approx \frac{hc}{\lambda k_B T},$$

which directly results in the Rayleigh-Jeans law.

The total radiant emittance of a black body is given by

$$M_e^{BB}(T) = \int_0^\infty \int_{2\pi} L_{e\lambda}^{BB}(\lambda; T) \cos\theta \, \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi \, \mathrm{d}\lambda = \sigma T^4, \tag{5.19}$$

where

$$\sigma = \frac{2\pi^5 k_B^4}{15 c^2 h^3} \approx 5.670 \times 10^{-8} \, \frac{\mathrm{W}}{\mathrm{m^2 K^4}} \tag{5.20}$$

is the *Stefan-Boltzmann* constant. Equation (5.19) is known as the *Stefan-Boltzmann law*. As a matter of fact, it already was discovered in 1879 and 1884 by Jožef Stefan and Ludwig Boltzmann, respectively, *i.e.* about twenty years prior to the derivation of Planck's law. This law is very important because it tells us that if the temperature of a body (in K) is doubled, it emits 16 times as much power. Little temperature variations thus have a large influence on the total emitted power.

Another important property of blackbody radiation is Wien's displacement law, which states that the wavelength of maximal radiance is inversely proportional to the temperature,

$$\lambda_{\max} T = b \approx 2.898 \times 10^{-3} \, \mathrm{mK}. \tag{5.21}$$

Figure 5.4 shows the spectra for three different temperatures. Note the strong increase in radiance with temperature and also the shift of the maximum to shorter wavelengths.

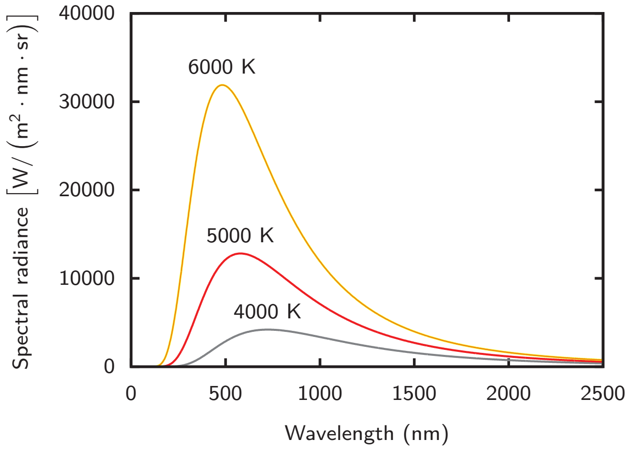

Figure 5.4: The blackbody spectrum at three different temperatures.

5.4 Wave-particle duality

In Planck's law, as stated in Eqs. (5.18), the constant h appeared for the first time. Its product with the frequency, $h\nu = hc/\lambda$ has the unit of an energy. Planck himself did not see the implications of h. It was Einstein, who understood in 1905 that Planck's law actually has to be interpreted such that light comes in quanta of energy with the size

$$E_{\text{ph}} = h\nu. \tag{5.22}$$

Nowadays, these quanta are called *photons*. In terms of classical mechanics we could say that *light shows the behaviour of particles*.

On the other hand, we have seen in Chapter 4 that light also shows *wave character* which becomes obvious when looking at the propagation of light through space or at reflection and refraction at a flat interface. It also was discovered that other particles, such as electrons, show wave-like properties.

This behaviour is called *wave-particle* duality and is a very intriguing property of *quantum mechanics* that was discovered and developed in the first quarter of the twentieth century. Many discussion was held on how this duality has to be interpreted - but this is out of the focus of this book. So we just will accept that depending on the situation light might behave as a wave or as a particle.

5.5 Solar spectra

As we already mentioned in chapter 3, only photons of appropriate energy can be absorbed and hence generate electron-hole pairs in a semiconductor material. Therefore, it is important to know the spectral distribution of the solar radiation, *i.e.* the number of photons of a particular energy as a function of the wavelength λ. Two quantities are used to describe the solar radiation spectrum, namely the *spectral irradiance* $I_{e\lambda}$ and the *spectral photon flux* $\Phi_{\text{ph}}(\lambda)$. We defined these quantities already in section 5.2.

The surface temperature of the Sun is about 6000 K. If it was a perfect black body, it would emit a spectrum as described by Eqs. (5.18), which give the spectral radiance. For calculating the spectral *irradiance* a blackbody with the size and position of the Sun would have on Earth, we have to multiply the spectral radiance with the solid angle of the Sun as seen from Earth,

$$I_{e\lambda}^{BB}(T;\lambda) = L_{e\lambda}^{BB}(T;\lambda)\Omega_{\text{Sun}}. \tag{5.23}$$

We can calculate Ω_{Sun} with

$$\Omega_{\text{Sun}} = \pi \left(\frac{R_{\text{Sun}}}{\text{AU} - R_{\text{Earth}}} \right)^2. \tag{5.24}$$

Using R_{Sun} = 696 000 km, an astronomical unit AU = 149 600 000 km, and R_{Earth} = 6370 km, we find

$$\Omega_{\text{Sun}} \approx 68.5\,\mu\text{sr}. \tag{5.25}$$

The blackbody spectrum is illustrated in Fig. 5.5. The spectrum outside the atmosphere of Earth is already very different. It is called the AM0 spectrum, because no (or "zero") atmosphere is traversed. AM0 also is shown in Fig. 5.5. The irradiance at AM0 is $I_e(\text{AM0}) = 1361\ \text{Wm}^{-2}$.

When solar radiation passes through the atmosphere of Earth, it is attenuated. The most important parameter that determines the solar irradiance under clear sky conditions is the distance that the sunlight has to travel through the atmosphere. This distance is the shortest when the Sun is at the zenith, *i.e.* directly overhead. The ratio of an actual path length of the sunlight to this minimal distance is known as the *optical air mass*. When the Sun is at its zenith the optical air mass is unity and the spectrum is called the air mass 1 (AM1) spectrum. When the Sun is at an angle θ with the zenith, the air mass is given by

$$\text{AM} := \frac{1}{\cos\theta}. \tag{5.26}$$

For example, when the Sun is 60° from the zenith, *i.e.* 30° above the horizon, we receive an AM2 spectrum. Depending on the position on the Earth and the position of the Sun in the sky, terrestrial solar radiation varies both in intensity and the spectral distribution. The attenuation of solar radiation is due to scattering and absorption by air molecules, dust particles and/or aerosols in the atmosphere. Especially, water vapour (H_2O), oxygen (O_2) and carbon dioxide (CO_2) cause absorption. Since this absorption is wavelength-selective, it results in gaps in the spectral distribution of solar radiation as apparent in Fig. 5.5. Ozone absorbs radiation with wavelengths below 300 nm. Depletion of ozone from the atmosphere allows more ultra-violet radiation to reach the Earth, with consequent harmful effects upon biological systems. CO_2 molecules contribute to the absorption of solar radiation at wavelengths above 1 µm. By changing the CO_2 content in the atmosphere the absorption in the infrared is enhanced, which has consequences for our climate.

Solar cells and photovoltaic modules are produced by many different companies and laboratories. Further, many different solar cell technologies are investigated and sold. It is therefore of utmost importance to define conditions that allow a comparison of all different solar cells and PV modules. These conditions are the *Standard Test Conditions* (STC), characterised by an irradiance of 1000 Wm^{-2}, an AM1.5 spectrum and a cell temperature

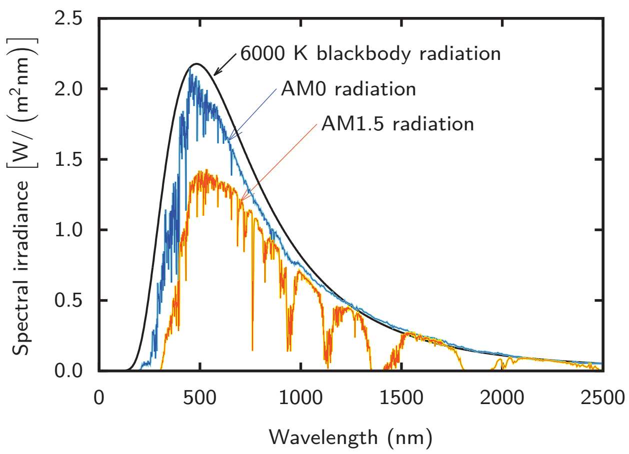

Figure 5.5: Different solar spectra: the blackbody spectrum of a blackbody at 6000 K, the extra-terrestrial AM0 spectrum and the AM1.5 spectrum.

of 25° C. The AM1.5 spectrum is a reference solar spectral distribution, being defined in the International Standard IEC 60904-3 [29]. This spectrum is based on the solar irradiance received on a sun-facing plane surface tilted at 37° to the horizontal. Both direct sun light, diffuse sun light and the wavelength-dependent albedo of light bare soil are being taken into account. Albedo, is the part of the solar radiation that is reflected by the Earth's surface and depends on the reflectivity of the environment. The total irradiance of the AM1.5 spectrum is 1000 Wm^{-2} and is close to the maximum received at the surface of the Earth at a cloudless day. Both STC and the AM1.5 spectrum are used all over the world in both industry and (test) laboratories. The power generated by a PV module at STC is thus expressed in the unit watt peak, Wp.

The actual amount of solar radiation that reaches a particular place on the Earth is extremely variable. In addition to the regular daily and annual variation due to the apparent motion of the Sun, irregular variations have to be taken into account that are caused by local atmospheric conditions, such as clouds. These conditions particularly influence the direct and diffuse components of solar radiation. The direct component of solar radiation is that part of the sunlight that directly reaches the surface. Scattering of the sunlight in the atmosphere generates the diffuse component. Albedo may also be present in the total solar radiation. We use the term global radiation to refer to the total solar radiation, which is made up of these three components.

The design of an optimal photovoltaic system for a particular location depends on the availability of the solar insolation data at the location. Solar irradiance integrated over a period of time is called solar irradiation. For example, the average annual solar irradiation in the Netherlands is 1 000 kWh/m^2, while in the Sahara the average value is 2 200 kWh/m^2, thus more than twice as high. We will discuss these issues in more detail in in Chapters 17 and 18..

A

Derivations in Electrodynamics

A.1 The Maxwell equations

The four Maxwell equations couple the electric and magnetic fields to their sources, *i.e.* electric charges and current densities, and to each other. They are given by

$$\nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho_F(\mathbf{r}), \tag{A.1a}$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t}, \tag{A.1b}$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}, t) = 0, \tag{A.1c}$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = +\frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t} + \mathbf{J}_F(\mathbf{r}), \tag{A.1d}$$

where $\mathbf{r}$ and t denote location and time, respectively. $\mathbf{D}$ is the *electric displacement*, $\mathbf{E}$ the *electric field*,[1] $\mathbf{B}$ is the *magnetic induction* and $\mathbf{H}$ is the *magnetic field*.[1] ρ_F is the *free charge density* and $\mathbf{J}_F$ is the *free current density*.

The electric displacement and field are related to each other via

$$\mathbf{D} = \epsilon\epsilon_0\mathbf{E}, \tag{A.2a}$$

where ϵ is the relative permittivity of the medium in that the fields are observed and $\epsilon_0 = 8.854 \times 10^{-12}$ As/(Vm) is the permittivity *in vacuo*. Similarly, the magnetic field and induction are related to each other via

$$\mathbf{B} = \mu\mu_0\mathbf{H}, \tag{A.2b}$$

[1] In this appendix we use for the electric field $\mathbf{E}$ instead of ξ and for the magnetic field $\mathbf{H}$ instead of ζ.

where μ is the relative permeability of the medium in that the fields are observed and $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am) is the permeability of vacuum. Eqs. (A.2) are only valid if the medium is *isotropic*, *i.e.* ϵ and μ are independent of the direction. We may assume all the materials important for solar cells to be *nonmagnetic*, *i.e.* $\mu \equiv 1$.

A.2 Derivation of the electromagnetic wave equation

We now derive de electromagnetic wave equations in source-free space, $\rho_F \equiv 0$ and $\mathbf{j}_F \equiv 0$. For the derivation of the electromagnetic wave equations we start with applying the rotation operator $\nabla\times$ to the second Maxwell equation, Eq. (A.1b),

$$\nabla \times (\nabla \times \mathbf{E}) = -\nabla \times \left(\frac{\partial \mathbf{B}}{\partial t}\right). \tag{A.3}$$

Now we take the fourth Maxwell equation, Eq. A.1d, with $\mathbf{j}_F = 0$ and Eqs. (A.2),

$$\frac{1}{\mu\mu_0}\nabla \times \mathbf{B} = \epsilon\epsilon_0 \frac{\partial \mathbf{E}}{\partial t},$$

and substitute it into Eq. (A.3),

$$\nabla \times (\nabla \times \mathbf{E}) = -\epsilon\epsilon_0\mu\mu_0 \left(\frac{\partial^2 \mathbf{E}}{\partial t^2}\right). \tag{A.4}$$

By using the relation

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \Delta\mathbf{E} = -\Delta\mathbf{E}, \tag{A.5}$$

we find

$$\Delta\mathbf{E} = \epsilon\epsilon_0\mu\mu_0 \left(\frac{\partial^2 \mathbf{E}}{\partial t^2}\right). \tag{A.6}$$

In Eq. (A.5) we used that we are in source free space, *i.e.* $\nabla\mathbf{E} = 0$. Equation (A.6) is the *wave equation* for the *electric field*. Note that the factor

$$\frac{1}{\epsilon\epsilon_0\mu\mu_0}$$

has the unit of $(\mathrm{m/s})^2$, *i.e.* a speed to the square. In easy terms, it is the squared propagation speed of the wave.[1] We now set

$$c_0^2 := \frac{1}{\epsilon_0\mu_0} \tag{A.7}$$

and

$$n^2 = \epsilon, \tag{A.8}$$

where c_0 is the speed of light *vacuo* and n is the *refractive index* of the medium. Since we also assume $\mu \equiv 1$, we finally obtain for the wave equation for the electric field

$$\Delta\mathbf{E} - \frac{n^2}{c_0^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2}\right) = 0. \tag{A.9a}$$

[1] In reality, if the medium is absorbing, things are getting much more complex.

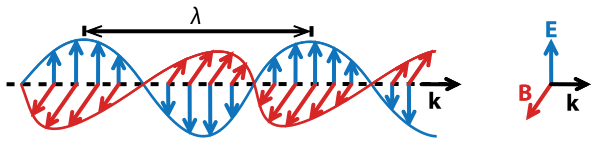

Figure A.1: Illustrating the **E** and **B** field and the **k** vector of a plane electromagnetic wave.

In a similar manner we can derive the wave equation for the *magnetic field*,

$$\Delta \mathbf{H} - \frac{n^2}{c_0^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2}\right) = 0. \tag{A.9b}$$

A.3 Properties of electromagnetic waves

In section A.2, we found that plane waves can be described by

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}_0 \cdot \mathrm{e}^{ik_z z - i\omega t}, \tag{A.10a}$$
$$\mathbf{H}(\mathbf{x}, t) = \mathbf{H}_0 \cdot \mathrm{e}^{ik_z z - i\omega t}. \tag{A.10b}$$

In this section we study some general properties of plane electromagnetic waves, illustrated in Fig. A.1. Substituting Eq. (A.10a) into the first Maxwell equation, Eq. (A.1a), yields

$$\begin{aligned}
\nabla \mathbf{D} = \epsilon\epsilon_0 \nabla \mathbf{E} &= 0,\\
\nabla \mathbf{E} &= 0,\\
\nabla\left(\mathbf{E}_0 \mathrm{e}^{ik_z z - i\omega t}\right) &= 0,\\
E_{x,0}\underbrace{\frac{\partial}{\partial x}\mathrm{e}^{ik_z z - i\omega t}}_{=0} + E_{y,0}\underbrace{\frac{\partial}{\partial y}\mathrm{e}^{ik_z z - i\omega t}}_{=0} + &\\
+ E_{z,0}\frac{\partial}{\partial z}\mathrm{e}^{ik_z z - i\omega t} &= 0,\\
iE_{z,0}k_z \mathrm{e}^{ik_z z - i\omega t} &= 0,\\
E_{z,0} &= 0,
\end{aligned} \tag{A.11a}$$

where we used the notation $\mathbf{E}_0 = \left(E_{x,0},\, E_{y,0},\, E_{z,0}\right)$. In a similar manner, by substituting Eq. (A.10b) into the first Maxwell equation, Eq. (A.1c), we obtain

$$H_{z,0} = 0. \tag{A.11b}$$

Thus, neither the electric nor the magnetic fields have components in the propagation direction but only components perpendicular to the propagation direction (the x- and y-directions in our case).

Without loss of generality we now assume that the electric field only has an x-component, $\mathbf{E}_0 = (E_{x,0},\, 0\, 0)$. Substituting this electric field into the left hand side of the second Maxwell equation, Eq. (A.1b), yields

$$\nabla \times \mathbf{E} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} \end{pmatrix} \times \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y \\ \frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_z \\ \frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x \end{pmatrix} \tag{A.12}$$

$$= \begin{pmatrix} 0 \\ \frac{\partial}{\partial z} E_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ E_{x,0} \frac{\partial}{\partial z} \mathrm{e}^{ik_z z - i\omega t} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ iE_{x,0} k_z \mathrm{e}^{ik_z z - i\omega t} \\ 0 \end{pmatrix}. \tag{A.13}$$

The right hand side of Eq. (A.1b) yields

$$\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} = \mu_0 \frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t} = \tag{A.14}$$

$$\mu_0 \begin{pmatrix} \frac{\partial}{\partial t} H_x \\ \frac{\partial}{\partial t} H_y \\ \frac{\partial}{\partial t} H_z \end{pmatrix} = \mu_0 \begin{pmatrix} H_{x,0} \frac{\partial}{\partial t} \mathrm{e}^{ik_z z - i\omega t} \\ H_{y,0} \frac{\partial}{\partial t} \mathrm{e}^{ik_z z - i\omega t} \\ 0 \end{pmatrix} = \mu_0 \begin{pmatrix} -i\omega H_{x,0} \mathrm{e}^{ik_z z - i\omega t} \\ -i\omega H_{y,0} \mathrm{e}^{ik_z z - i\omega t} \\ 0 \end{pmatrix}. \tag{A.15}$$

Thus, we obtain for Eq. (A.1b)

$$\begin{pmatrix} 0 \\ iE_{x,0} k_z \mathrm{e}^{ik_z z - i\omega t} \\ 0 \end{pmatrix} = -\mu_0 \begin{pmatrix} -i\omega H_{x,0} \mathrm{e}^{ik_z z - i\omega t} \\ -i\omega H_{y,0} \mathrm{e}^{ik_z z - i\omega t} \\ 0 \end{pmatrix}, \tag{A.16}$$

i.e. only the y-component of the magnetic field survives. The electric and magnetic components are related to each other via

$$E_{x,0} k_z = \mu_0 \omega H_{y,0} \tag{A.17}$$

By substituting Eq. (4.4) into Eq. (A.17), we find

$$H_{y,0} = \frac{n}{c\mu_0} E_{x,0} = \frac{n}{Z_0} E_{x,0}, \tag{A.18}$$

where

$$Z_0 = c\mu_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.7\,\Omega \tag{A.19}$$

is the *impedance of free space*.

In summary, we found the following properties of the electromagnetic field:

- The electric and magnetic field vectors are perpendicular to each other and also perpendicular to the propagation vector,

$$\mathbf{k} \cdot \mathbf{E}_0 = \mathbf{k} \cdot \mathbf{H}_0 = \mathbf{H}_0 \cdot \mathbf{E}_0 = 0. \tag{A.20}$$

- The electric and magnetic fields are proportional to the propagation direction, hence electromagnetic waves are *transverse waves*.

- The electric and magnetic vectors have a constant, material dependent ratio. If the electric field is along the x-direction and the magnetic field is along the y-direction, this ratio is given by

$$H_{y,0} = \frac{n}{c\mu_0} E_{x,0} = \frac{n}{Z_0} E_{x,0}, \tag{A.21}$$

  where

$$Z_0 = c\mu_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.7\,\Omega \tag{A.22}$$

  is the *impedance of free space*.

The derivations in this section were done for plane waves. However, it can be shown that the properties of electromagnetic waves summarised in the itemisation above are valid for electromagnetic waves in general.