9

Solar Cell Parameters and Equivalent Circuit

9.1 External solar cell parameters

The main parameters that are used to characterise the performance of solar cells are the *peak power* P_{max}, the *short-circuit current density* J_{sc}, the *open circuit voltage* V_{oc}, and the *fill factor FF*. These parameters are determined from the illuminated *J-V* characteristic as illustrated in Fig. 8.10. The *conversion efficiency* η can be determined from these parameters.

9.1.1 Standard test conditions

For a reliable measurement of the *J-V* characteristics, it is vital to perform the measurements under *standard test conditions* (STC). This means, that the total irradiance on the solar cell that should be measured is equal to 1000 W/m². Further, the spectrum should resemble the AM1.5 spectrum that we discussed in Section 5.5. Additionally, the temperature of the solar cell should be kept constant at 25°C. As we will see in Section 20.3, the performance of a solar cell strongly depends on the temperature.

9.1.2 Short-circuit current density

The *short-circuit current* I_{sc} is the current that flows through the external circuit when the electrodes of the solar cell are short circuited. The short-circuit current of a solar cell depends on the photon flux incident on the solar cell, which is determined by the spectrum of the incident light. For standard solar cell measurements, the spectrum is standardised to the AM1.5 spectrum. The I_{sc} depends on the area of the solar cell. In order to remove

the dependence of the solar cell area onto I_{sc}, often the *short-circuit current density* is used to describe the maximum current delivered by a solar cell. The maximum current that the solar cell can deliver strongly depends on the optical properties of the solar cell, such as absorption in the absorber layer and reflection.

In the ideal case, J_{sc} is equal to J_{ph}, which can be easily derived from Eq. (8.33). J_{ph} can be approximated by Eq. (8.34), which shows that in case of an ideal diode (for example no surface recombination) and uniform generation, the critical material parameters that determine J_{ph} are the diffusion lengths of minority carriers. Crystalline silicon solar cells can deliver under an AM1.5 spectrum a maximum possible current density of 46 mA/cm^2. In laboratory c-Si solar cells the measured J_{sc} is above 42 mA/cm^2, while commercial solar cell have an J_{sc} exceeding 35 mA/cm^2.

9.1.3 Open-circuit voltage

The *open-circuit voltage* is the voltage at which no current flows through the external circuit. It is the maximum voltage that a solar cell can deliver. V_{oc} corresponds to the forward bias voltage, at which the dark current density compensates the photocurrent density. V_{oc} depends on the photo-generated current density and can be calculated from Eq. (8.33) assuming that the net current is zero,

$$V_{oc} = \frac{k_B T}{q} \ln \left(\frac{J_{ph}}{J_0} + 1 \right) \approx \frac{k_B T}{q} \ln \left(\frac{J_{ph}}{J_0} \right),$$ (9.1)

where the approximation is justified because of $J_{ph} \gg J_0$

Equation 9.1 shows that V_{oc} depends on the saturation current density of the solar cell and the photo-generated current. While J_{ph} typically has a small variation, the key effect is the saturation current, since this may vary by orders of magnitude. The saturation current density, J_0, depends on the recombination in the solar cell. Therefore, V_{oc} is a measure of the amount of recombination in the device. Laboratory crystalline silicon solar cells have a V_{oc} of up to 720 mV under the standard AM1.5 conditions, while commercial solar cells typically have V_{oc} exceeding 600 mV.

9.1.4 Fill factor

The fill factor is the ratio between the maximum power ($P_{max} = J_{mpp} V_{mpp}$) generated by a solar cell and the product of V_{oc} with J_{sc} (see Fig. 8.10),

$$FF = \frac{J_{mpp} V_{mpp}}{J_{sc} V_{oc}}.$$ (9.2)

The subscript "mpp" in Eq. (9.2) denotes the *maximum power point* (MPP) of the solar cell, *i.e.* the point on the *J-V* characteristic of the solar cell, at which the solar cell has the maximal power output. To optimise the operation of PV systems, it is very important, to operate the solar cells (or PV modules) at the MPP. This is ensured with *maximum power point tracking* (MPPT), which is discussed in great detail in Section 19.1.
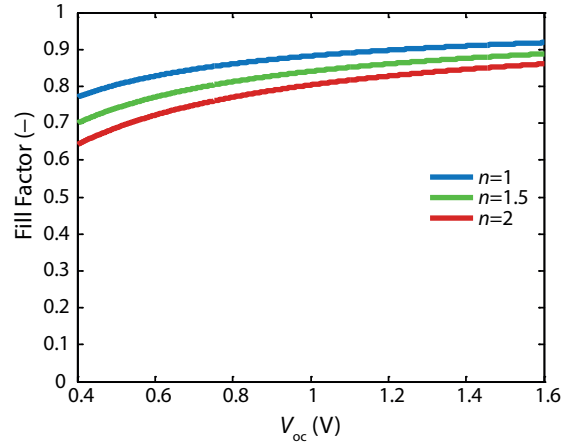
Figure 9.1: The *FF* as a function of V_{oc} for a solar cell with ideal diode behaviour.

Assuming that the solar cell behaves as an ideal diode, the fill factor can be expressed as a function of open-circuit voltage V_{oc}[35],

$$FF = \frac{v_{oc} - \ln{(v_{oc} + 0.72)}}{v_{oc} + 1}, \quad (9.3)$$

where

$$v_{oc} = V_{oc}\frac{q}{k_B T} \quad (9.4)$$

is a normalised voltage. Eq. (9.3) is a good approximation of the ideal value of *FF* for $v_{oc} > 10$. The *FF* as a function of V_{oc} is illustrated in Fig. 9.1. This figure shows that *FF* does not change drastically with a change in V_{oc}. For a solar cell with a particular absorber, large variations in V_{oc} are not common. For example, at standard illumination conditions, the difference between the maximum open-circuit voltage measured for a silicon laboratory device and a typical commercial solar cell is about 120 mV, giving a maximal *FF* of 0.85 and 0.83, respectively. However, the variation in maximum *FF* can be significant for solar cells made from different materials. For example, a GaAs solar cell may have a *FF* approaching 0.89.

However, in practical solar cells the dark diode current Eq. (8.23) does not obey the Boltzmann approximation. The non-ideal diode is approximated by introducing an *ideality factor n*, into the Boltzmann factor,

$$\exp{\frac{qV_a}{nk_B T}}.$$

Fig. 9.1 also demonstrates the importance of the diode ideality factor when introduced into the normalised voltage in Eq. (9.3). The ideality factor is a measure of the junction quality and the type of recombination in a solar cell. For the ideal junction where the recombination is represented by the recombination of the minority carriers in the quasi-neutral regions the *n* is equal to 1. However, when other recombination mechanisms occur, the *n* can have a value of 2. A high *n* value not only lowers the *FF*, but since it signals a

high recombination, it leads to a low V_{oc}. Eq. 9.3) describes a maximum achievable *FF*. In practice the *FF* is often lower due to the presence of parasitic resistive losses.

9.1.5 Conversion efficiency

The *conversion efficiency* is calculated as the ratio between the maximal generated power and the incident power. As mentioned above, solar cells are measured under the STC, where the incident light is described by the AM1.5 spectrum and has an irradiance of $I_{in} = 1000\ \text{W/m}^2$,

$$\eta = \frac{P_{max}}{I_{in}} = \frac{J_{mpp}\ V_{mpp}}{I_{in}} = \frac{J_{sc}\ V_{oc}\ FF}{I_{in}}. \tag{9.5}$$

Typical external parameters of a crystalline silicon solar cell as shown are; $J_{sc} \approx 35$ mA/cm^2, V_{oc} up to 0.65 V and *FF* in the range 0.75 to 0.80. The conversion efficiency lies in the range of 17 to 18%.

Example

A crystalline silicon solar cell generates a photo-current density of $J_{ph} = 35\ \text{mA/cm}^2$. The wafer is doped with 10^{17} acceptor atoms per cubic centimetre and the emitter layer is formed with a uniform concentration of 10^{19} donors per cubic centimetre. The minority-carrier diffusion length in the p-type region and n-type region is $500 \cdot 10^{-6}$ m and $10 \cdot 10^{-6}$ m , respectively. Further, the intrinsic carrier concentration in silicon at 300 K is $1.5 \cdot 10^{10}$ cm^{-3}, the mobility of electrons in the p-type region is $\mu_n = 1000\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$ and holes in the n-type region is $\mu_p = 100\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$. Assume that the solar cell behaves as an ideal diode. Calculate the built-in voltage, the open-circuit voltage and the conversion efficiency of the cell.

$$J_{ph} = 350\ \text{Am}^{-2}.$$
$$N_A = 10^{17}\ \text{cm}^{-3} = 10^{23}\ \text{m}^{-3}.$$
$$N_D = 10^{19}\ \text{cm}^{-3} = 10^{25}\ \text{m}^{-3}.$$
$$L_N = 500 \cdot 10^{-6}\ \text{m}.$$
$$L_P = 10 \cdot 10^{-6}\ \text{m}.$$
$$D_N = (k_B T/q)\mu_n = 0.0258\ \text{V} \cdot 1000 \cdot 10^{-4}\ \text{cm}^2\text{V}^{-1}\text{s}^{-1} = 2.58 \cdot 10^{-3}\ \text{m}^2\text{s}^{-1}.$$
$$D_P = (k_B T/q)\mu_p = 0.0258\ \text{V} \cdot 100 \cdot 10^{-4}\ \text{cm}^2\text{V}^{-1}\text{s}^{-1} = 2.58 \cdot 10^{-4}\ \text{m}^2\text{s}^{-1}.$$

Using Eq. (8.16) we calculate the built-in voltage of the cell,

$$\psi_0 = \frac{k_B T}{q}\ \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.0258\ \text{V} \cdot \ln\left[\frac{10^{23}10^{25}}{(1.5 \cdot 10^{16})^2}\right] = 0.93\ \text{V}.$$

cell performance. With the help of filters and monochromators a very narrow wavelength band of photon energies can be selected that then can be incident on the solar cell.

As already seen in Eq. (9.6), $\text{EQE}(\lambda)$ is proportional to the the current divided by the photon flow. While the current can be easily determined using an Ampere meter, the photon flow must be determined indirectly. This is done by performing a measurement with a calibrated photodetector (or solar cell), of which the EQE is known. Via this measurement we find

$$\Psi_{\text{ph},\lambda} = \frac{I_{\text{ph}}^{\text{ref}}(\lambda)}{q\,\text{EQE}^{\text{ref}}(\lambda)}, \tag{9.7}$$

By combining Eqs. (9.6) and (9.7) we therefore obtain

$$\text{EQE}(\lambda) = \text{EQE}^{\text{ref}}(\lambda)\frac{I_{\text{ph}}(\lambda)}{I_{\text{ph}}^{\text{ref}}(\lambda)}. \tag{9.8}$$

Hence, the EQE can be determined by performing two current measurements. Of course it is very important that the light source is sufficiently stable during the whole measurement as we assume that the photon flow in the reference measurement and the actual measurement is unchanged.

If we perform the EQE measurement under short circuit conditions, the measurement can be used to determine the *short circuit current density* J_{sc}. Determining J_{sc} via the EQE has the advantage that it is independent of the spectral shape of the used light source, in contrast to determining the J_{sc} via an *J-V* measurement. Secondly, on lab scale the real contact area of solar cells is not accurately determined during *J-V* measurements. When using shading masks, the EQE measurement is independent of the contact area. Hence, for accurately measuring the short circuit current density, it is not sufficient to rely on *J-V* measurements only, but a spectral response setup have to be used.

For determining J_{sc} we combine the photon flow at a certain wavelength with the EQE at this wavelength, leading to the flow of electrons leaving the solar cell at this wavelength. J_{sc} then is obtained by integrating across all the relevant wavelength,

$$J_{\text{sc}} = -q \int_{\lambda_1}^{\lambda_2} \text{EQE}(\lambda)\Phi_{\text{ph},\lambda}^{\text{AM1.5}}\,\mathrm{d}\lambda, \tag{9.9}$$

with the spectral photon flux $\Phi_{\text{ph},\lambda}$. For crystalline silicon, the important range would be from 300 to 1200 nm.

9.3 The equivalent circuit

The *J-V* characteristic of an illuminated solar cell that behaves as the ideal diode is given by Eq. (8.33),

$$J(V) = J_{\text{rec}}(V) - J_{\text{gen}}(V) - J_{\text{ph}}$$
$$= J_0\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right] - J_{\text{ph}}.$$

This behaviour can be described by a simple equivalent circuit, illustrated in Fig. 9.3 (a), in which a diode and a current source are connected in parallel. The diode is formed by a
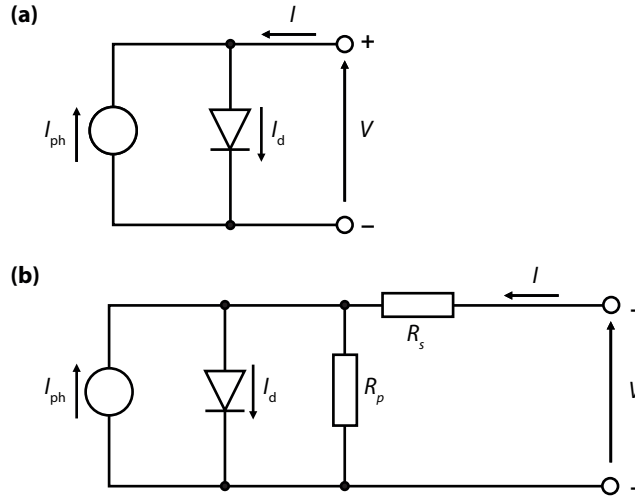
Figure 9.3: The equivalent circuit of (a) an ideal solar cell and (b) a solar cell with series resistance R_s and shunt resistance R_p.

p-n junction. The first term in Eq. (8.33) describes the dark diode current density while the second term describes the photo-generated current density. In practice the *FF* is influenced by a series resistance R_s, and a shunt resistance R_p. The influence of these parameters on the *J-V* characteristic of the solar cell can be studied using the equivalent circuit presented in Fig. 9.3 (b). The *J-V* characteristic of the one-diode equivalent circuit with the series resistance and the shunt resistance is given by

$$J = J_0 \left\{ \exp\left[\frac{q\,(V - AJR_s)}{k_B T} \right] - 1 \right\} + \frac{V - AJR_s}{R_p} - J_{ph}, \tag{9.10}$$

where A is the area of the solar cell. The effect of R_s and R_p on the *J-V* characteristic is illustrated in Fig. 9.4.

In real solar cells the *FF* is influenced by additional recombination occurring in the *p-n* junction. This non-ideal diode is often represented in the equivalent circuit by two diodes, an ideal one with an ideality factor equal to unity and a non-ideal diode with an ideality factor larger than one. The equivalent circuit of a real solar cell is presented in Fig. 9.5. The *J-V* characteristic of the two-diode equivalent circuit is given by

$$\begin{aligned} J =&\, J_{01} \left\{ \exp\left[\frac{q\,(V - AJR_s)}{n_1 k_B T} \right] - 1 \right\} \\ &+ J_{02} \left\{ \exp\left[\frac{q\,(V - AJR_s)}{n_2 k_B T} \right] - 1 \right\} \\ &+ \frac{V - AJR_s}{R_p} - J_{ph}, \end{aligned} \tag{9.11}$$

where J_{01} and J_{02} are the saturation current densities of the two diodes, respectively. n_1 and n_2 are the ideality factors of the two diodes.
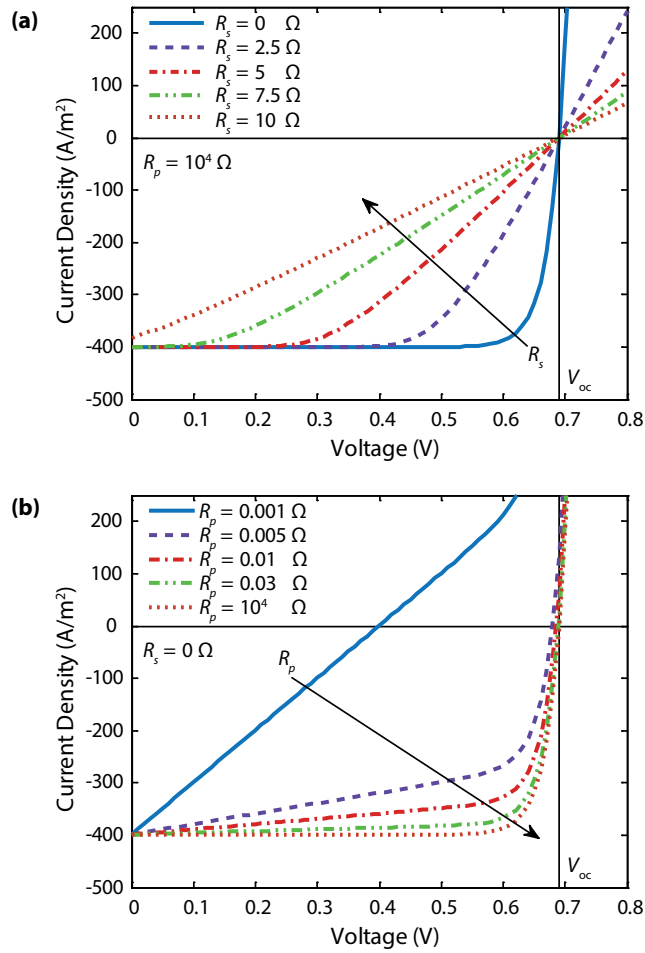
(a)

(b)

Figure 9.4: Effect of the (a) series resistance and (b) parallel resistance on the *J-V* characteristic of a solar cell.
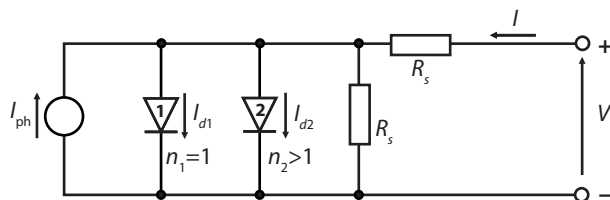

Figure 9.5: The equivalent circuit of a solar cell based on the two-diode model.