Delft Applied Mechanics Course:
Statics

AE1-914-I

18 August 2004, 9:00–12:00

ANSWERFORM

Studentnumber:

Name:

The information below is NOT to be provided by the student

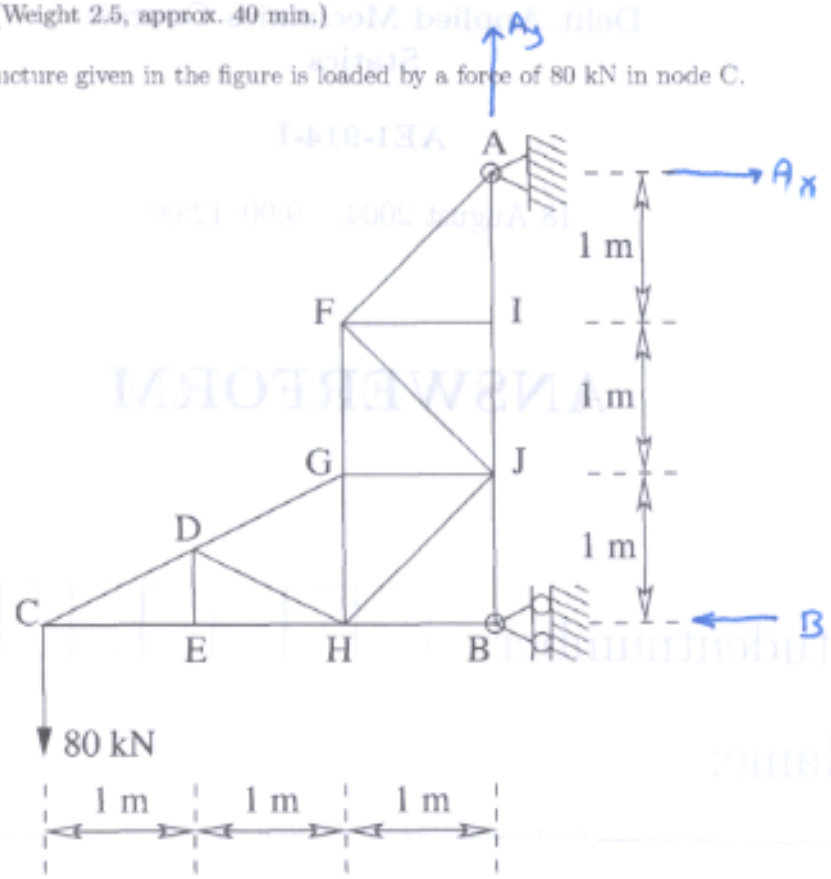
Grading:

Tentamen AE1-914-I Studentnummer:

18 Augustus 2004 Naam:

Problem 1 (Weight 2.5, approx. 40 min.)

The truss structure given in the figure is loaded by a force of 80 kN in node C.



Question a

Calculate the reaction forces in the supports A and B.

Answer

ΣM_A : $80 \cdot 3 - B \cdot 3 = 0 \Rightarrow B = 80$ kN.

ΣF_x : $-B + A_x = 0 \Rightarrow A_x = 80$ kN

ΣF_y : $-80 + A_y = 0 \Rightarrow A_y = 80$ kN

Question b

Somebody claims that bar IJ is a zero-force member. Is this correct? Motivate your answer.

Answer

Node A: 

$-\frac{1}{2}\sqrt{2}\,AF + 80 = 0 \Rightarrow AF = 80\sqrt{2}$

$80 - \frac{1}{2}\sqrt{2}\,AF - AI = 0 \Rightarrow AI = 0.$

Since $AI = 0$, we must have $IJ = 0$.

Question c

Indicate which bars are zero-force members.

Answer

AI, IJ, FI, BJ, DE, DH.

Question d

Calculate the forces in all bars with the correct sign for tension and compression. Collect your results in the table provided on the answerform.

Answer

C: 

$\frac{2}{\sqrt{5}}\,CD + CE = 0$

$\frac{1}{\sqrt{5}}\,CD - 80 = 0$

$CD = 80\sqrt{5}$ kN

$CE = -160$ kN.

$\Rightarrow EH = CE,\ DG = CD$

$HB = -B = -80$ kN.

$160 - 80 + \frac{1}{2}\sqrt{2}\, HJ = 0 \Rightarrow HJ = -80\sqrt{2}$ kN

$GH + \frac{1}{2}\sqrt{2}\, HJ = 0 \Rightarrow GH = 80$ kN

$AF = \sqrt{A_x^2 + A_y^2} = 80\sqrt{2}$ kN

$-FG - FJ \cdot \frac{1}{2}\sqrt{2} + 80 = 0$ $\quad 80 + \frac{1}{2}\sqrt{2}\, FJ = 0$ $\Rightarrow$ $FJ = -80\sqrt{2}$ kN, $FG = 160$ kN

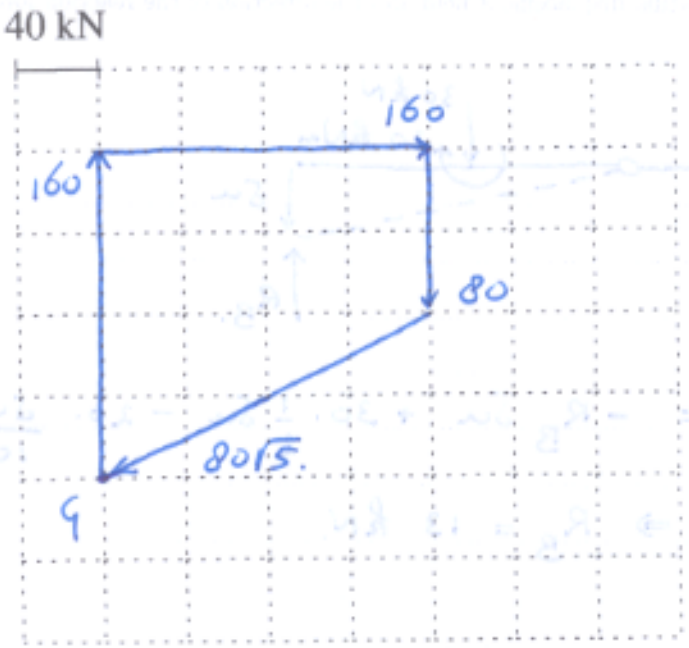
$-GJ + 80 + 80 = 0 \Rightarrow GJ = 160$ kN

bar	force (in kN)	bar	force (in kN)
CE	-160	HJ	$-80\sqrt{2}$
CD	$80\sqrt{5}$	BJ	0
DE	0	GJ	160
DG	$80\sqrt{5}$	FG	160
DH	0	FJ	$-80\sqrt{2}$
EH	-160	IJ	0
GH	80	FI	0
BH	-80	AF	$80\sqrt{2}$
AI	0		

Question e

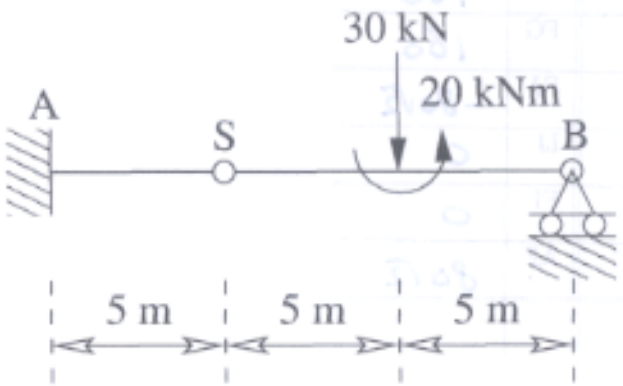
Draw the force polygon for node G.

Answer



Problem 2 (Weight 1, approx. 30 min.)

The structure depicted in the figure consists of a beam AS of length 5 m connected with a pin to a beam BS of length 10 m. The structure is loaded by a force of 30 kN and a couple of 20 kNm halfway along beam BS.



Question a

Using the principle of virtual work, calculate the reaction force in support B. Clearly indicate the virtual displacement field and the direction of the reaction force.

Answer

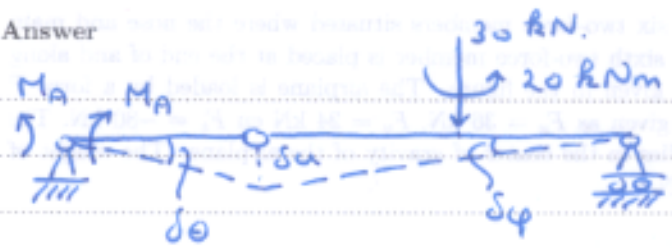
$$\delta W = -R_B\,\delta u + 30 \cdot \frac{1}{2}\,\delta u - 20 \cdot \frac{\delta u}{10} = 0.$$

$$\Rightarrow R_B = 13 \text{ kN}.$$

Question b

Using the principle of virtual work, calculate the bending moment in support A. Clearly indicate the virtual displacement field and the sign convention used.

Answer



$$\delta W = M_A \cdot \delta\theta + 30 \cdot 5\,\delta\varphi + 20 \cdot \delta\varphi = 0.$$

where $\delta\theta \cdot 5 = \delta u = \delta\varphi \cdot 10$

$\Rightarrow \delta\theta = 2\delta\varphi$

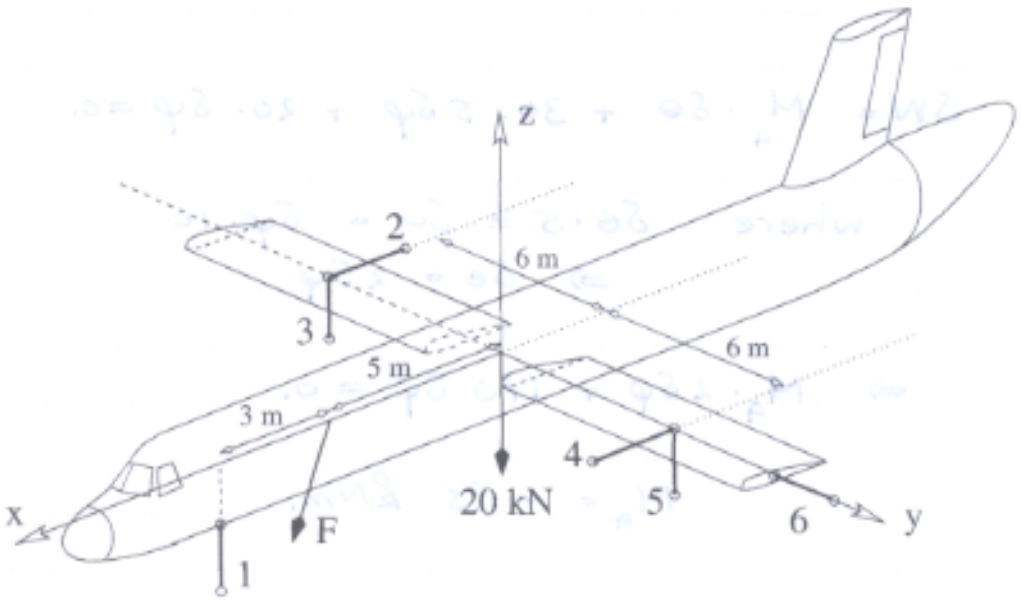
$\Rightarrow M_A \cdot 2\delta\varphi + 170\,\delta\varphi = 0.$

$M_A = -85$ kNm.

Problem 3 (Weight 2, approx. 30 min.)

In order to certify an airplane a loading situation must be simulated. The simulated loading situation is one in which the airplane must break hard and change direction simultaneously the moment the airplane enters the taxi lane after landing.

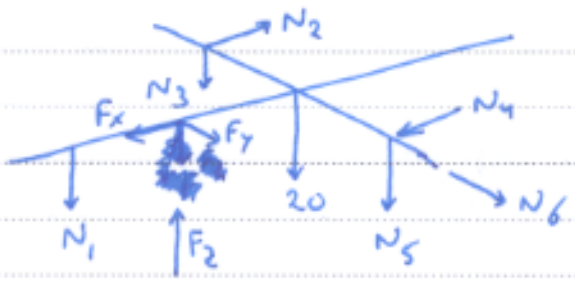
The airplane is supported by six two-force members situated where the nose and main landinggear are to come. The sixth two-force member is placed at the end of and along the wing. De dimensions are given in the figure. The airplane is loaded by a force **F** of which the components are given as $F_x = 36$ kN, $F_y = 24$ kN en $F_z = -80$ kN. The origin of the coordinate axes lies in the center of gravity of the airplane. The weight of the airplane equals $W = 20$ kN.



Question a

Calculate the forces in the six two-force members with the correct sign for tension and compression.

Answer



$\Sigma M_x = 0$: $N_3 \cdot 6 - N_5 \cdot 6 = 0 \Rightarrow N_3 = N_5$

$\Sigma M_y = 0$: $F_2 \cdot 5 - N_1 \cdot 8 = 0 \Rightarrow N_1 = F_2 \cdot \frac{5}{8}$

$\Sigma M_z = 0$: $N_2 \cdot 6 + N_4 \cdot 6 - F_y \cdot 5 = 0$

$\Sigma F_x = 0$: $F_x - N_2 + N_4 = 0.$

$\Sigma F_y = 0$: $F_y + N_6 = 0$

$\Sigma F_z = 0$: $-N_1 + F_z - 20 - N_3 - N_5 = 0.$

$\Rightarrow \quad N_1 = \frac{5}{8} \cdot (-80) = -50\ kN.$

$\Rightarrow \quad 50 + (-80) - 20 - N_3 - N_3 = 0 \Rightarrow N_3 = N_5 = -25\ kN.$

$\Rightarrow \quad N_6 = -F_y = -24\ kN.$

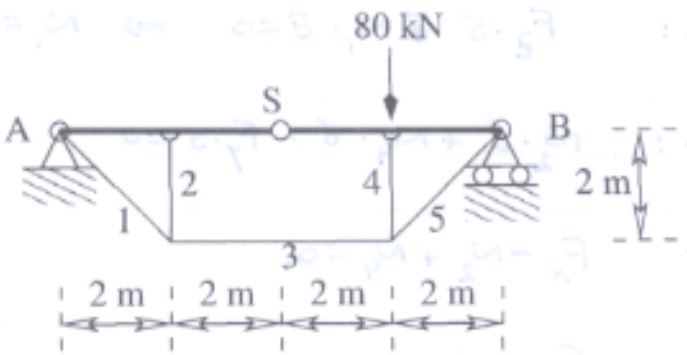
$\Rightarrow \quad N_2 = F_x + N_4 \Rightarrow 6(F_x + N_4) + 6N_4 - 5 \cdot 24 = 0$

$N_4 = -8\ kN$

$N_2 = 36 - 8 = 28\ kN.$

Problem 4 (Weight 2.5, approx. 40 min.)

The structure ASB given in the figure is loaded by a force of 80 kN halfway along beam BS. The dimensions are indicated in the figure.



Question a

Calculate the reaction forces in the supports A and B.

Answer

ΣM_B : $A \cdot 8 - 80 \cdot 2 = 0$

$A = 20$ kN.

ΣM_A : $B \cdot 8 - 80 \cdot 6 = 0$

$B = 60$ kN.

$(A + B = 80$ kN ↓.)

Question b

Calculate the connection forces in pin S and the normal force in bar 3 with the correct sign for tension or compression.

Answer

$\Sigma M_S: \quad N_3 \cdot 2 - 20 \cdot 4 = 0$

$N_3 = 40$ kN.

$\Sigma F_x: \quad S_x + N_3 = 0, \quad S_x = -40$ kN

$\Sigma F_y: \quad S_y + 20 = 0, \quad S_y = -20$ kN.

Question c

Calculate the forces in the remaining bars with the correct sign for tension or compression.

Answer

$N_1 = N_5$ (symmetry)

$N_2 = N_4$

$N_4 + \frac{1}{2}\sqrt{2}\, N_5 = 0$

$-40 + \frac{1}{2}\sqrt{2}\, N_5 = 0$

$\Rightarrow N_5 = 40\sqrt{2}$ kN, $N_4 = -40$ kN.

Question d

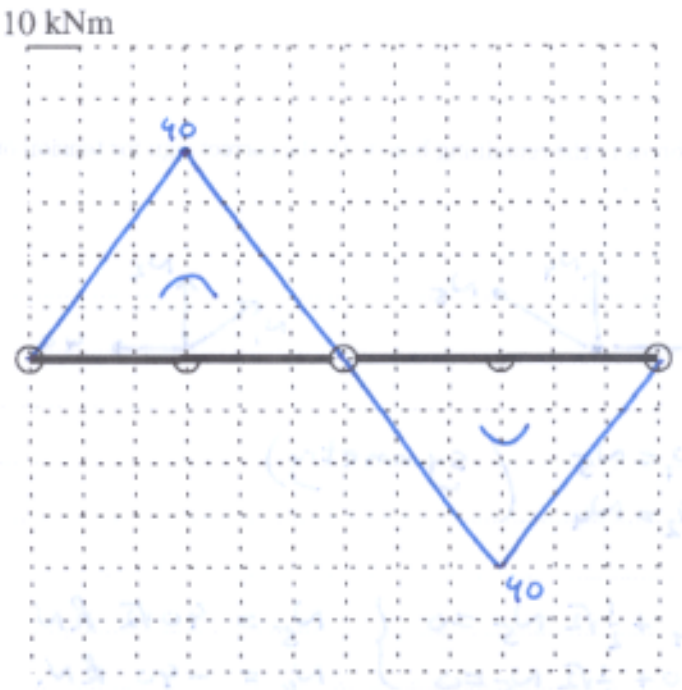
Disconnect beam ASB from its surroundings and draw all forces acting on it.

Answer

Question e

Draw the bending moment diagram (M-diagram) of beam ASB with the deformation signs.

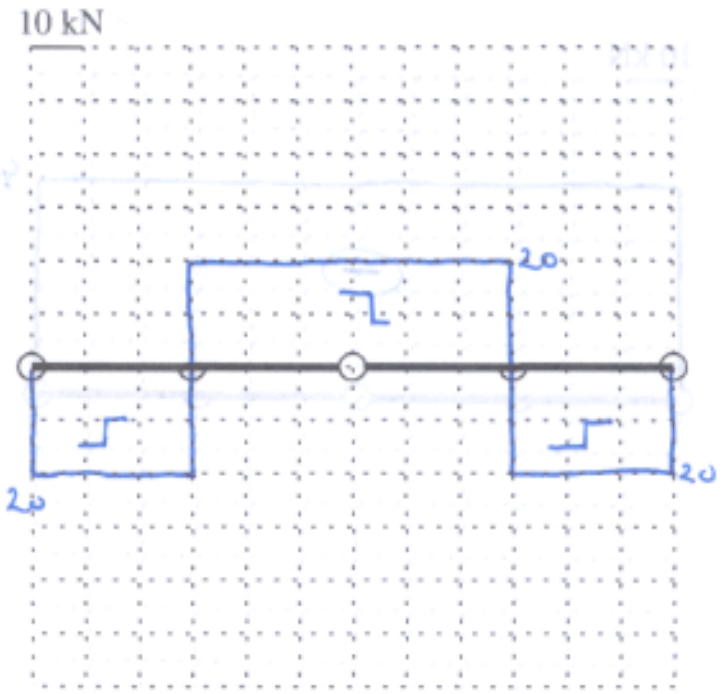
Answer



Question f

Draw the shear force diagram (V-diagram) of beam ASB with the deformation signs.

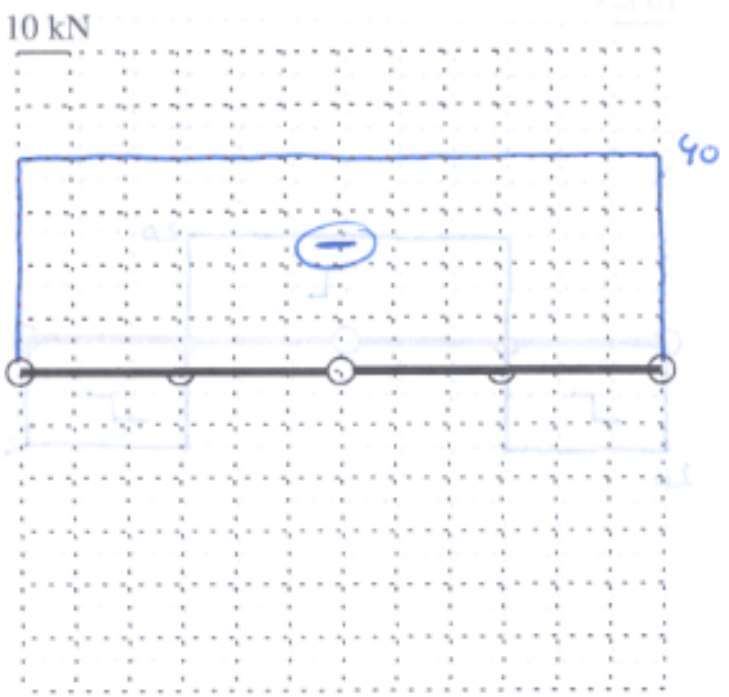
Answer



Question g

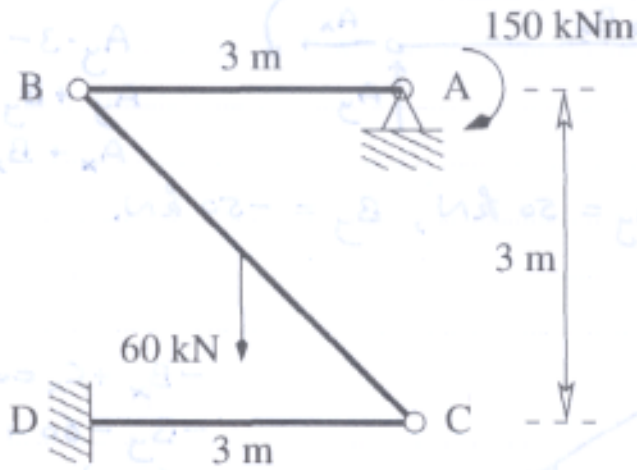
Draw the normal force diagram (N-diagram) for beam ASB with the correct sign for tension and compression.

Answer



Problem 5 (Weight 2, approx. 40 min.)

The structure given in the figure consists of two horizontal beams AB and CD of length $L = 3$ m connected with pins to a diagonal beam BC. The latter is at an angle of 45° with respect to the horizontal beams and is loaded halfway along its length by a vertical force of 60 kN. The structure is loaded in A by a moment of 150 kNm.



Question a

Is beam BC a two-force member? Motivate your answer.

Answer

A two-force member has two co-linear forces acting on it. BC has three forces acting on it; i.e. the pin force in B and C and the external force of 60 kN.

Question b

Calculate the connection forces in pins B and C and the reaction forces and moments in A and D.

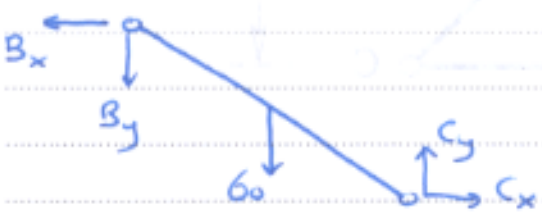
Answer

$A_y \cdot 3 - 150 = 0$

$A_y + B_y = 0$

$A_x + B_x = 0.$

$\Rightarrow A_y = 50\ kN,\ B_y = -50\ kN.$



$-B_x + C_x = 0$

$-B_y - 60 + C_y = 0.$

$60 \cdot \frac{3}{2} - C_y \cdot 3 - C_x \cdot 3 = 0.$

$\Rightarrow C_y = 10\ kN,\ C_x = 20\ kN,\ B_x = 20\ kN$

$A_x = -B_x = -20\ kN.$

$-M_D + C_y \cdot 3 = 0$

$D_y - C_y = 0$

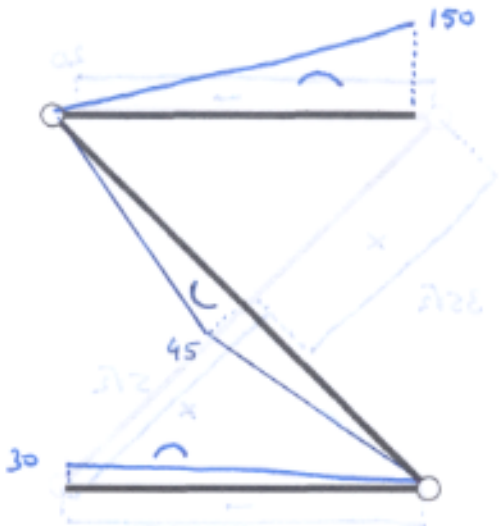
$D_x - C_x = 0.$

$\Rightarrow D_x = 20\ kN,\ D_y = 10\ kN,\ M_D = 30\ kNm.$

Question c

Draw the bending moment diagram (M-diagram) of the entire structure with the deformation signs.

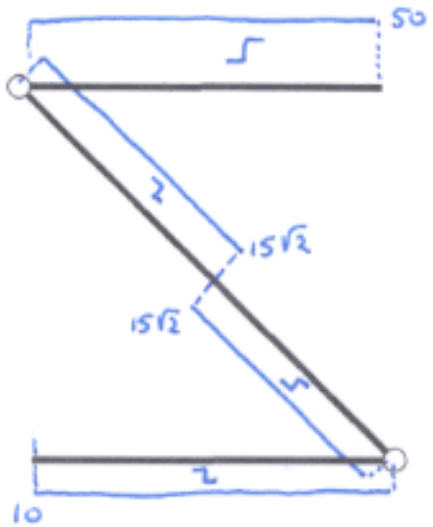
Answer



Question d

Draw the shear force diagram (V-diagram) of the entire structure with the deformation signs.

Answer



B:

$V + \frac{1}{2}\sqrt{2}\cdot 50 - \frac{1}{2}\sqrt{2}\cdot 20 = 0$

$V = -\frac{30}{2}\sqrt{2}$ kN.

$N - 50\cdot\frac{1}{2}\sqrt{2} - 20\cdot\frac{1}{2}\sqrt{2} = 0$

$N = \frac{70}{2}\sqrt{2}$ kN.

C:

$V - \frac{1}{2}\sqrt{2}\cdot 10 - \frac{1}{2}\sqrt{2}\cdot 20 = 0$

$V = \frac{30}{2}\sqrt{2}$ kN.

$N + \frac{1}{2}\sqrt{2}\cdot 10 - 20\cdot\frac{1}{2}\sqrt{2} = 0$

$N = 5\sqrt{2}$ kN

Question e

Draw the normal force diagram (N-diagram) of the entire structure with the correct signs for tension and compression.

Answer