Delft Applied Mechanics Course:
Statics

AE1-914-I

October 27, 2006, 9:00–12:00

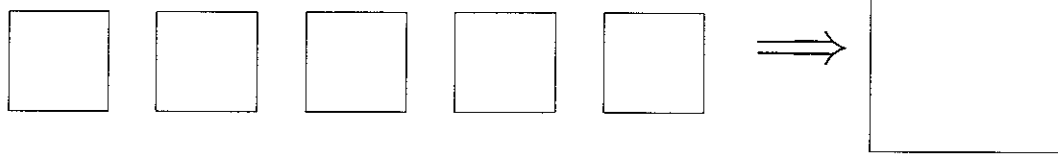
ANSWER SHEETS

Student number:

Name: Answer Model

Please do not write below this line

Grading:

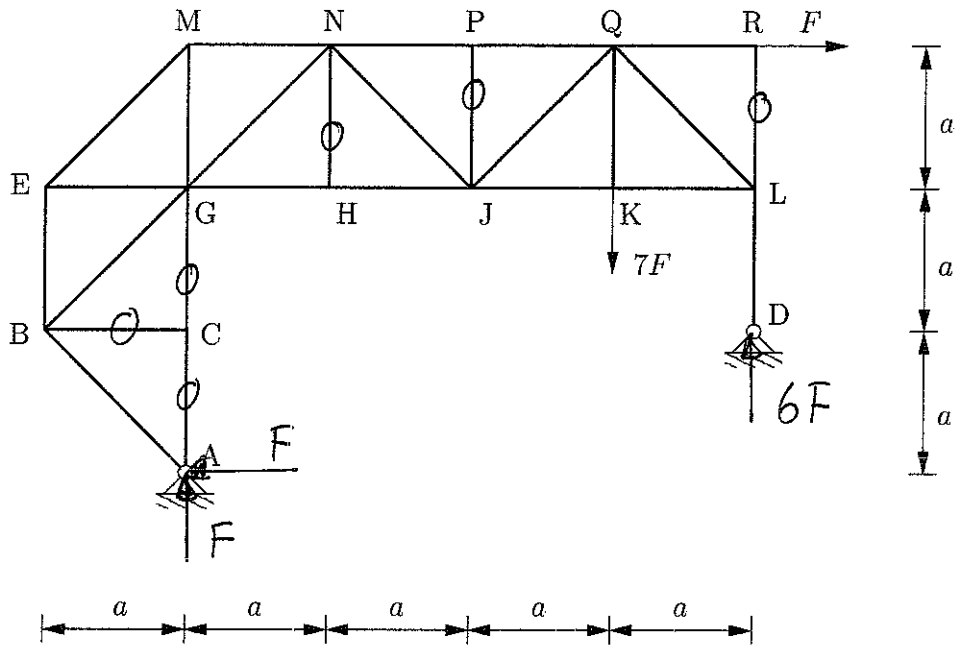


Exam AE1-914-II **Student number:**

October 27, 2006 **Name:**

Problem 1 (Weight 2, approx 35 min)

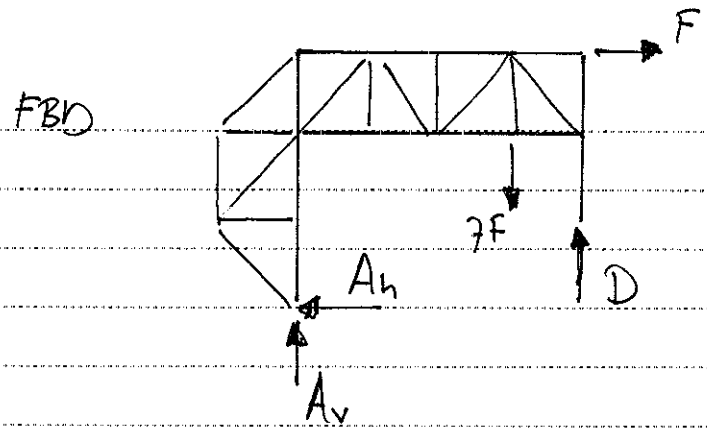
The truss structure in the figure is subjected to a horizontal force F in node R and a vertical force $7F$ in node K.



Question a

Calculate the reactions at A and D. Draw these forces as they act on the structure in reality in the figure above

Answer



$\Sigma T_A \circlearrowleft^{+}: \quad 4aD - 3a \cdot 7F - 3a \cdot F = 0$

$4aD - 24aF = 0 \quad \Rightarrow \quad D = 6F$

$\Sigma F_y \uparrow^+$: $A_v + D - 7F = 0$ $\quad A_v = F$

$\Sigma F_x \overset{+}{\rightarrow}$: $-A_h + F = 0$ $\quad A_h = F$

Check: $\Sigma T_D \circlearrowleft^+$: $-4a \cdot A_v - a \cdot A_h + a \cdot 7F - 2a \cdot F = 0$

$-4a \cdot F - a \cdot F + 7a \cdot F - 2a \cdot F = 0$ ✓

Question b

Determine the zero force members, indicate them in the figure.

Answer

BC, HN, PJ, RL, AC, CG

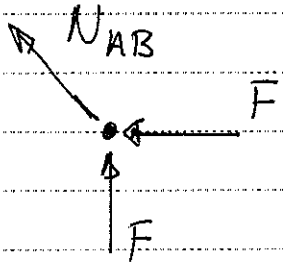
(AC and CG are zero force members because of the reaction in A).

Question c

Calculate the forces in the members AB, BC, BE, BG and the members JK, JQ, PQ and KQ. Use the correct signs for tension and compression. Collect the results in the table.

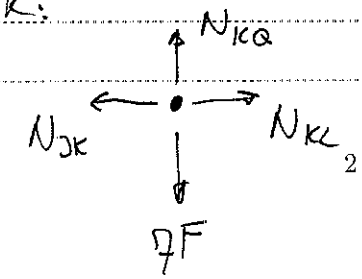
Answer

Node A:



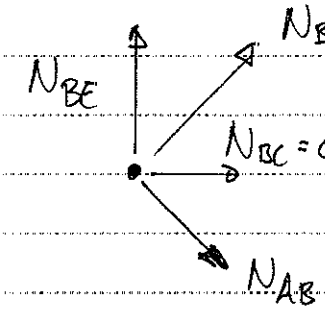
$N_{AB} = -\sqrt{2}\,F$

Node K:



$\Sigma F_y \uparrow$: $N_{KQ} = 7F$.

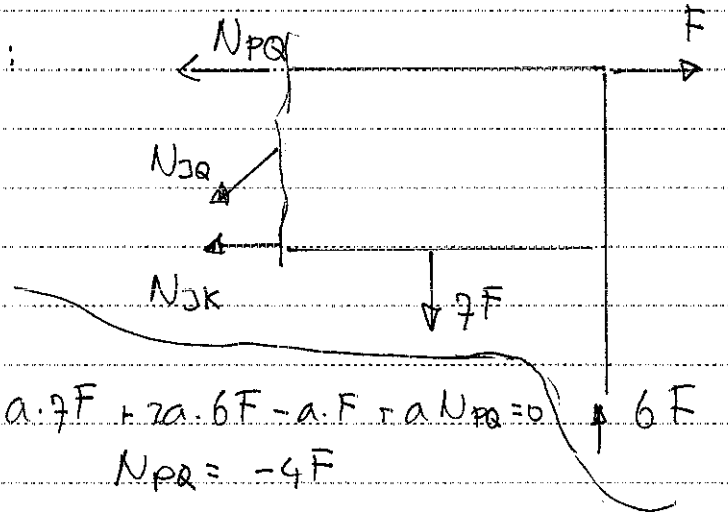
Node B:



$\Sigma F_x \rightarrow$: $\frac{1}{2}\sqrt{2}\, N_{AB} + \frac{1}{2}\sqrt{2}\, N_{BG} = 0$ $\quad N_{BG} = \sqrt{2}\, F$

$\Sigma F_y \uparrow$: $N_{BE} + \frac{1}{2}\sqrt{2}\, N_{BG} - \frac{1}{2}\sqrt{2}\, N_{AB}$ $\quad N_{BE} = -2F$

Sectum:



$\Sigma T_J \circlearrowleft^{+}$: $-a \cdot 7F + 2a \cdot 6F - a \cdot F + a N_{PQ} = 0$ $\quad N_{PQ} = -4F$

$\Sigma F_y \uparrow$: $-\frac{1}{2}\sqrt{2}\, N_{JQ} - 7F + 6F = 0$ $\quad N_{JQ} = -\sqrt{2}\, F$

$\Sigma F_x \rightarrow$: $-N_{JK} - \frac{1}{2}\sqrt{2}\, N_{JQ} - N_{PQ} + F = 0$ $\quad N_{JK} = 6F$

Check: $\Sigma T_Q \circlearrowleft^{+}$: $-a\, N_{JK} + a\, 6F = 0$ $\quad N_{JK} = 6F$ ✓

N_{AB}	N_{BC}	N_{BE}	N_{BG}
$-\sqrt{2}\, F$	0	$-2F$	$\sqrt{2}\, F$

N_{JK}	N_{JQ}	N_{PQ}	N_{KQ}
$6F$	$-\sqrt{2}\, F$	$-4F$	$7F$

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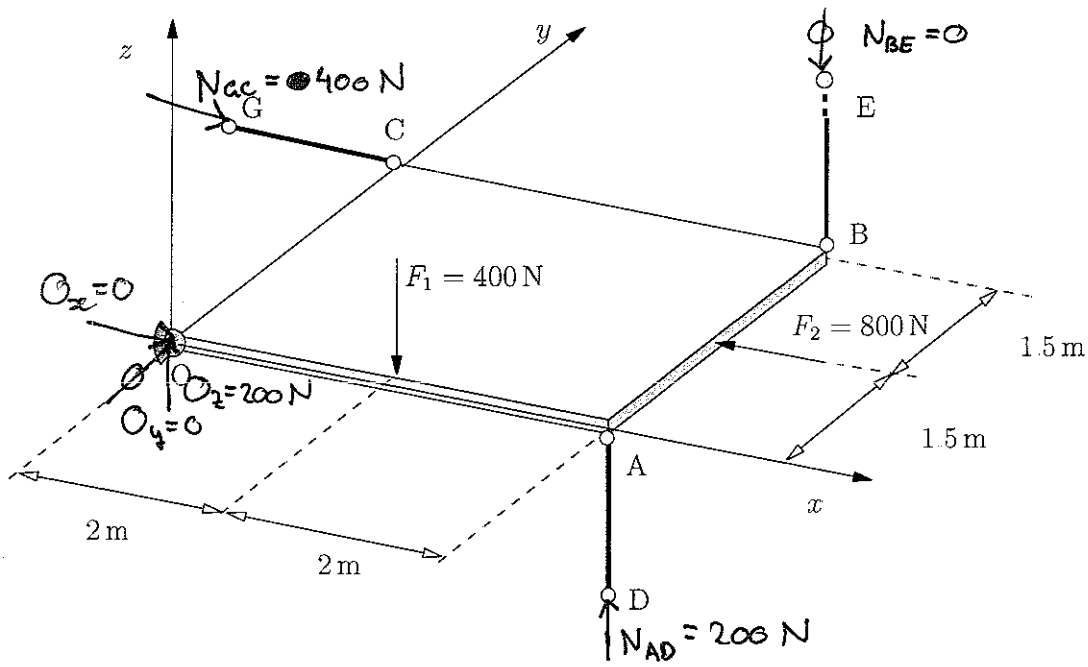
Problem 2 (Weight 2, approx. 35 min.)

The mass-less plate OABC in the figure is supported by a ball-and-socket joint O and the three links AD, BE and CG. The plate is subjected to a force $F_1 = 400\,\text{N}$ acting in the negative z-direction and a force $F_2 = 800\,\text{N}$ acting in the negative x-direction



Question

Determine all reaction forces and draw them in the figure as they act on the structure in reality.

Answer

$\sum T_{x|O} \overset{+\downarrow}{\Rightarrow} \quad N_{BE} \cdot 3 = 0 \qquad N_{BE} = 0\,\text{N}$

$\sum T_{y|O} \overset{+\nwarrow}{\Rightarrow} \quad N_{AD} \cdot 4 - F_1 \cdot 2 = 0 \qquad N_{AD} = 200\,\text{N}$

$\sum T_{z|O} \overset{+\nwarrow}{\Rightarrow} \quad -N_{GC} \cdot 3 + F_2 \cdot 1.5 = 0 \qquad N_{GC} = 400\,\text{N}$

$\sum F_y \overset{+\rightarrow}{\Rightarrow} \quad O_y = 0\,\text{N}$

$\sum F_x \overset{+\rightarrow}{\Rightarrow} \quad O_x - F_2 + N_{GC} = 0 \qquad O_x = 400\,\text{N}$

$\sum F_z \overset{+\uparrow}{\Rightarrow} \quad O_z - F_1 + N_{AD} = 0 \qquad O_z = 200\,\text{N}$

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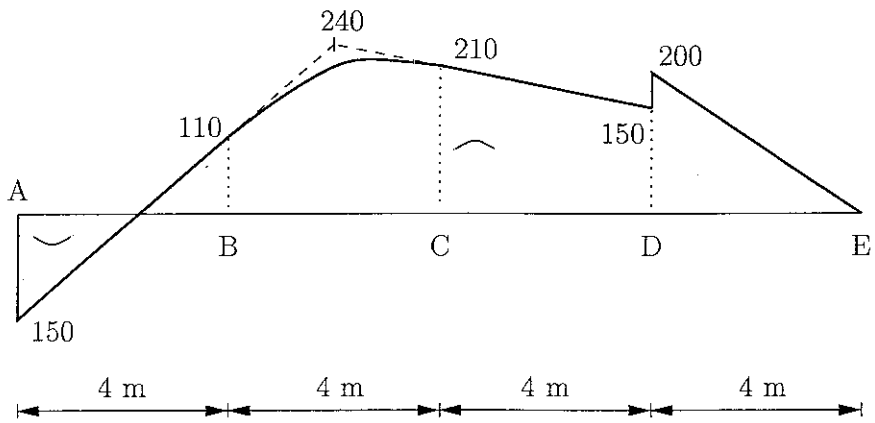
Problem 3 (Weight 2, approx 35 min.)

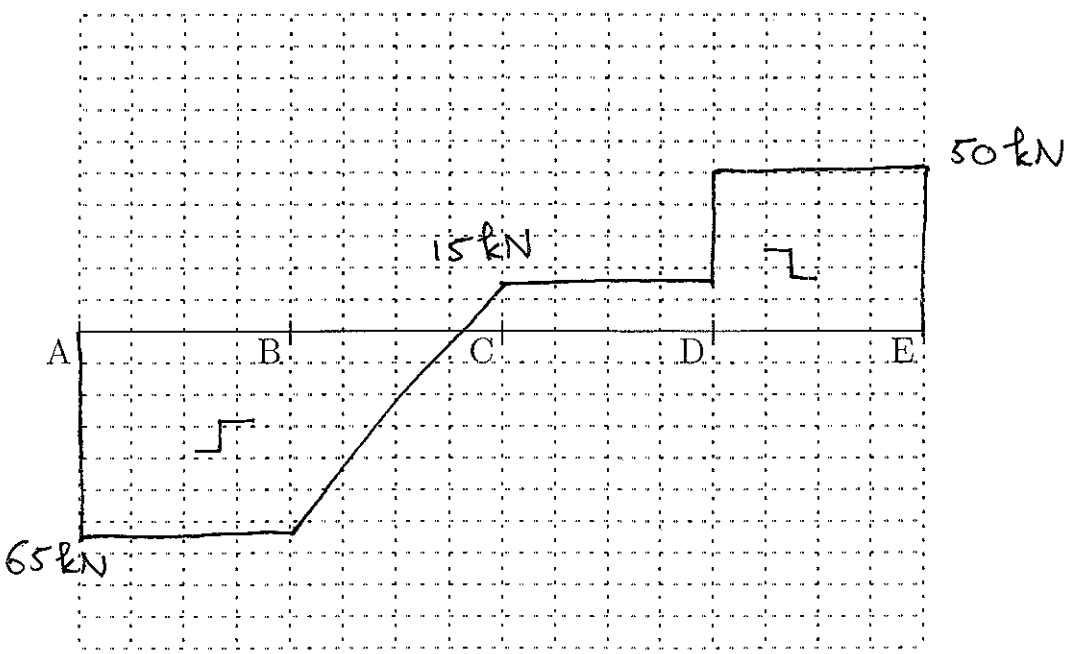
Beam ABCDE is subjected to point forces, constant distributed forces and/or couples. All force act perpendicular to the beam's axis. The corresponding M-diagram is shown in the figure below. Note that the curve between point B and C is a parabola. The dashed lines denote the tangents in B and C. All moments are in kNm.



Question a

Draw the corresponding shear force diagram (V-diagram) using the correct deformation signs. Mention all relevant values.

Answer



Slope AB = $V_{AB} = \frac{150 + 110}{4} = 65$ kN ⌐

Slope DE = $V_{DE} = \frac{200}{4} = 50$ kN ⌙

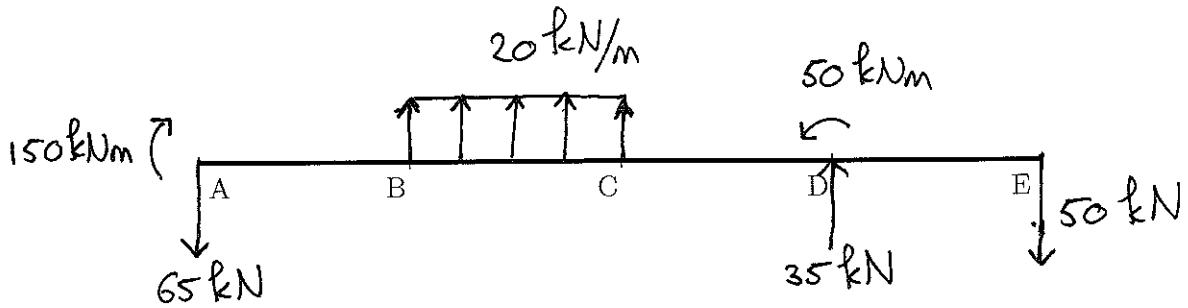
Slope CD = $V_{CD} = \frac{60}{4} = 15$ kN ⌙

BC is distr. load → −65 → +15

Question b

Draw all forces, distributed forces and couples that act on the beam in the right direction and indicate their magnitudes

Answer



Question c

Show that the beam is in equilibrium

Answer

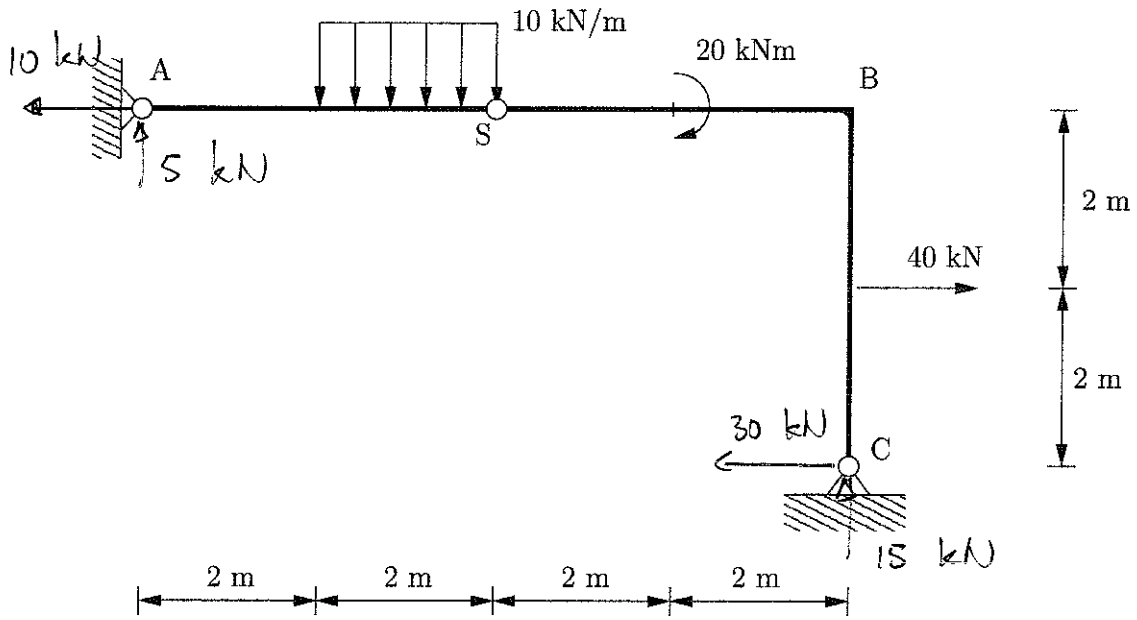
$\sum \vec{F}_x \rightarrow : 0 = 0$ QED

$\sum F_y \uparrow + : -65 + 20 \cdot 4 + 35 - 50 = 0$ QED

$\sum \vec{T}_A \circlearrowleft + : -150 + 20 \cdot 4 \cdot 6 + 35 \cdot 12 + 50 - 50\ 16 = 0$ QED.

Problem 4 (Weight 2 5, approx 45 min.)

The frame ASBC in the figure is loaded by a vertical distributed force of 10 kN/m, a couple of 20 kNm and a horizontal force of 40 kN. S is a hinge and corner B is rigid.

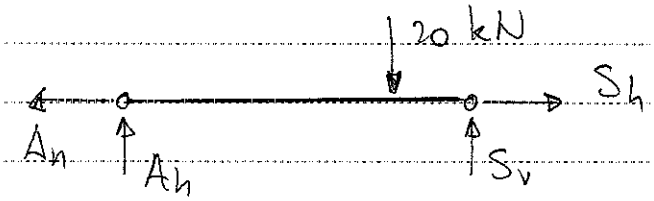


Question a

Calculate the reaction forces in A and C. Draw these forces in the figure as they act on the structure in reality.

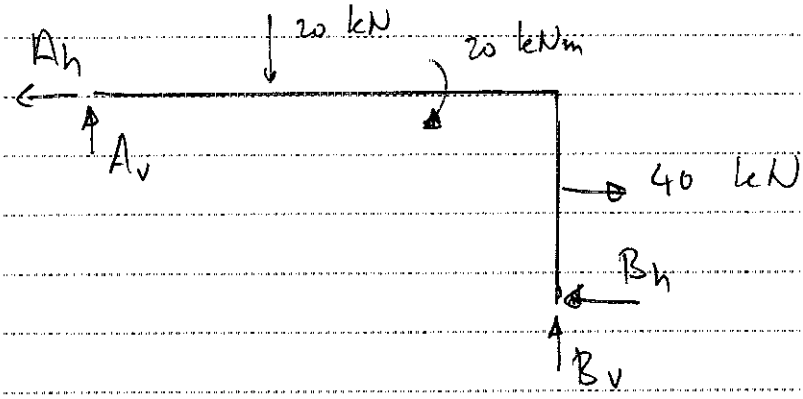
Answer

FBD. (A-S)



ΣT_S, $1 \cdot 20 - 4A_v = 0$ $\quad A_v = 5$ kN

FBD (complete structure)



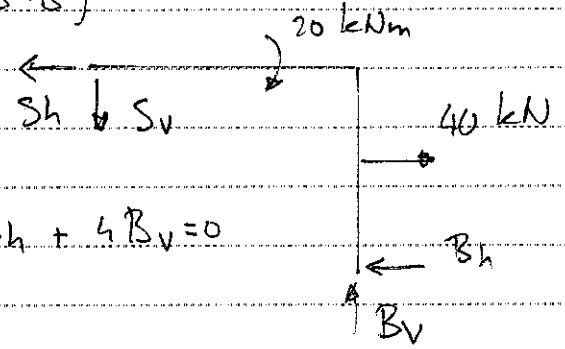
$\Sigma T_B \overset{+}{\circlearrowleft}$: $4A_h - 8A_v + 5 \cdot 20 - 20 - 2 \cdot 40 = 0$

$A_h = 10$ kN

$\Sigma F_x \overset{+}{\rightarrow}$: $-A_h + 40 - B_h = 0$ $\quad B_h = 30$ kN

$\Sigma F_y \overset{+}{\uparrow}$: $A_v - 20 + B_v = 0$ $\quad B_v = 15$ kN

Check. FBD (S-B)

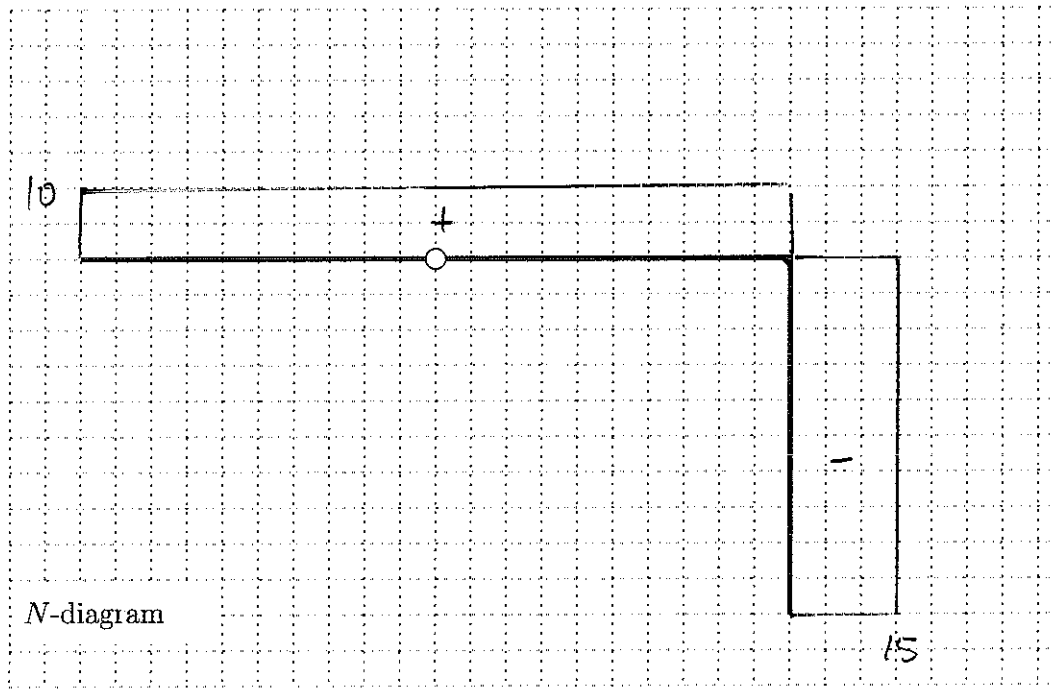


$\Sigma T_S \overset{+}{\circlearrowleft}$: $-20 + 2 \cdot 40 - 4B_h + 4B_v = 0$

Question b

Draw the normal force diagram (N-diagram) of the structure using the correct deformation signs. Mention all relevant values.

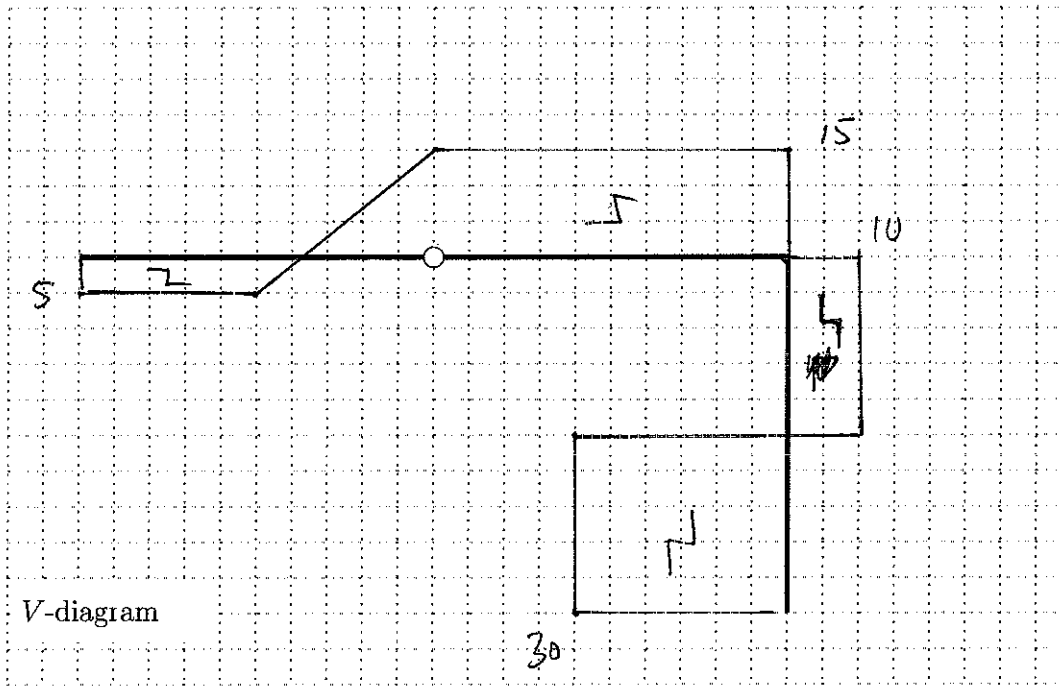
Answer



Question c

Draw the shear force diagram (V-diagram) of the structure using the correct deformation signs. Mention all relevant values.

Answer

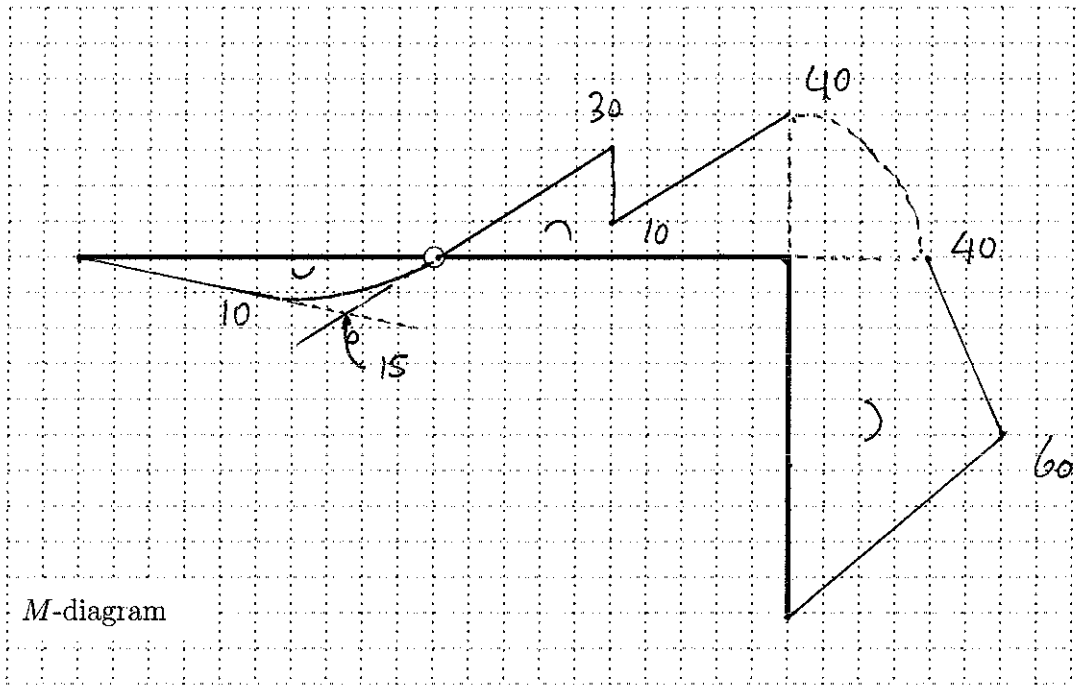


V-diagram

Question d

Draw the moment diagram (M-diagram) of the structure using the correct deformation signs Mention all relevant values and draw the tangents when necessary

Answer



M-diagram

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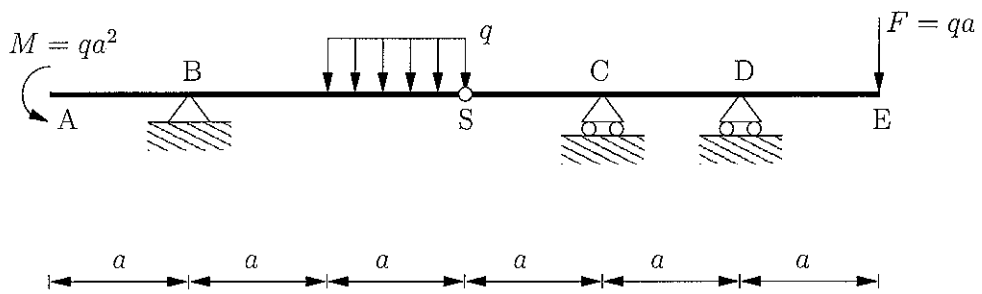
Problem 5 (Weight 1 5, approx 30 min.)

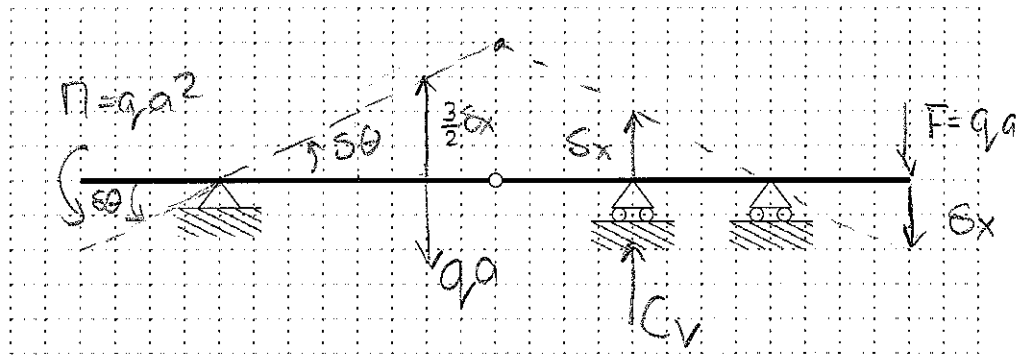
Question a

The structure ABSCDE in the figure is subjected to a couple $M = qa^2$ in point A, a distributed force q and a force $F = qa$ in E S is a hinge



Use the principles of virtual work to calculate the reacion force in C Clearly indicate what virtual displacement field and which sign conventions have been used

Answer



$\delta A = 0 \qquad \delta\theta = \frac{\delta x}{a}$

$$M\delta\theta - \tfrac{3}{2}\delta x\, qa + C_v \delta x + F\delta x = 0$$

$$qa^2\delta\theta - \tfrac{3}{2}qa^2\delta\theta + C_v a\delta\theta + qa^2\delta\theta = 0$$

$$C_v = -\tfrac{1}{2}qa$$

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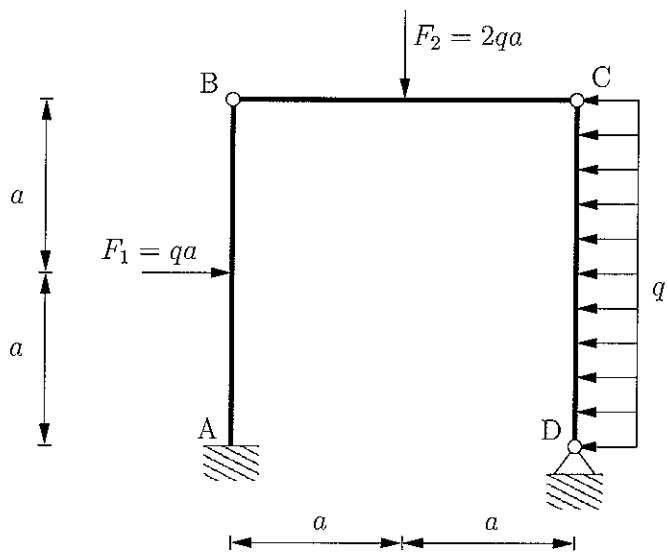
Question b

The structure in the figure is subjected to a horizontal force $F_1 = qa$, a vertical force $F_2 = 2qa$ and a distributed force q. B and C are hinges.



Use the principles of virtual work to calculate the reaction moment in A. Clearly indicate what virtual displacement field and which sign conventions have been used.

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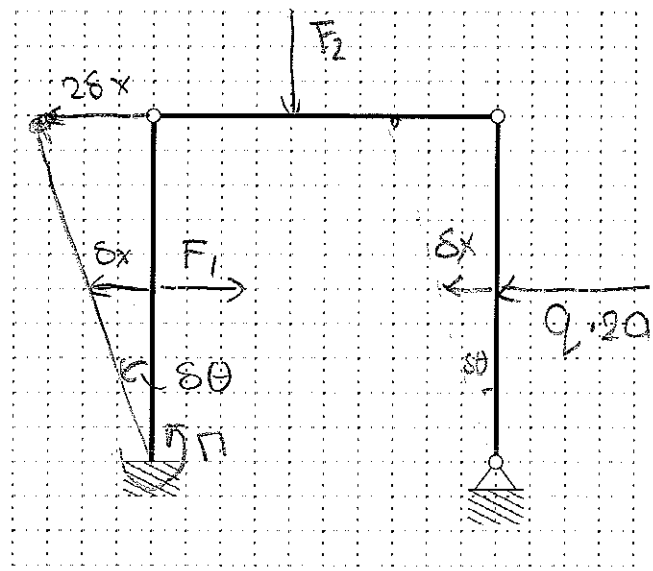
Answer



$\delta A = 0$ $\qquad \delta x = a\,\delta\theta$

$$\Pi\,\delta\theta - F_1\,\delta x + 2qa\,\delta x = 0$$
$$\Pi\,\delta\theta - qa^2\delta\theta + 2qa^2\delta\theta = 0$$
$$\Pi = -qa^2$$

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