Delft Applied Mechanics Course:
Statics

AE1-914-I

January 28, 2005, 9:00–12:00

ANSWER SHEETS

Studentnumber: | | | | | | | |

Name:

The information below is NOT to be provided by the student

Grading:

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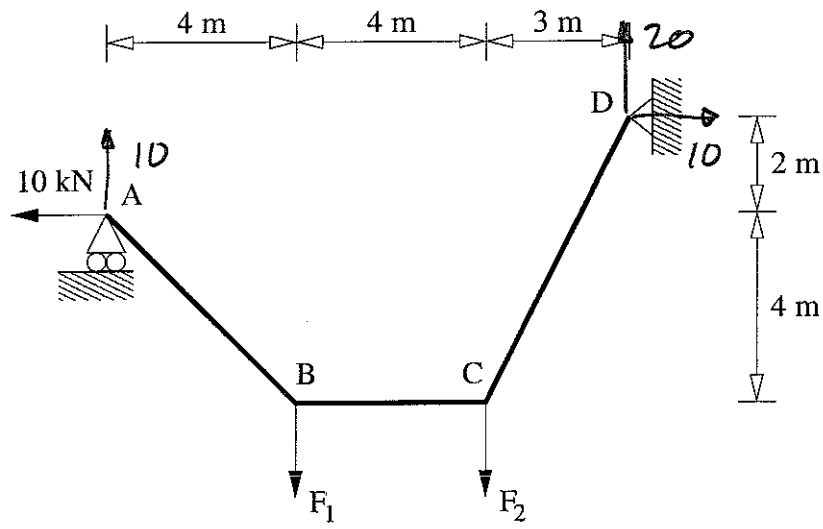
Problem 1 (Weight 1.0, approx. 20 min.)

A cable is attached to a horizontal roller in A and to a hinge in D and is loaded in B and C. There is a horizontal force of 10 kN in horizontal direction acting on the support in A as drawn in the figure

N.B.: The questions may be answered in a different order



Question a

Determine the cable forces in the sections AB, BC and CD.

Answer

Node A: 10, A_V, F_{AB}

ΣF_x: $-10 + \frac{1}{2}\sqrt{2}\, F_{AB} = 0$.

$F_{AB} = 10\sqrt{2}$ kN

Section AB: 10, A_V, F_1, F_{BC}

ΣF_x: $-10 + F_{BC} = 0$

$F_{BC} = 10$ kN

Total: A_V, D_V, D_h, 10, F_1, F_2

ΣF_x: $-10 + D_h = 0$

$D_h = 10$ kN.

Node D: D_V, $D_h = 10$, F_{CD}

ΣF_x: $-\frac{1}{5}\sqrt{5}\, F_{CD} + D_h = 0$

$F_{CD} = \sqrt{5}\, D_h = 10\sqrt{5}$ kN.

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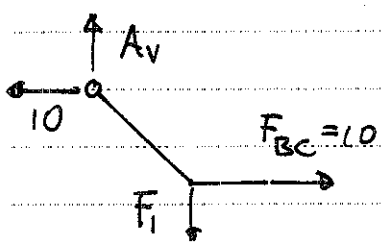
Question b

Determine the forces F_1 and F_2

Answer

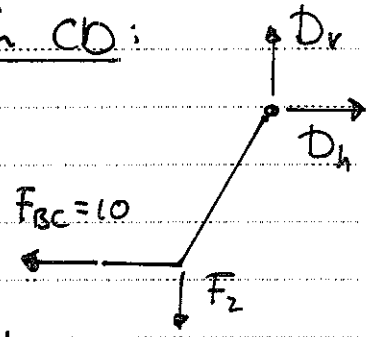
Section AB:



$\Sigma M_A \overset{+}{\circlearrowright}: \quad 4F_1 - 4F_{BC} = 0$

$F_1 = F_{BC} = 10\ kN.$

Section CD:



$\Sigma M_D \overset{+}{\circlearrowright}: \quad 6F_{BC} - 3F_2 = 0$

$F_2 = 2F_{BC} = 20\ kN$

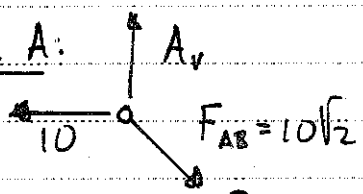
Question c

Determine the reaction forces and draw them in the figure on the previous page in the right direction.

Answer

D_h: see a.

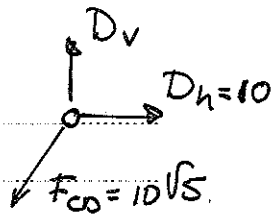
Node A:



$\Sigma F_y \overset{+}{\uparrow}: \quad A_V - \frac{1}{2}\sqrt{2}\, F_{AB} = 0$

$A_V = 10\ kN.$

Node D:



$\Sigma F_y \overset{+}{\uparrow}: \quad D_V - \frac{2}{5}\sqrt{5}\, F_{CD} = 0$

$D_V = 20\ kN.$

Please collect the results in the table below.

F_1	F_2	F_{AB}	F_{BC}	F_{CD}	A_v	D_h	D_v
10	20	$10\sqrt{2}$	10	$10\sqrt{5}$	10	10	20

all in kN

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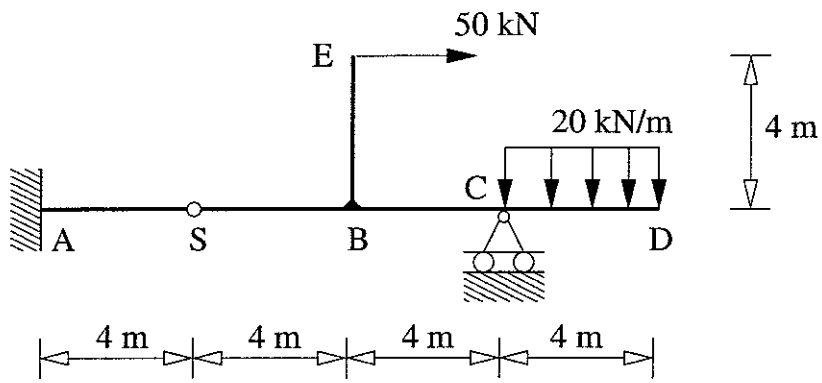
Problem 2 (Weight 2.0, approx. 35 min)

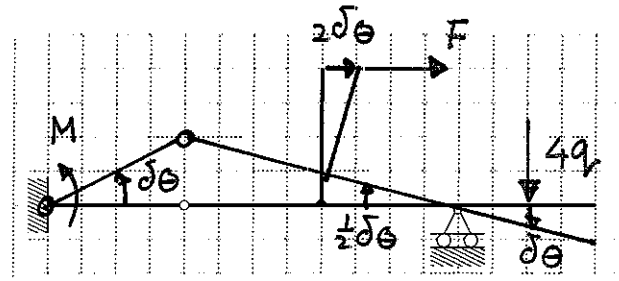
The structure in the figure consists of a part AS and a part SBCDE which are connected by a hinge in S. The structure is clamped in A and is supported by a horizontal roller in C. A horizontal force of 50 kN is applied at point E and a vertically distributed force of 20 kN/m is applied at segment CD.



Question a

Using the principle of virtual work, calculate the reaction moment in support A. Clearly indicate the virtual displacement field and sign convention used.

Answer



$$\delta W = M\delta\theta + F\,2\delta\theta + 4q\cdot\delta\theta = 0$$

$$(M + 2F + 4q)\,\delta\theta = 0$$

$$M + 2F + 4q = 0 \Rightarrow M = -180 \text{ kNm}$$

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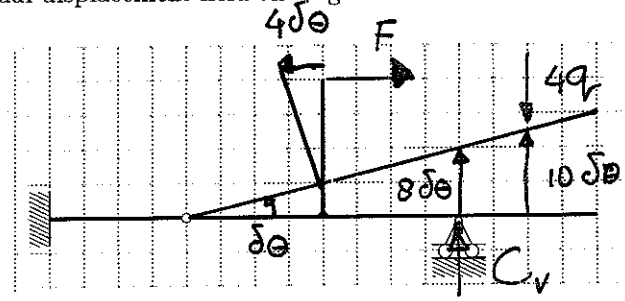
Question b

Using the principle of virtual work, calculate the reaction force in support C. Clearly indicate the virtual displacement field and sign convention used

Answer



$$\delta W = -F\,4\delta\theta + C_v\,8\delta\theta - 4q\,10\delta\theta = 0$$

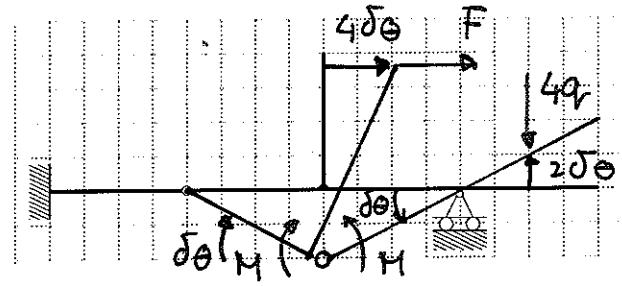
$$(-4F + 8C_v - 40q)\,\delta\theta = 0.$$

$$8C_v = 1000 \qquad C_v = 125\ kN.$$

Question c

Using the principle of virtual work, calculate the bending moment in the structure just *right* of B. Clearly indicate the virtual displacement field and sign convention used.

Answer



$$\delta W = F\,4\delta\theta + M\delta\theta + M\delta\theta - 4q\,2\delta\theta = 0$$

$$(4F + 2M - 8q)\,\delta\theta = 0$$

$$2M = 100 - 200 \qquad M = -20\ kNm.$$

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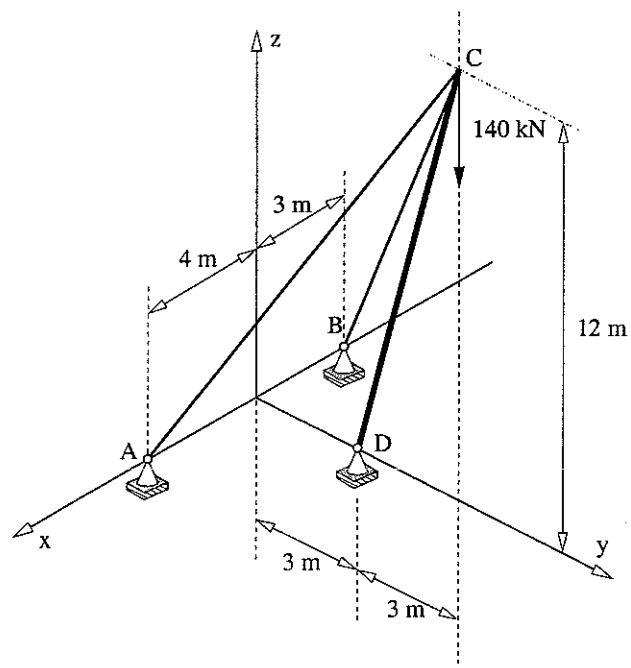
Problem 3 (Weight 2 5, approx 45 min.)

A massless pole CD in the y–z plane is secured by two cables AC and BC. The pole is supported by a hinge in D. Point C is loaded by a vertical load of 140 kN. The position of C is $(0, 6, 12)$ meter.



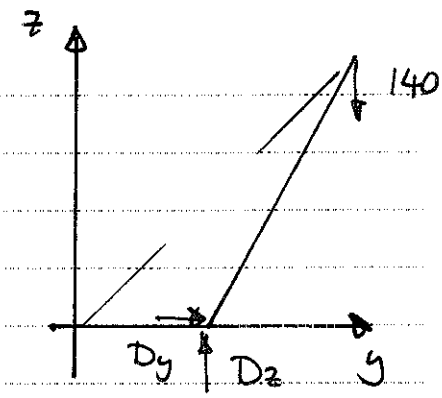
Question a

Calculate the vertical reaction force D_z in point D.

Answer

$\Sigma M_{x|A}: \quad 3D_z - 6 \cdot 140 = 0$

$D_z = 280 \text{ kN}$



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Question b

Calculate the force F_{CD} in the pole and the reaction forces D_x and D_y in D.

Answer

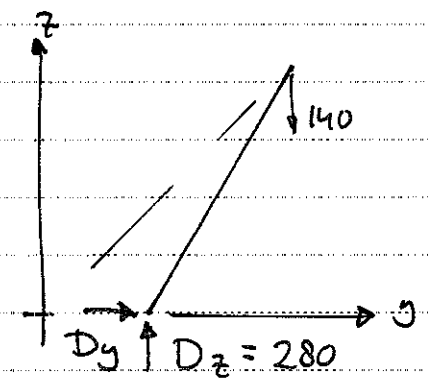
$\overset{+}{\curvearrowright}\ \Sigma M_{x|C} \quad -3\cdot 280 + 12 D_y = 0$

$D_y = 70$ kN.

$\overset{+}{\curvearrowright}\ \Sigma M_{y|C} \quad 12 D_x = 0$

$D_x = 0$ kN.

$F_{CD} = \sqrt{280^2 + 70^2} = 288.6$ kN.



Question c

Calculate the reaction forces in A and B in the $x-$, $y-$ and $z-$ direction.

Answer

$\vec{CA} = 4\underline{i} - 6\underline{j} - 12\underline{k} \quad |\vec{CA}| = 14.$

$\underline{n}_{CA} = \frac{4}{14}\underline{i} - \frac{6}{14}\underline{j} - \frac{12}{14}\underline{k}$

$\underline{F}_{CA} = \underline{n}_{CA} |F_{CA}| \quad A_x = \frac{4}{14} F_{CA} \quad A_y = -\frac{6}{14} F_{CA} \quad A_z = -\frac{12}{14} F_{CA}$

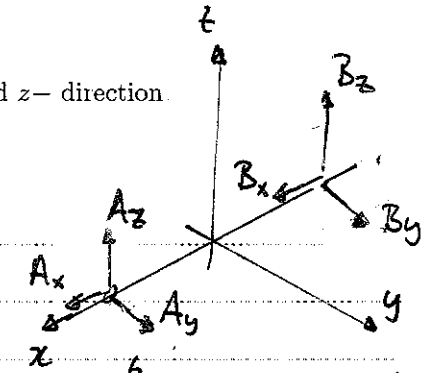
$\overset{+}{\curvearrowright}\ \Sigma M_{y|B} \quad -7 A_z - 3\cdot 280 + 3\cdot 140 = 0 \quad A_z = -60$ kN

$F_{AC} = -\frac{14}{12} A_z = 70$ kN $\quad A_x = 20$ kN $\quad A_y = -30$ kN.

$\Sigma F_x: \quad A_x + B_x = 0 \quad B_x = -20$ kN.

$\Sigma F_y: \quad A_y + B_y + 70 = 0 \quad B_y = -40$ kN

$\Sigma F_z: \quad A_z + B_z + 280 - 140 = 0 \quad B_z = -80$ kN.



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Question d

Calculate the cable-forces F_{AC} and F_{BC}.

Answer

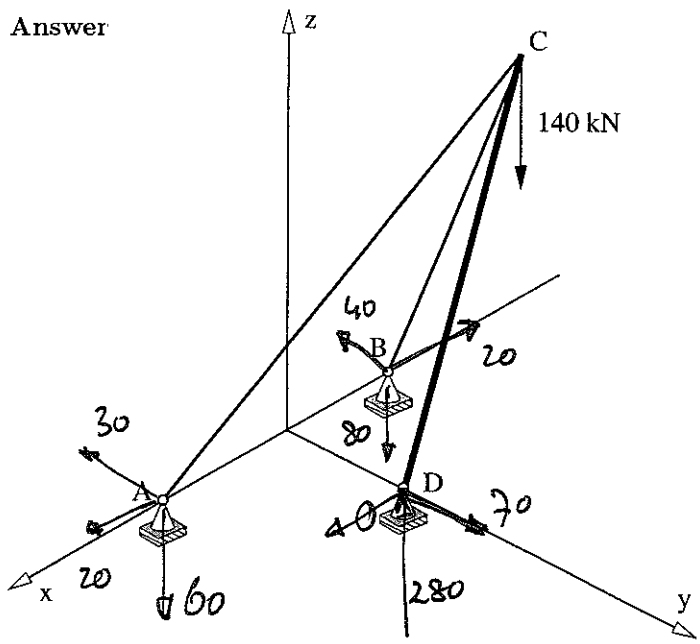
$F_{AC} = 70$ kN (see c.)

$F_{BC} = \sqrt{20^2 + 40^2 + 80^2} = \sqrt{8400} = 91.65$ kN.

Question e

Draw all components of the reactions in A, B and D in the figure below in the right direction and collect all the answers in the table

Answer



A_x	A_y	A_z
20	30	60
B_x	B_y	B_z
20	40	80
D_x	D_y	D_z
0	70	280
F_{AC}	F_{BC}	F_{CD}
70	91.65	288.6

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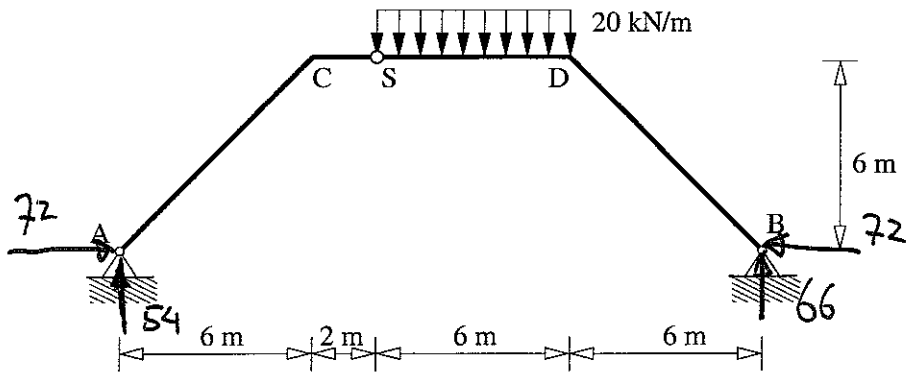
Problem 4 (Weight 2.5, approx. 45 min.)

The structure ASB as shown in the figure (note that S is a hinge) is loaded by a uniformly distributed loaded on section SD.



Question a

Calculate the reaction forces and draw them in the figure as they act on the structure in reality.

Answer

Total: $\sum M_A \overset{+}{\circlearrowleft}$: $11 \cdot 120 - 20 B_v = 0$ $\quad B_v = 66$ kN

$\sum F_y \uparrow$: $A_v + B_v - 120 = 0$ $\quad A_v = 54$ kN

Section ACS

$\sum M_S \overset{+}{\circlearrowleft}$ $\quad -8 \cdot A_v + 6 A_h = 0$ $\quad A_h = 72$ kN.

Total: $\sum F_x \overset{+}{\rightarrow}$ $\quad A_h + B_h = 0$ $\quad B_h = -72$ kN.

Check Section SDB

$\sum M_S \overset{+}{\circlearrowleft}$ $\quad 3 \cdot 120 - 12 \cdot 66 + 6 \cdot 72 = 0$ ✓

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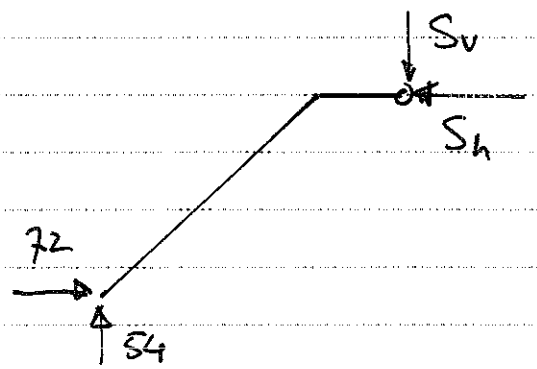
Question b

Calculate the forces in the hinge S

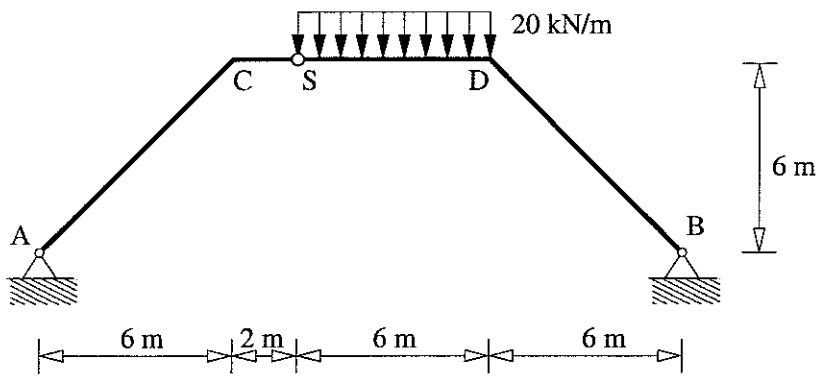
Answer

Section ACS



ΣF_x: $72 - S_h = 0$ $S_h = 72$ kN

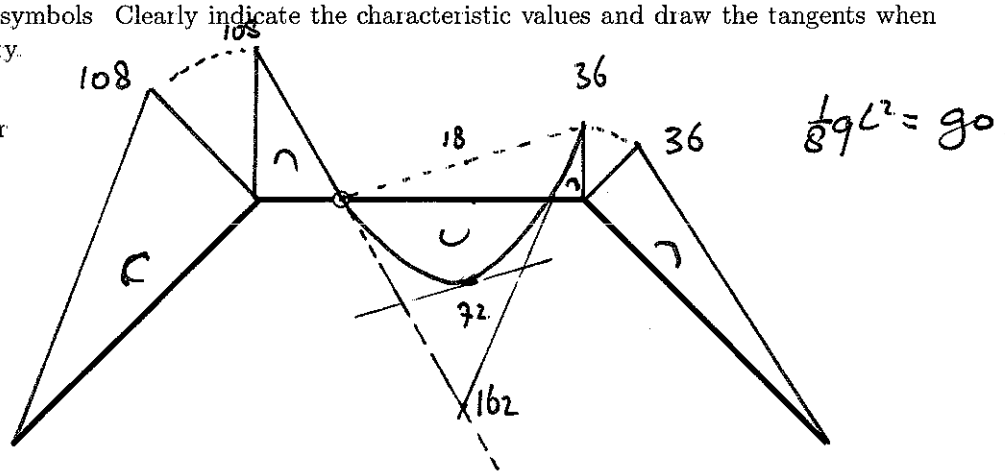
ΣF_y: $54 - S_v = 0$ $S_v = 54$ kN.



Question c

Draw the bending moment diagram (M-diagram) of the structure with the correct deformation symbols. Clearly indicate the characteristic values and draw the tangents when necessary.

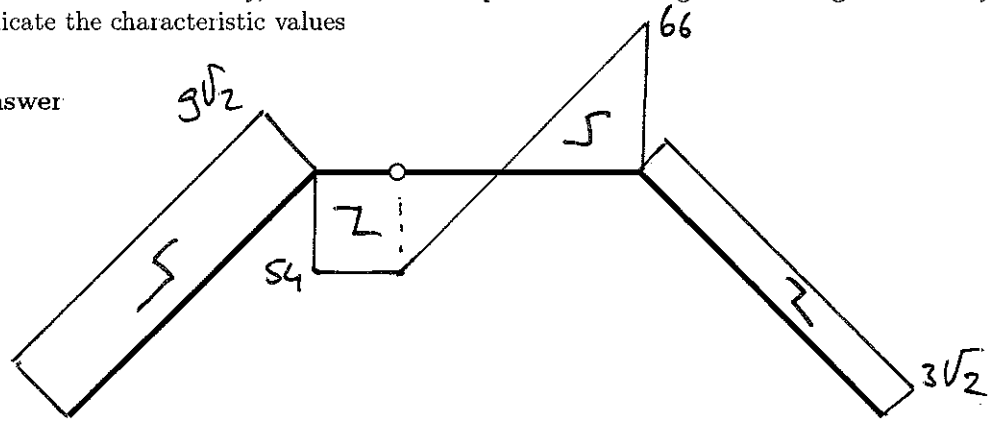
Answer



Question d

Draw the shear force diagram (V-diagram) of the structure with the correct deformation symbols. When necessary, make use of the slopes of the bending moment diagram. Clearly indicate the characteristic values

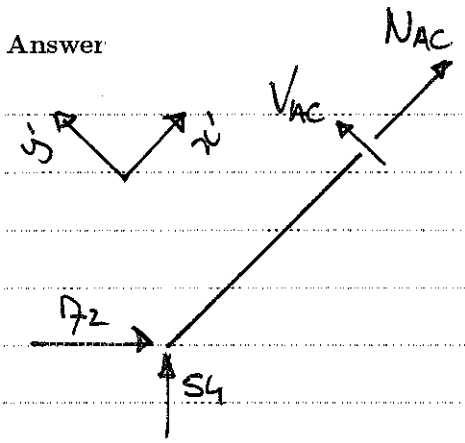
Answer



Question e

Determine the shear force V_{AC} and the normal force N_{AC} in section AC.

Answer



$\Sigma F_{y'}$ (+) $\quad V_{AC} - \frac{1}{2}\sqrt{2}\,72 + \frac{1}{2}\sqrt{2}\,54 = 0$

$V_{AC} = 9\sqrt{2}$ kN.

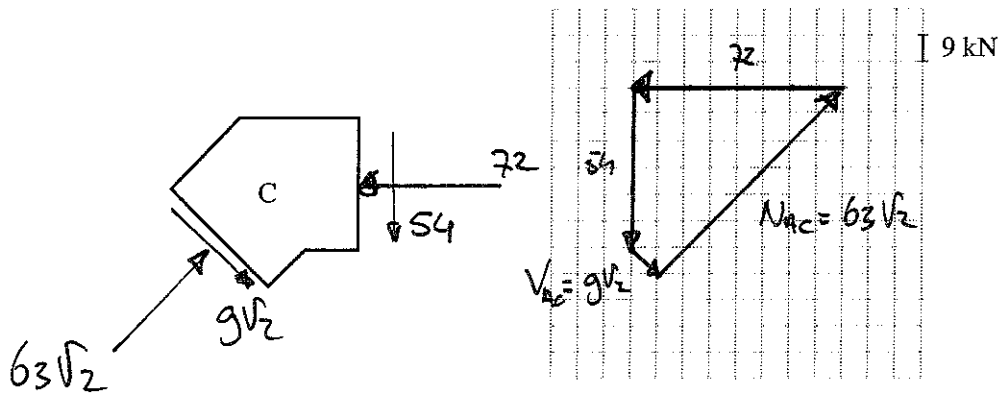
$\Sigma F_{x'}$ (+) $\quad N_{AC} + \frac{1}{2}\sqrt{2}\,72 + \frac{1}{2}\sqrt{2}\,54 = 0$

$N_{AC} = -63\sqrt{2}$ kN.

Question f

Consider force equilibrium of point C. Draw all forces in C in the blown up picture on the answer sheet and demonstrate by using a force polygon that the forces are in equilibrium

Answer



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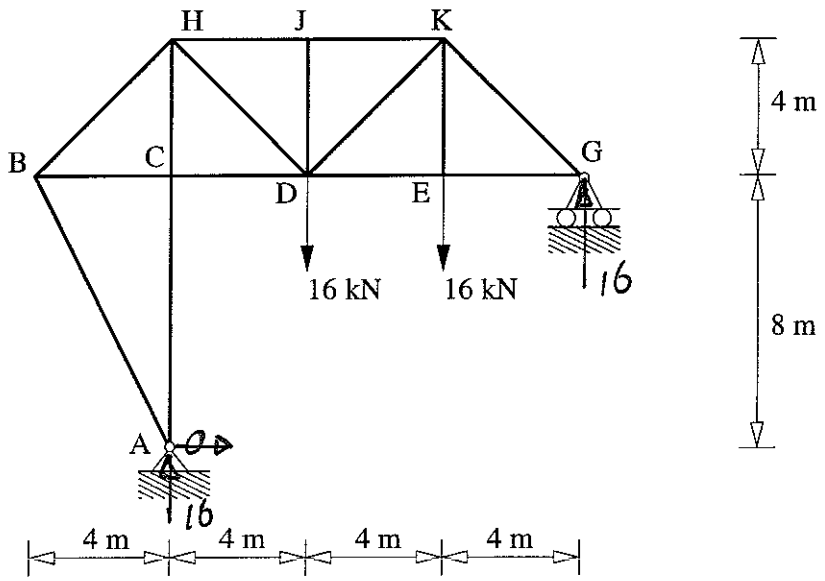
Problem 5 (Weight 2.0, approx. 35 min.)

The truss structure in the figure is loaded by two forces. The structure is supported by a hinge in A and a horizontal roller in G.



Question a

Determine the reactions in A and G and draw them in the figure on the previous page as they act on the structure in reality

Answer

ΣM_A): $12G_V - 4\cdot 16 - 8\cdot 16 = 0$ $G_V = 16$ kN

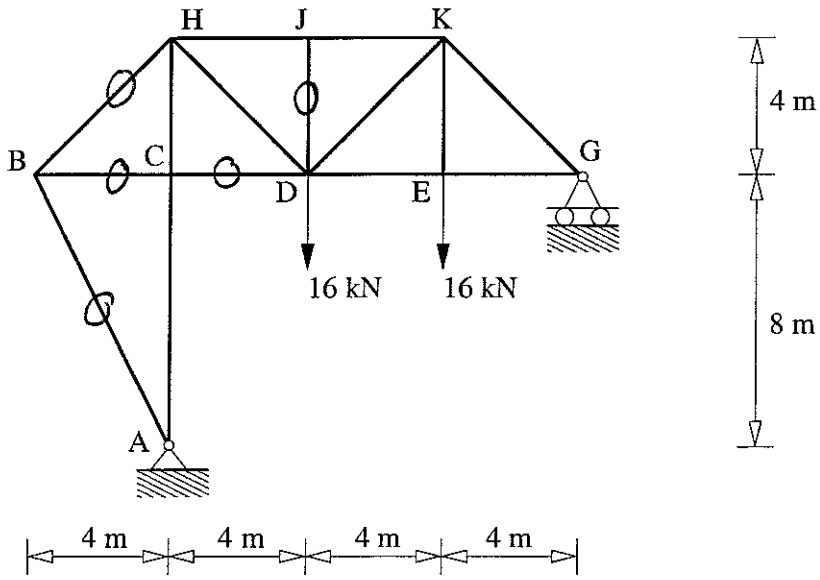
ΣF_y: ↑+ $A_V + G_V - 16 - 16 = 0$ $A_V = 16$ kN

ΣF_x: → $A_h = 0$

Question b

Indicate the zero-force-members in the figure below.

Answer

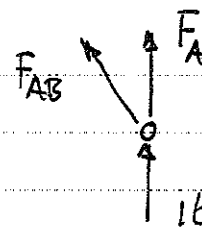


Question c

Determine the forces in the members AB, AC, JK, DK, DE, EG and GK, using the correct signs for tension or compression. Collect the results in the table.

Answer

Node A:



$\rightarrow$ ΣF_x: $-\frac{1}{5}\sqrt{5}\,F_{AB} = 0$

$F_{AB} = 0$

$+\uparrow$ ΣF_y: $16 + F_{AC} + \frac{2}{5}\sqrt{5}\,F_{AB} = 0$

$F_{AC} = -16$ kN

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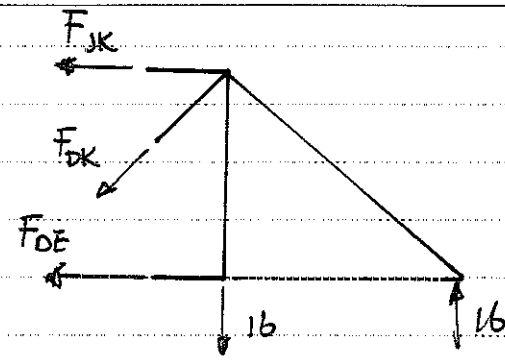


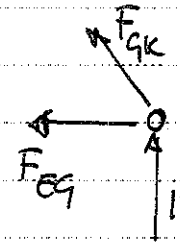
$\Sigma M_D \overset{+}{\circlearrowleft}$ $\quad 4F_{JK} - 4\cdot 16 + 8\cdot 16 = 0$

$F_{JK} = -16$ kN

$\Sigma M_K \overset{+}{\circlearrowleft}$ $\quad 4\cdot 16 - 4F_{DE} = 0 \quad F_{DE} = 16$ kN

ΣF_x $\quad -F_{JK} - \frac{1}{2}\sqrt{2}\,F_{DK} - F_{DE} = 0 \quad F_{DK} = 0$ kN.

Node G:



$\Sigma F_y \uparrow$: $\quad 16 + \frac{1}{2}\sqrt{2}\,F_{GK} = 0$

$F_{GK} = -16\sqrt{2}$ kN.

$\Sigma F_x \rightarrow$: $\quad -F_{EG} + 16 = 0 \quad F_{EG} = 16$ kN

F_{AB}	F_{AC}	F_{JK}	F_{DK}	F_{DE}	F_{EG}	F_{GK}
0	-16	-16	0	16	16	$-16\sqrt{2}$

all in kN.