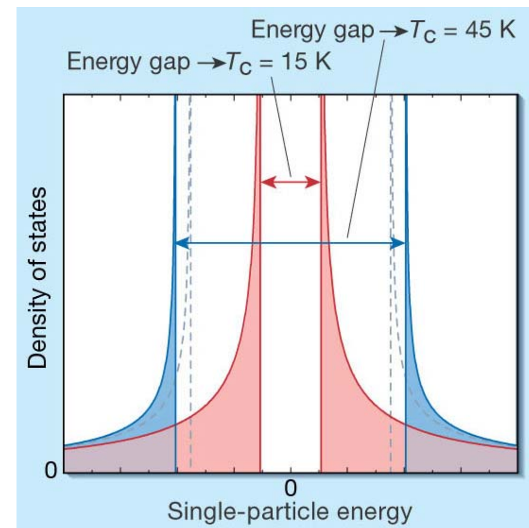
mesoscopic superconductivity

superconductivity in the bulk: Cooper pairs

• electron-phonon interaction leads to an attraction of electrons

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta}$$

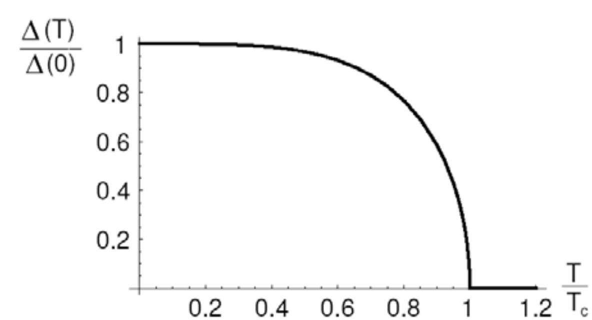
• Cooper pairs: bound states of two electrons with opposite momentum and spin (size of Cooper pair: coherence length)



• the net spin is zero and as a consequence they obey Bose-Einstein statistics: at low T all pairs condense in the lowest energy state (no Pauli exclusion !)

• the superconducting state can then be described with a single, macroscopic wave function: $\Psi = |\Psi| \exp(i\phi)$
$|\Psi|^2$: density of Cooper pairs; ϕ: phase of the condensate

• the pairing leads to an energy gap Δ in the spectrum; the density of states is $N_s(E) = N_N(E)\, E/(E^2-D^2)^{1/2}$

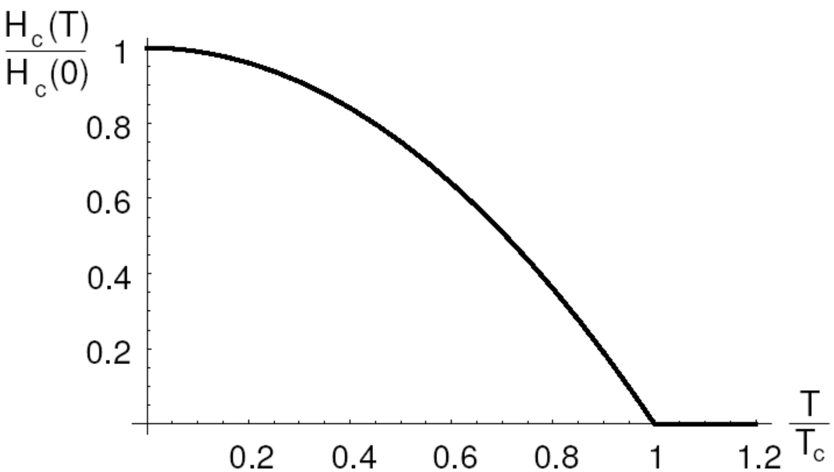


• energy gap $\Delta \approx 1.75 k_B T_C$ needed to excite a quasiparticle from the ground state (condensate)

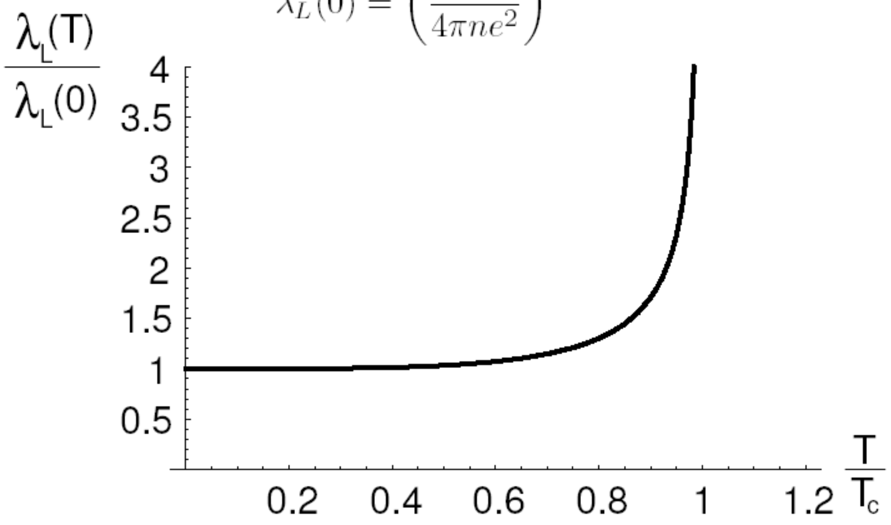
penetration depth and critical magnetic field

$$H_c(T) \approx H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right].$$

$$\lambda_L(T) = \frac{\lambda_L(0)}{\left[1-\left(\frac{T}{T_c}\right)^4\right]^{1/2}}.$$



$$\lambda_L(0) = \left(\frac{mc^2}{4\pi n e^2}\right)^{1/2}$$



some numbers aluminum:

T_c = 1.2 K; Δ = 0.18 meV
$\lambda_L(0)$ = 50 nm
v_F = 2 10^6 m/s
ξ_0 = 1.6 μm
n = 1.8 10^{29} m^3

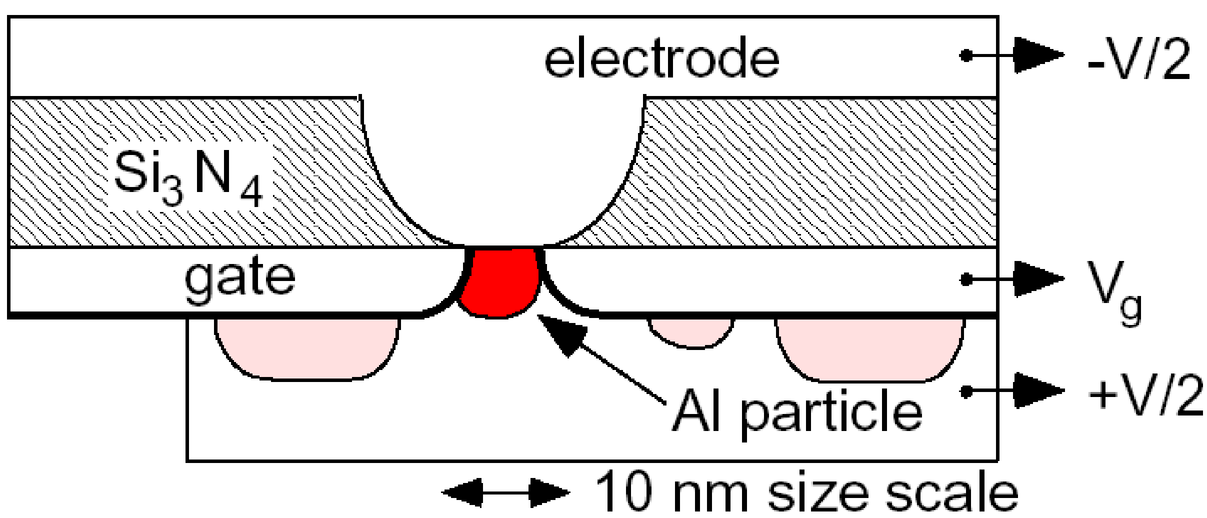
superconductivity in particles << λ,ξ ?

VOLUME 74, NUMBER 16 PHYSICAL REVIEW LETTERS 17 APRIL 1995

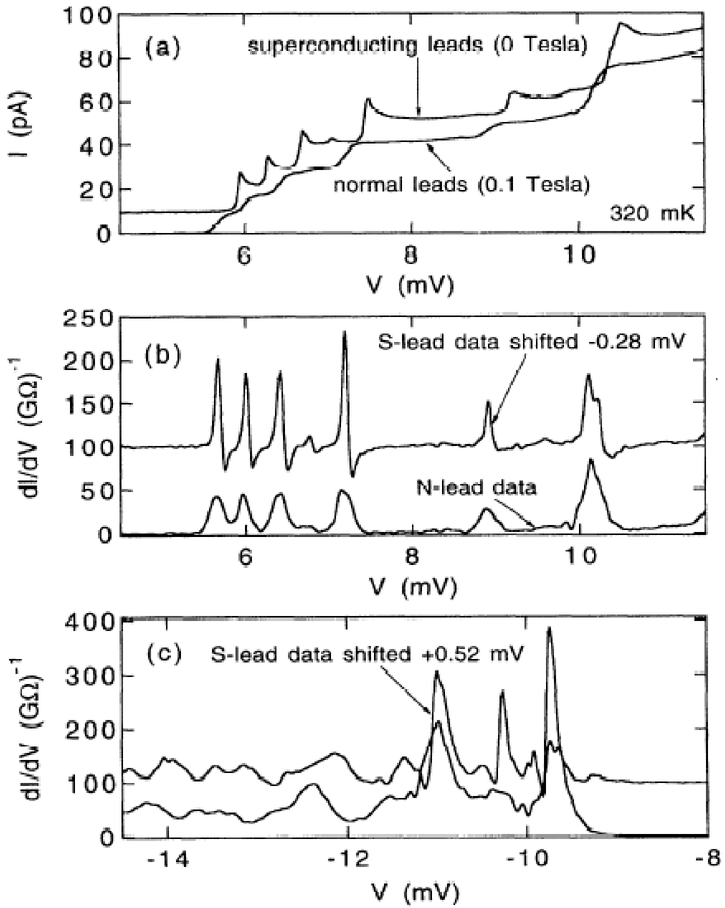
Spectroscopic Measurements of Discrete Electronic States in Single Metal Particles

D. C. Ralph, C. T. Black, and M. Tinkham
Department of Physics and Division of Applied Sciences, Harvard University, Cambridge, Massachusetts 02138
(Received 21 November 1994)

We have made tunnel junctions containing one Al particle of diameter <10 nm. Tunneling via discrete electronic states in the particle produces steps in the current-voltage (*I-V*) curve, providing, for the first time, a spectroscopic measurement of the electronic energy levels in a metal particle. With superconducting leads, the *I-V* contribution from each discrete state has the form of the BCS density of states. We can determine the parity of the electron number in the particle's ground state through the effects of an applied magnetic field on the *I-V* curve.



spectroscopic measurements on a small superconducting in particle



The I-V curve changes dramatically if the Al leads are superconducting (S) [9], as occurs when no magnetic field is applied. Each current step is shifted to higher V, relative to the N-lead data, and takes the form of a spike, with a region of negative dI/dV [upper curve, Fig. 2(a)].

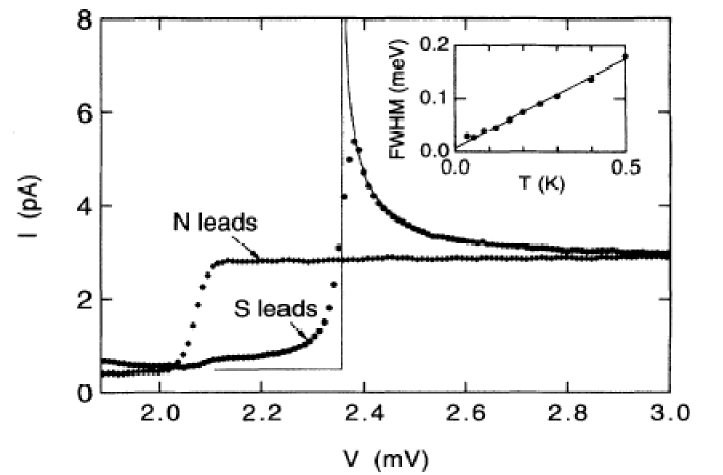
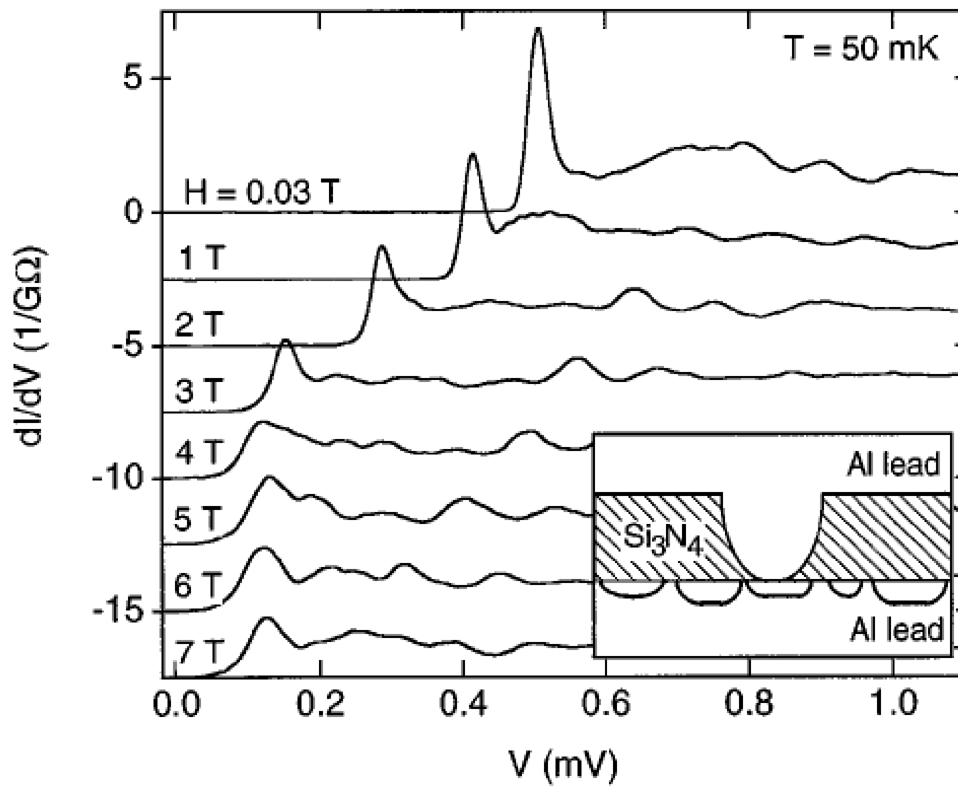


FIG. 3. Points: Tunneling current via one electronic state at 30 mK, for superconducting and normal leads. Line: Fit of the S-lead data to the BCS density of states. $C_A/C_B = 0.656 \pm 0.006$, $\Delta = 0.172 \pm 0.004$ meV, $V_N = 2.070$ mV, and $e\Gamma_{1N}\Gamma_{2N}/(\Gamma_{1N} + \Gamma_{2N}) = 2.16$ pA. Inset: FWHM of dI/dV for the first transition beyond the Coulomb threshold in a different sample, after correcting for capacitance ratio. The fit to the data has slope $3.7k_B$.

Spectroscopy of the Superconducting Gap in Individual Nanometer-Scale Aluminum Particles

C. T. Black, D. C. Ralph, and M. Tinkham
Department of Physics and Division of Applied Sciences, Harvard University, Cambridge, Massachusetts 02138
(Received 31 August 1995)



Determine the superconducting gap from the experiment?

a theoretical answer

PHYSICAL REVIEW B VOLUME 59, NUMBER 14 1 APRIL 1999-II

Superconductivity in ultrasmall metallic grains

Fabian Braun and Jan von Delft
Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany
(Received 16 January 1998; revised manuscript received 27 April 1998)

What happens to superconductivity when the sample is made very very small? Anderson[1] addressed this question already in 1959: he argued that if the sample is so small that its electronic eigenspectrum becomes discrete, with a mean level spacing $d = 1/\mathcal{N}(\varepsilon_F) \sim 1/\mathrm{Vol}$, ''superconductivity would no longer be possible'' when d becomes larger than the bulk gap $\tilde{\Delta}$. Heuristically, this is obvious (see Fig. 1 below): $\tilde{\Delta}/d$ is the number of free-electron states that pair correlate (those with energies within $\tilde{\Delta}$ of ε_F), i.e., the ''number of Cooper pairs'' in the system; when this becomes $\lesssim 1$, it clearly no longer makes sense to call the system ''superconducting.''

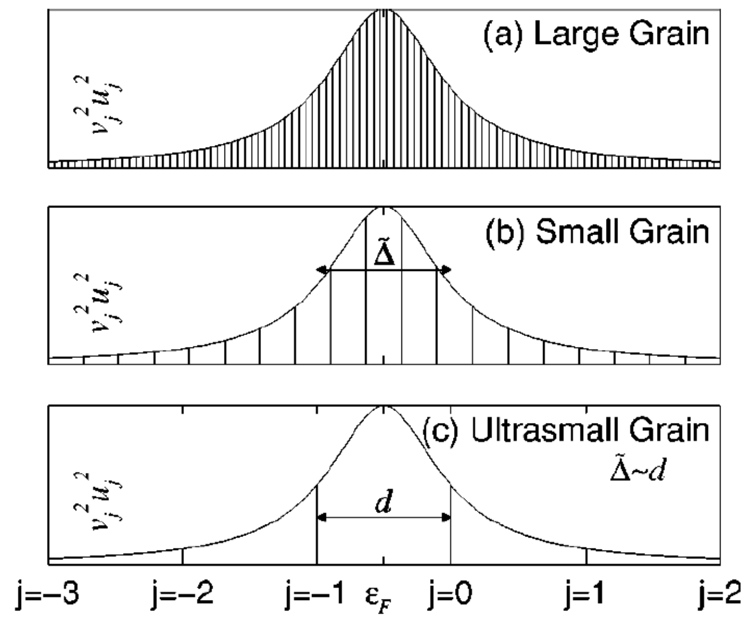


FIG. 1. An illustration of why ''superconductivity breaks down'' when the sample becomes sufficiently small. Each vertical line represents a pair of single-particle state $|j\pm\rangle$ with energy ε_j,

even-odd effect

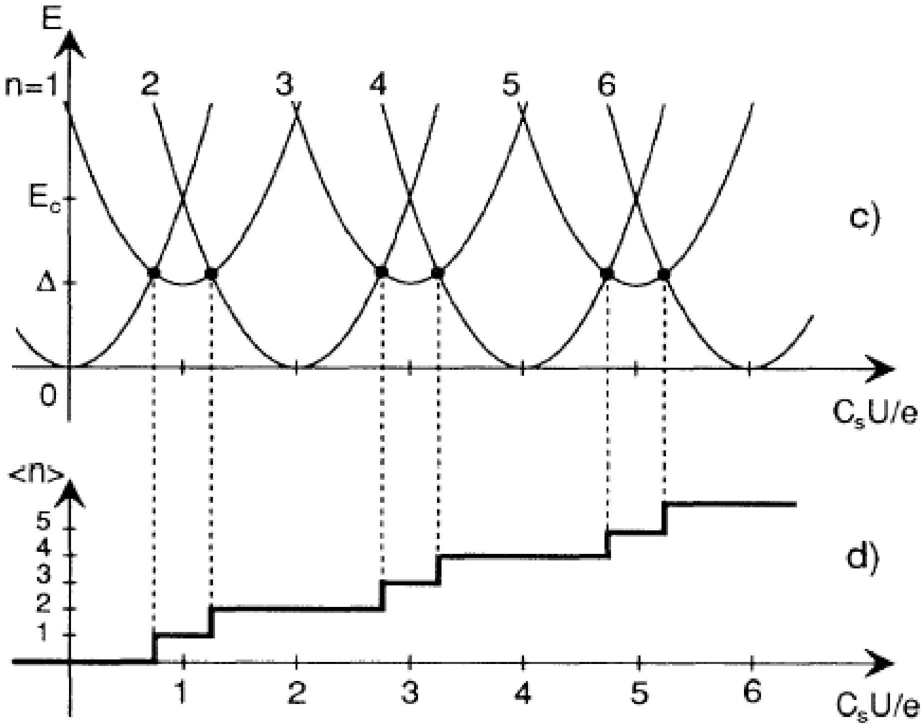
Energy depends on the parity of the superconductors:

$U = E_C(Ne - \alpha V_G)^2 + \Delta_i$

$\Delta_i = 0$ **if** $N = 2n$
$\Delta_i = \Delta$ **if** $N = 2n+1$

The ground state energy for odd n is Δ above the minimum energy for even n.

Even in an experiment on aluminum islands with 10^9 electrons, the parity of such a big number can be measured!!



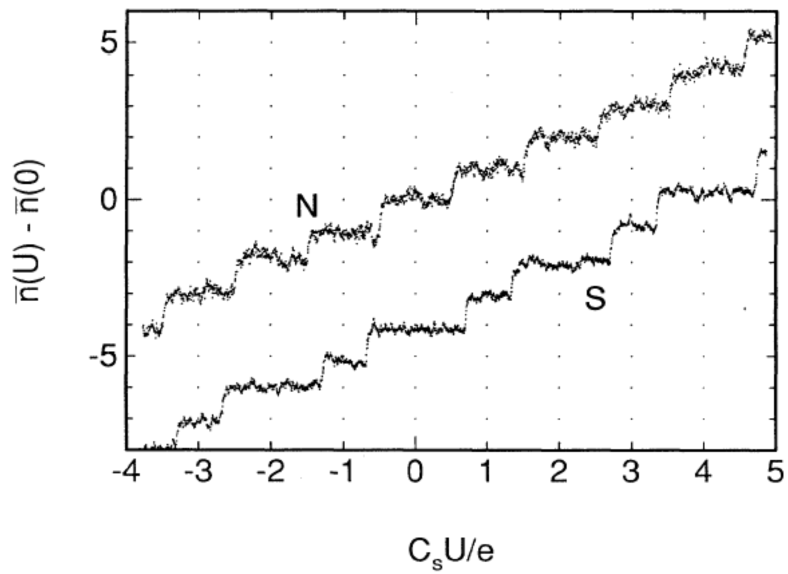
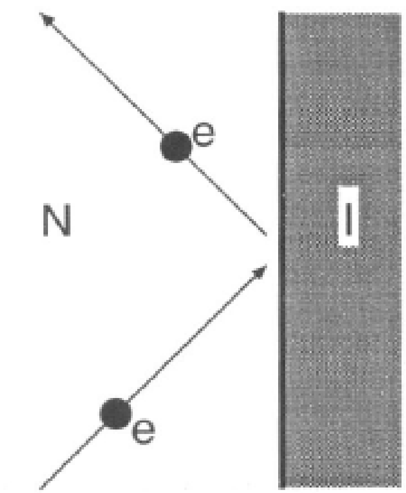
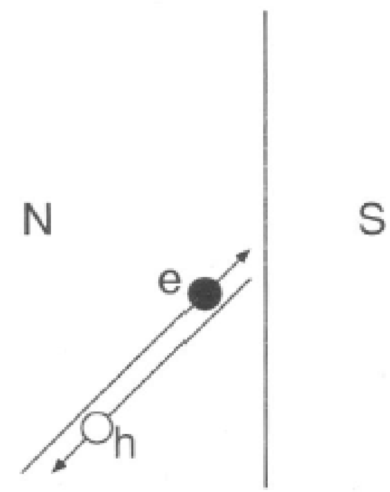


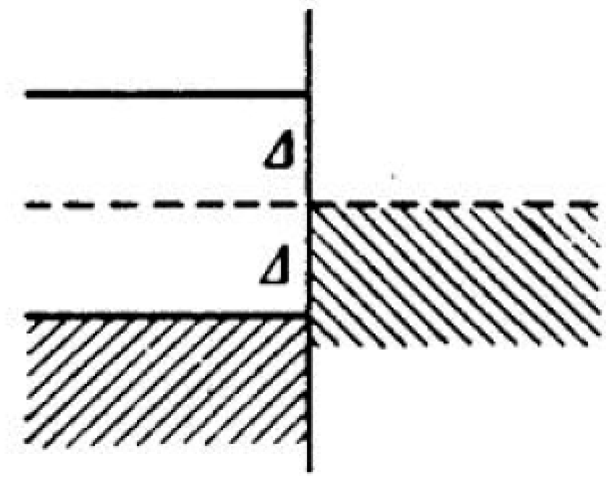
FIG. 4. Variations of the average value $\bar{n}$ of the number of extra electrons in the box as a function of the polarization C_sU/e, at $T = 25$ mK. Trace *N*: normal island. Trace *S*: superconducting island. For clarity, trace *S* has been offset vertically by 4 units.

P. Lafarge *et al.*
Phys. Rev. Lett. 70 (1993) 994

Andreev reflection at a N-S interface







- Clean N-S interface: $E>\Delta$ quasiparticle transport is possible; $E<\Delta$ single-particle tunneling is suppressed exponentially.
- ordinary reflection at a clean SN interface requires a momentum change of the the charge carriers of about $2p_F$.
- Δp_{max} = (dU/dx) Δt, with $dU \approx \Delta$, $dx \approx 2\xi_0$ and $\Delta t = 2\xi_0/v_F$ so that $\Delta p_{max} = 2p_F (\Delta/E_F) << 2p_F$
- An electron can, however, be reflected as a hole with opposite group velocity. In this way a charge 2e is transferred – Andreev reflection (so there is no charge conservation)
- In Andreev reflection Cooper pairs are transferred into a superconductor from a normal conductor in a coherent way

Andreev reflection at a N-S interface

• energy conservation: $E_e = E$; $E_h = -E$; $E_{pair} = 0$ (E = energy with respect to E_F)

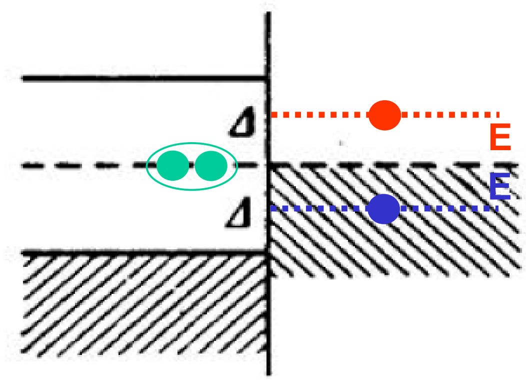
• momentum is (almost) conserved: electron-hole symmetry is not exact, only exact for charge carriers at the Fermi energy

$$\Delta k = (dk/dE)_{k_F}\, \Delta E$$

$$\Delta E = 2E \text{ so that } (dk/dE)_{k_F} = 2E/\hbar v_F$$

$$k_e = k_F + E/\hbar v_F$$

$$k_h = k_F - E/\hbar v_F$$



• Andreev reflection is phase coherent which means that there is a well defined relation between the phase of the electron and the reflected hole

$$\varphi_h = \varphi_e + \varphi_s - \arccos(E/\Delta)$$

$$\varphi_e = \varphi_h - \varphi_s - \arccos(E/\Delta)$$

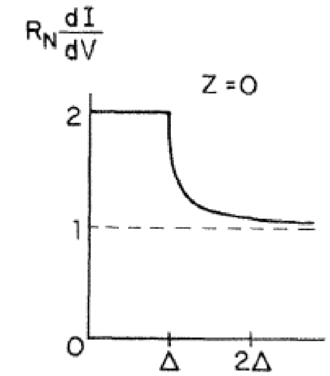
$E = 0$ then $\arccos(e/\Delta) = \pi/2$

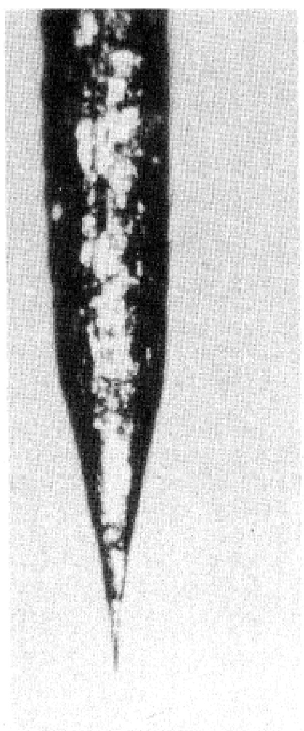
• when there is only one superconductor, φ_s does not play a role and can be chosen to be zero by an appropriate gauge transformation; with more than one superconductor involved, phase differences start to play a role

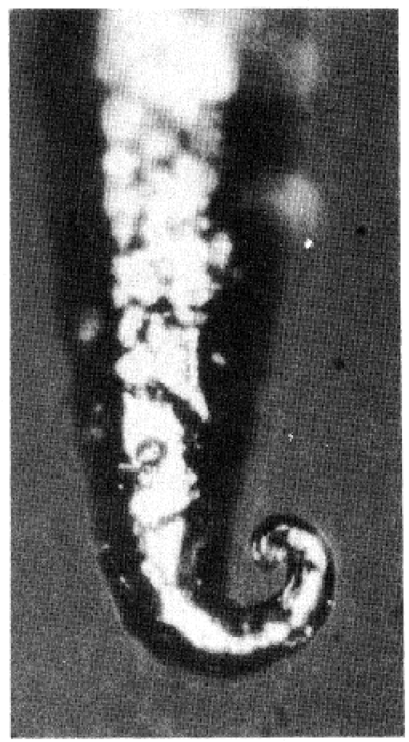
conductance of an ideal N-S interface

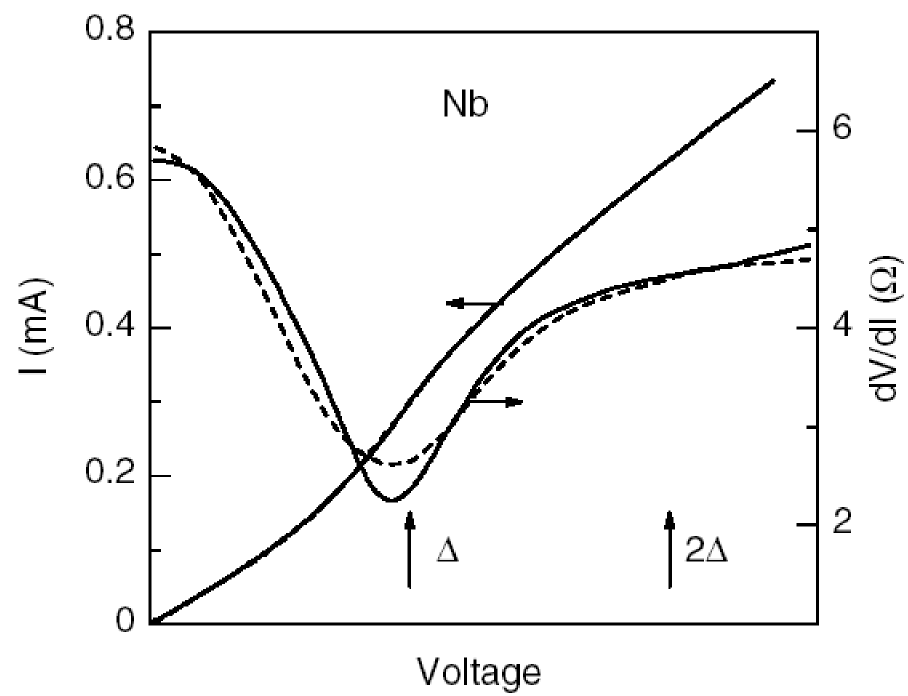
• ideal Andreev reflection doubles the conductance of an N-S system compared to that of an N-N system (e.g. by applying a magnetic field or large voltage)

• N-S point contacts are widely used to measure the gap and its temperature dependence for a large variety of superconductors and other materials with a energy gap (point contact spectroscopy)









PHYSICAL REVIEW B VOLUME 27, NUMBER 1 1 JANUARY 1983

Metallic to tunneling transition in Cu-Nb point contacts

G. E. Blonder and M. Tinkham
Department of Physics, Harvard University, Cambridge, Massachusetts 02138

conductance of a non-ideal N-S interface

• a non-ideal interface destroys Andreev reflection, i.e. ordinary reflection becomes more important

• this is usually described as introducing a delta function tunnel barrier between N and S of strength H; the parameter Z characterizes this barrier and is defined as $2\pi H/hv_F$.

• large Z: tunnel contact (S-I-N interface)

• excess current indicates the presence of Andreev reflection

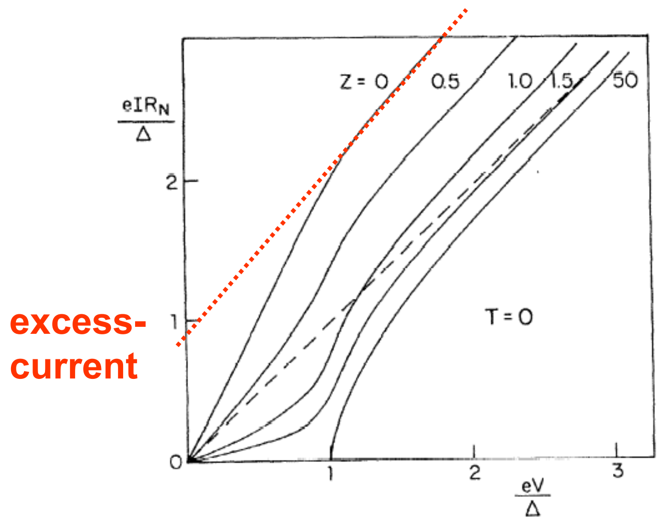


FIG. 6. Current vs voltage for various barrier strengths Z at $T=0$. These curves attain their asymptotic limits only for very high voltages. For example, the tunnel junction ($Z=50$) curve will be within 1% of the normal-state curve (dotted line) only when $eV \geq 7\Delta$.

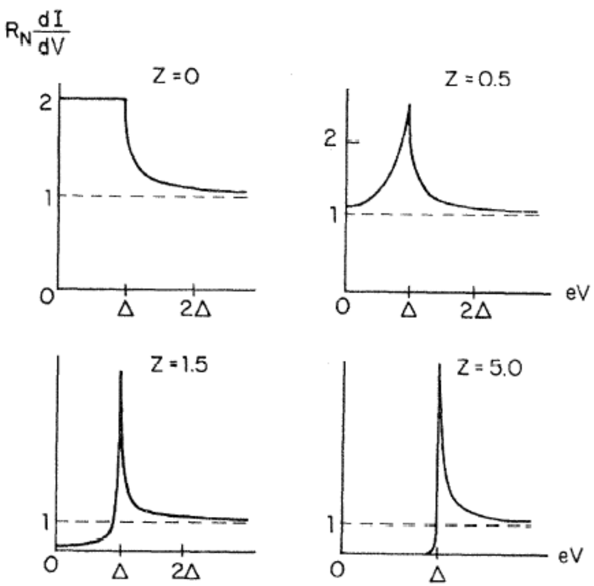


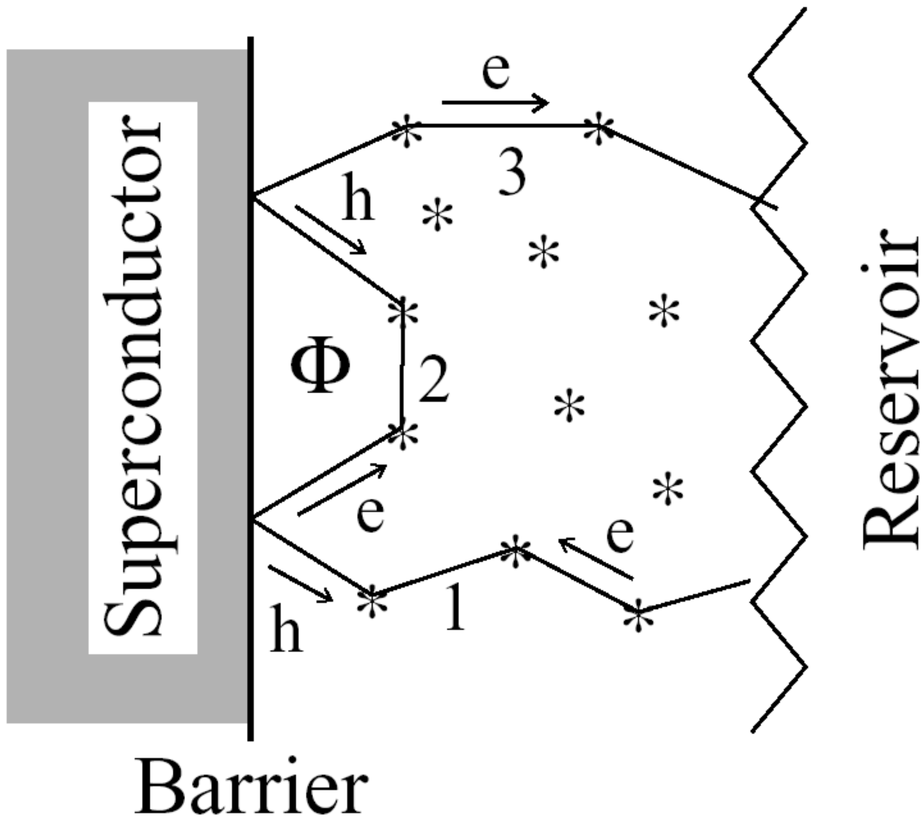
FIG. 7. Differential conductance vs voltage for various barrier strengths Z at $T=0$. This quantity is proportional to the transmission coefficient for electric current for particles at $E=eV$.

Can you understand why ordinary reflection becomes more important given the discussion about momentum change a few slides ago?

Blonder, Tinkham and Klapwijk, Phys. Rev. B 25 (1982) 4515

enhancement of interference effects

The presence of a superconductor enhances any particle interference effect in the normal state since the (dynamical) phase of the original electron and reflected hole cancel (at E = 0)



the phase an electron acquires is $$\varphi_e = \int_{-L}^{0} k_e \, dx = k_F L + \frac{E}{\hbar v_F} L$$

the phase a reflected hole acquires is $$\varphi_h = \int_{0}^{-L} k_h \, dx = -k_F L + \frac{E}{\hbar v_F} L$$

enhanced weak localization

- since the reflected hole and electron do not acquire additional phase shifts along their time reversed paths, there is an enhanced probability for coherent backscattering (effect is roughly the same size as in the normal state)
- important for contacts with a high transparency (for samples with many Andreev reflection events)
- “t is as if Andreev reflection effectively doubles the length of the disordered region”

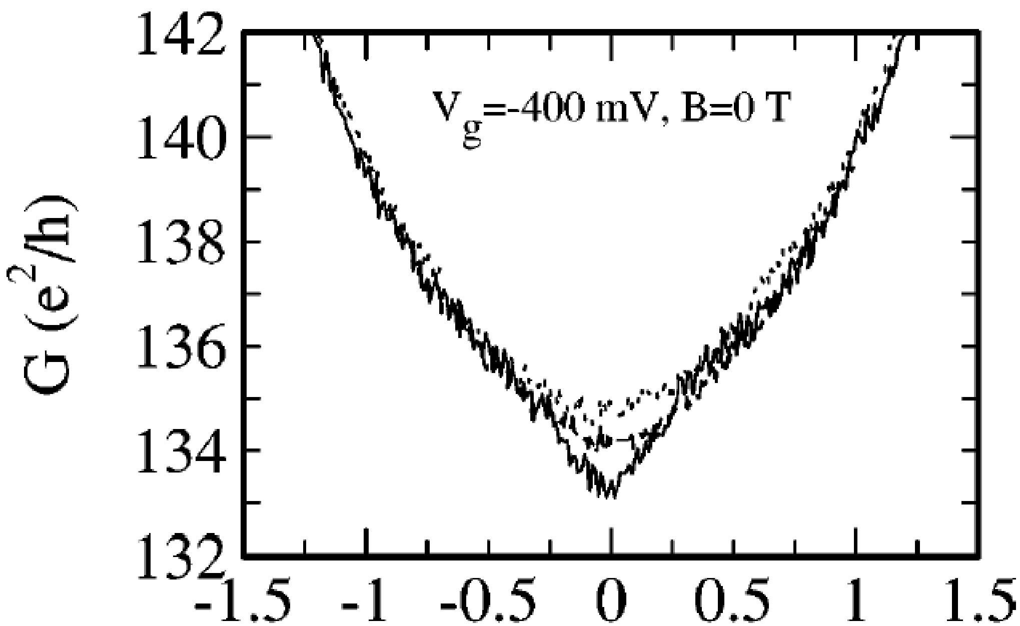


FIG. 3. Differential conductance as a function of the bias voltage at several temperatures (without quantum point contact; $V_g = -400$ mV; from top to bottom $T = 0.9$, 0.7, and 0.01 K).

Lensen et al., Phys. Rev. B 58 (1998) 4888

enhanced weak localization

- temperature, magnetic field and a voltage destroy the enhanced weak localization

$$B_c = \frac{h}{e l_\varphi^2}$$

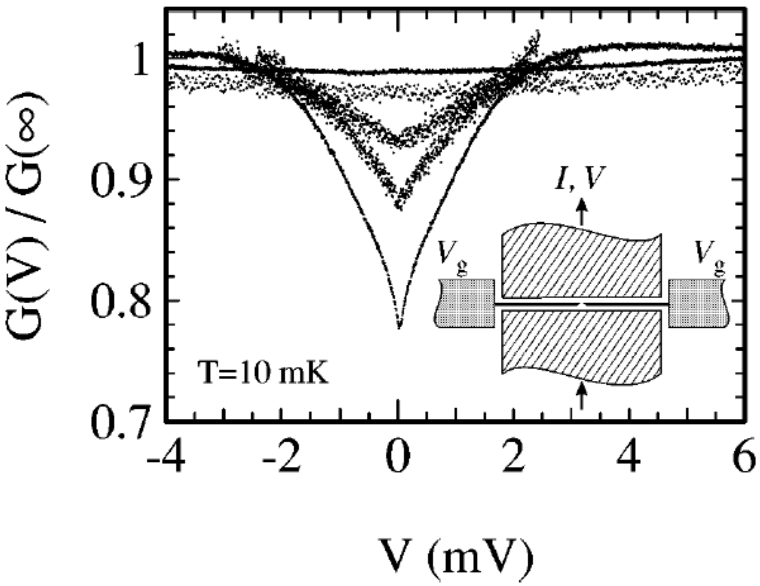


FIG. 1. Normalized differential conductance as a function of the bias voltage at several magnetic fields ($V_g = 0$ mV; from top to bottom $B = 60$, 35, 20, 15, and 0 mT); the inset shows the layout of the center of the sample on scale (the hatched areas are Ti/Sn contacts, the gray and black areas are gold gates).

$$eV_c = \frac{\pi}{2} \frac{\hbar v_F l_e}{l_\varphi^2}$$

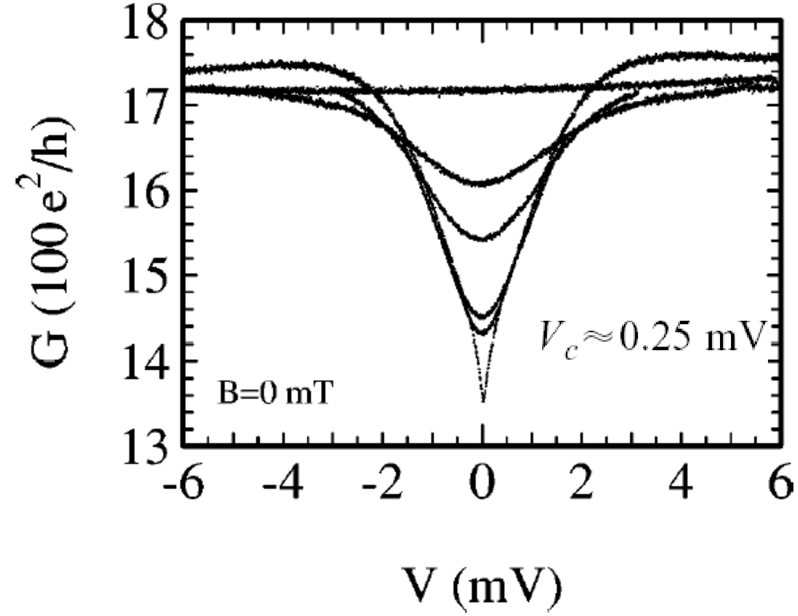
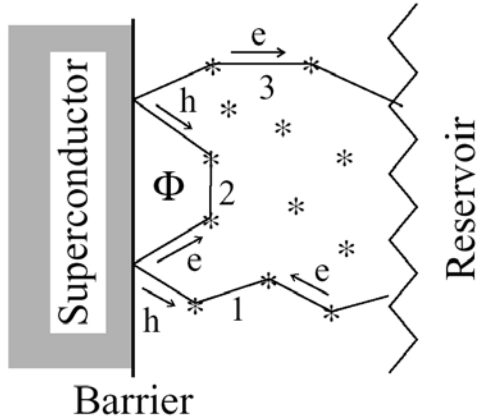


FIG. 2. Differential conductance as a function of the bias voltage at several temperatures ($V_g = 0$ mV; from top to bottom $T = 4.2$, 2.5, 1.8, 0.9, 0.75, and 0.01 K).

Lensen et al., Phys. Rev. B 58 (1998) 4888

reflectionless tunneling

- because of Andreev reflection, an electron in a disordered system gets more than one opportunity to undergo Andreev reflection; this results in a enhancement of the conductance around zero bias since the bias voltage adds additional phases (see previous slides)
- important for S-N contacts with a low transparency



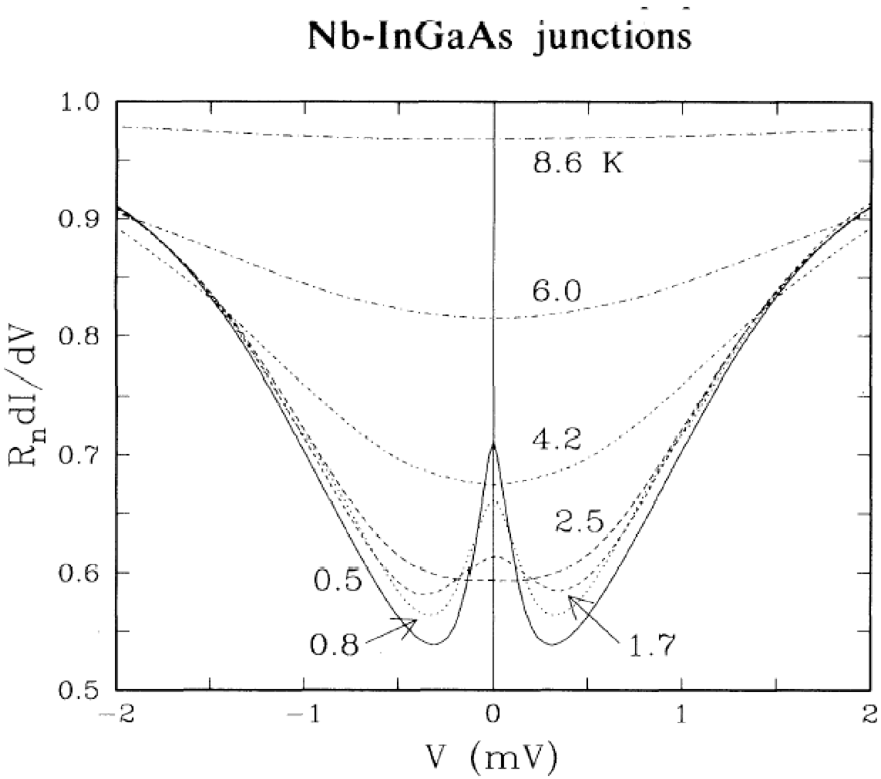


FIG. 2. Normalized conductance-voltage characteristics at temperatures of 8.6, 6.0, 4.2, 2.5, 1.7, 0.8, and 0.5 K and zero magnetic field for a 2.5×10^{19}-cm^{-3} device ($R_n = 0.27\ \Omega$).

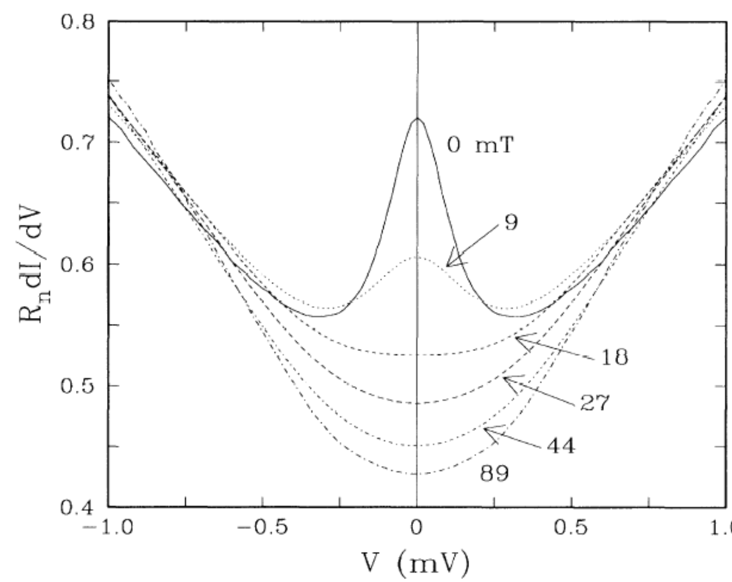
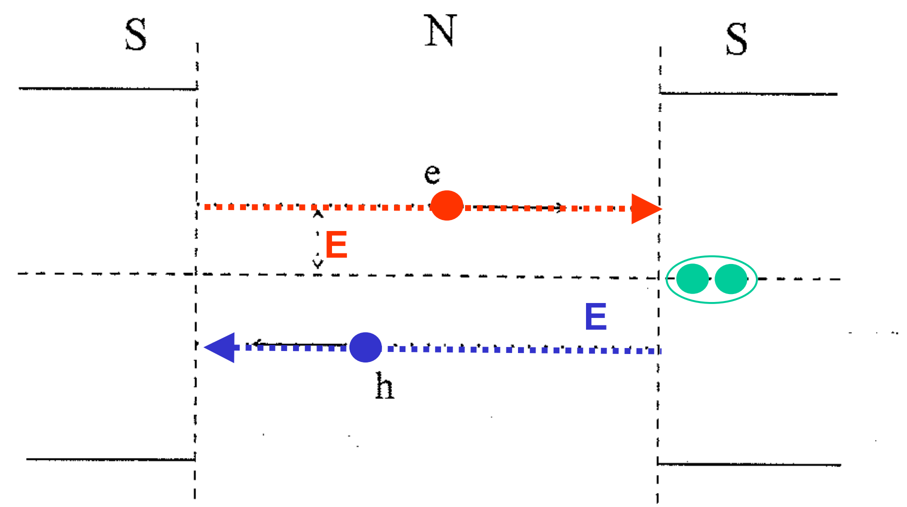


FIG. 3. Normalized conductance-voltage characteristics at 0.42 K in parallel magnetic fields of 0, 9, 18, 27, 44, and 89 mT for a 2.5×10^{19}-cm^{-3} device ($R_n = 0.24\ \Omega$).

Kastalsky et al., Phys. Rev. Lett. 67 (1991) 3026

Andreev bound states in a S-N-S junction

- an electron with energy $E<\Delta$ can not enter superconductor 2
- it will be reflected as an hole with energy $-E$ in the opposite direction, retracing the original path of the electron
- the hole reaches the left superconductor and will be reflected as an electron with energy E' (the energy with respect to the Fermi energy of superconductor 1; with no bias applied E = E')
- full quantum mechanical description of this process: Andreev bound state



- condition for the formation of a bound state (1D picture): total phase acquired during one cycle is a multiple of 2π: $\varphi_{s2} - \varphi_{s1} + (k_e - k_h) L - 2\arccos(E/\Delta) = 2\pi n$

- long junction: arccos term << $(k_e - k_h) L$ and using $k_e = k_F + 2E/\hbar v_F$ one finds:

$$E_n = \hbar v_F/2L \,(2\pi (n+1/2) \pm \varphi) \qquad \text{with } \varphi = \varphi_{s2} - \varphi_{s1}$$

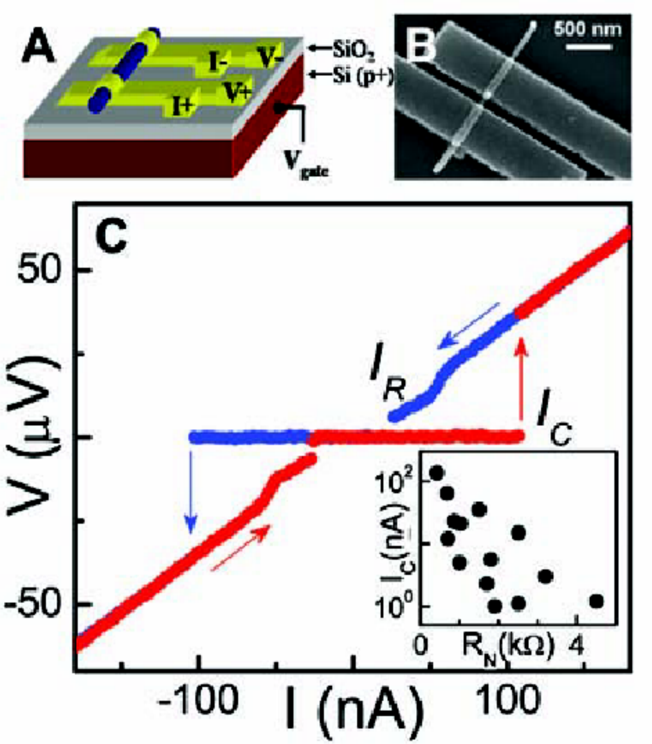
- + sign: starting with a right-moving electron (left-moving hole); – sign starting with a left-moving electron (right-moving hole)

supercurrent in a S-N-S junction

- Andreev bound states are confined states which carry a net supercurrent (1D long junction):

$$I_n = -(2e/\hbar) \sum dE_n/d\varphi \qquad \text{with} \qquad E_n = \hbar v_F/2L\,(\,2\pi\,(n+1/2) \pm \varphi\,)$$

- each state carriers a supercurrent of ev_F/L and each subsequent state carriers a supercurrent in the opposite direction
- short junction (see lecture notes ξ_n=$hv_F/4\Delta$ >L)): $I_c = 2e\Delta/\pi\hbar$
- often one looks at the I_cR_n product which for a short junction equals $2\Delta/e$ (check this for a point contact with one channel, but the result is independent on the number of channels)
- Andreev reflection at the S-N interface and phase-coherent propagation in the normal conductor can thus be viewed as the microscopic origin of the proximity effect: induced superconductivity in a normal metal which occurs over a distance ξ_n

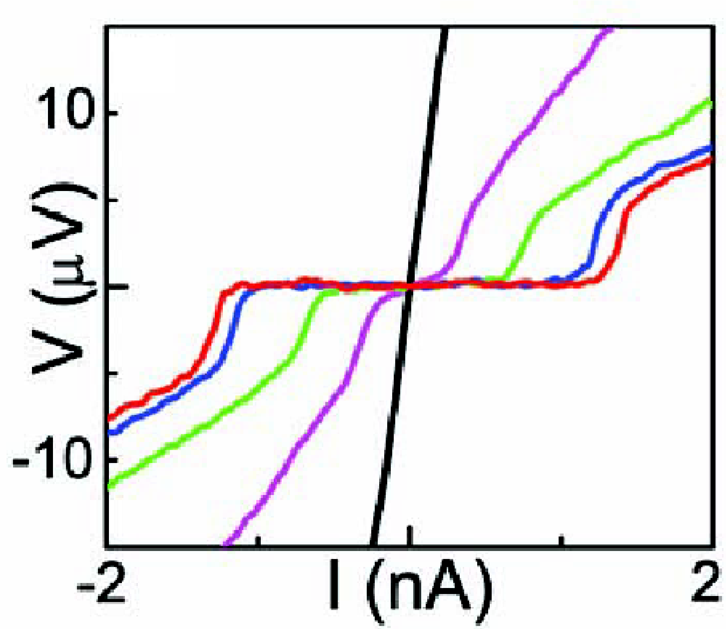


Doh et al., Science 309 (2005)

tunable supercurrent through a S-semiconducting nanowire-S junction

- In this case is the normal metal a diffusive semiconducting nanowire: gate tunable superconducting properties

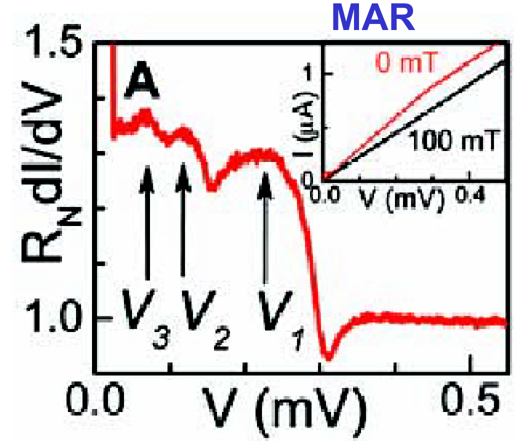
- very difficult to make clean normal metal-semiconductor interfaces; InAs is one of the materials for which it works



V_g = 0 (red), –10 (blue), –50 (green), –60 (purple), and –71 V (black)

Nanoscale superconductor/semiconductor hybrid devices are assembled from indium arsenide semiconductor nanowires individually contacted by aluminum-based superconductor electrodes. Below 1 kelvin, the high transparency of the contacts gives rise to proximity-induced superconductivity. The nanowires form superconducting weak links operating as mesoscopic Josephson junctions with electrically tunable coupling. The supercurrent can be switched on/off by a gate voltage acting on the electron density in the nanowire. A variation in gate voltage induces universal fluctuations in the normal-state conductance, which are clearly correlated to critical current fluctuations. The alternating-current Josephson effect gives rise to Shapiro steps in the voltage-current characteristic under microwave irradiation.

MAR



Doh et al., Science 309 (2005) 272

multiple Andreev reflection (MAR)

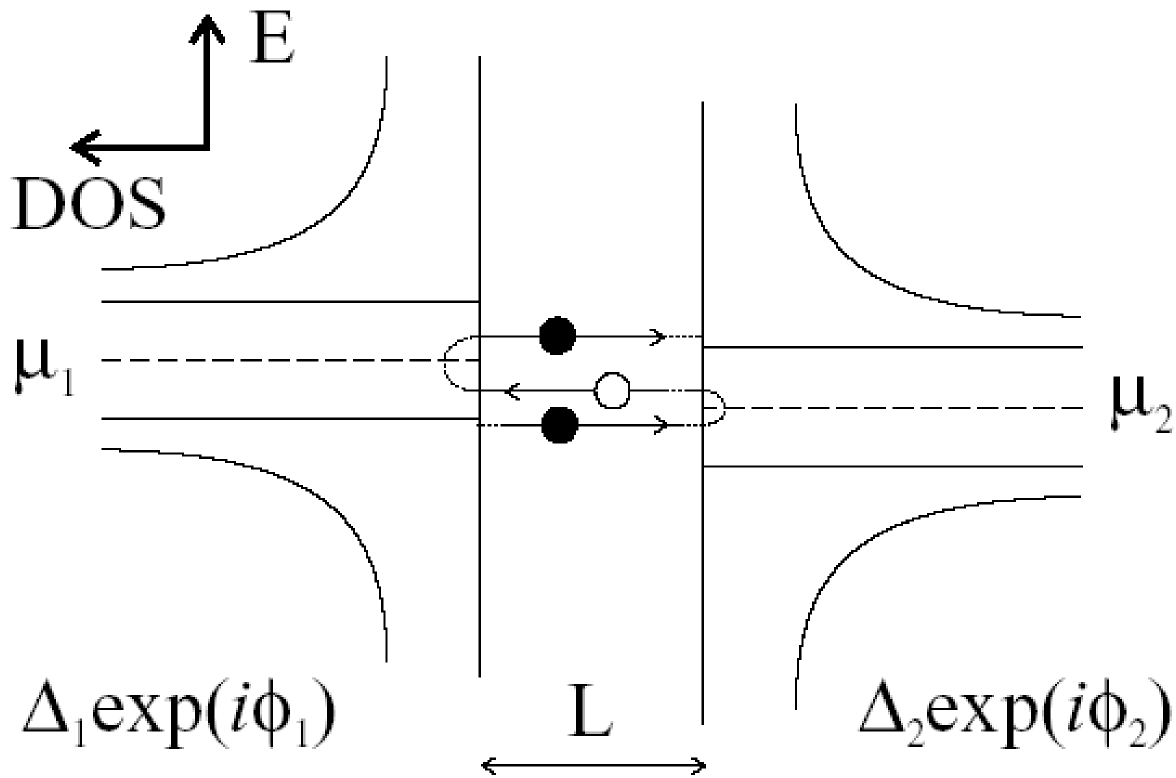
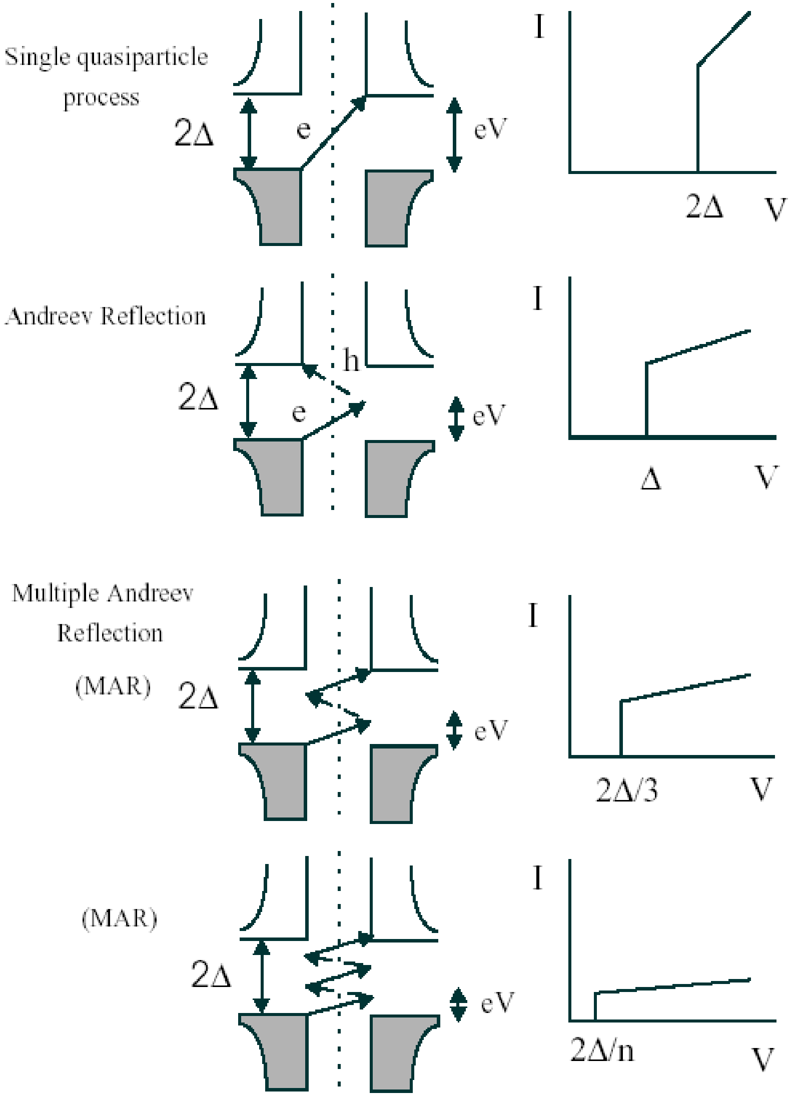
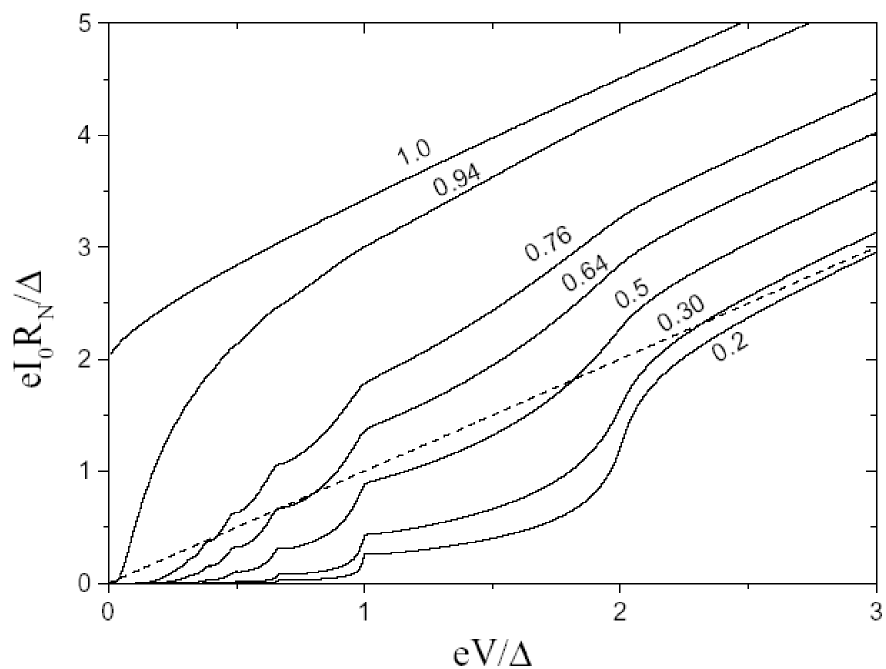


Figure 1.1: An electron injected from superconductor 1 can not enter superconductor 2, since the density of states (DOS) is zero for $|E| < |\Delta_{1,2}|$. Current transport can only occur if the electron is Andreev reflected into a hole. The number of Andreev reflections needed before the particle can enter the superconductor as an excitation depends on the applied voltage $eV = \mu_1 - \mu_2$ between both superconductors.

multiple Andreev reflection (MAR): sub-harmonic gap structure



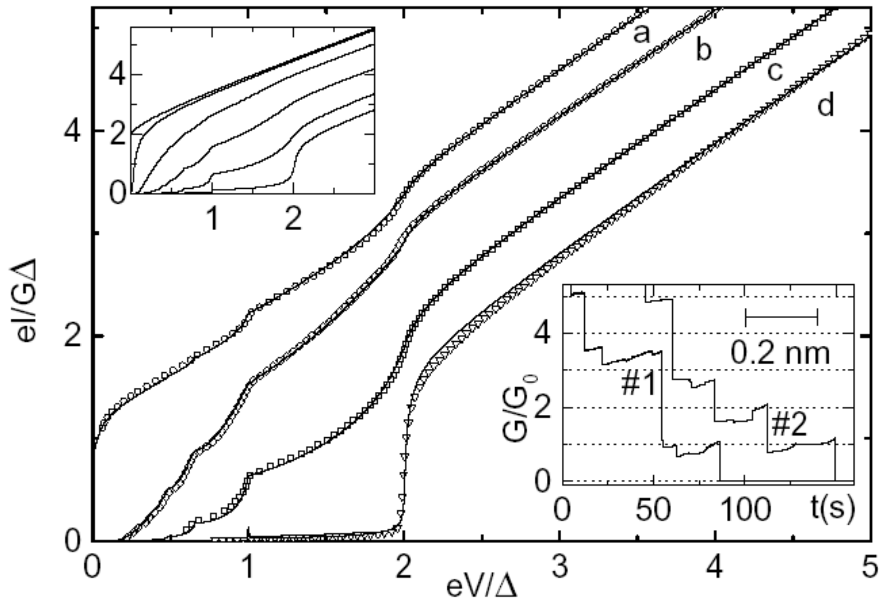
- The dc current exhibits **sub-harmonic gap structure** at voltages $eV_n = 2\Delta/n$ (n integer)
- Increase in current every time the next MAR process (i.e. n) becomes available



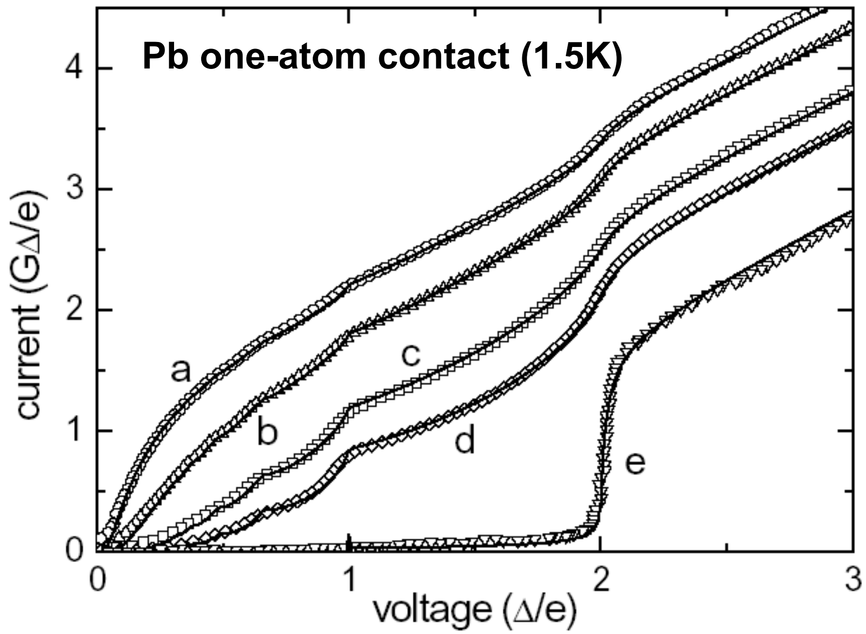
subharmonic gap structure: atomic contacts

The extended quantum states that carry the current from one bank to the other necessarily proceed through the valence orbitals of the constriction atom. It thus seems reasonable to conjecture that the number of current-carrying modes (or 'channels') of a one-atom contact is determined by the number of available valence orbitals, and so should strongly differ for metallic elements in different series of the periodic table.

Al one-atom contact (30 mK)



(a)	(b)	(c)	(d)
0.997	0.74	0.46	0.025
0.46	0.11	0.35	
0.29		0.07	



(a)	(b)	(c)	(d)	(e)
0.955	0.89	0.76	0.65	0.026
0.355	0.36	0.34	0.34	
0.085	0.145	0.27	0.29	
0.005	0.005	0.02	0.12	

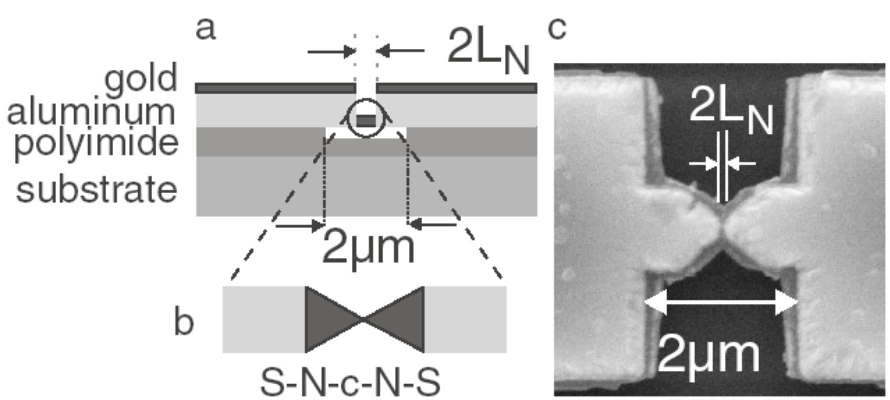
The mesoscopic PIN code, $\{\tau_i\}$, can be measured!!!

E. Scheer et al., Phys. Rev. Lett. 78 (1997) 3535

E. Scheer et al., Nature 394 (1998) 154

Proximity Effect and Multiple Andreev Reflections in Gold Atomic Contacts

E. Scheer,[1,*] W. Belzig,[2] Y. Naveh,[3,†] M. H. Devoret,[4] D. Esteve,[4] and C. Urbina[4]



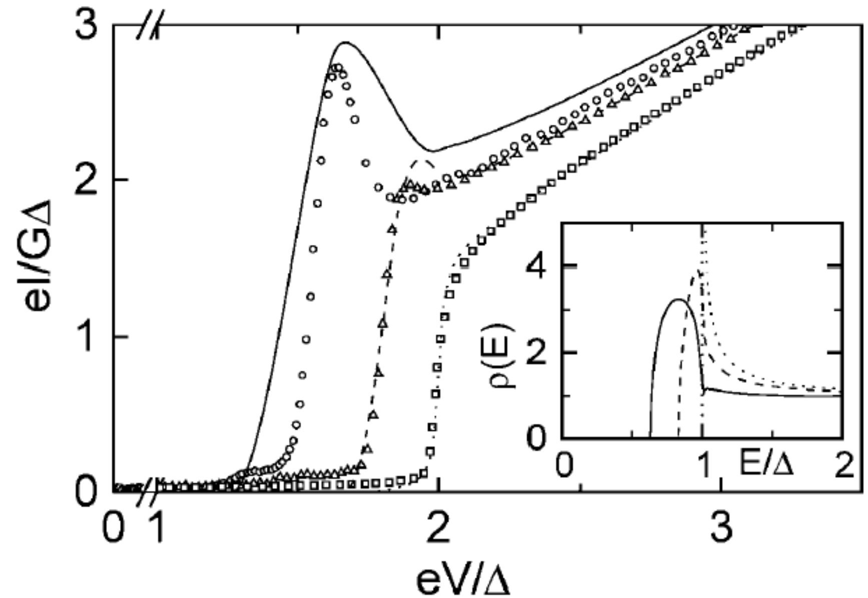
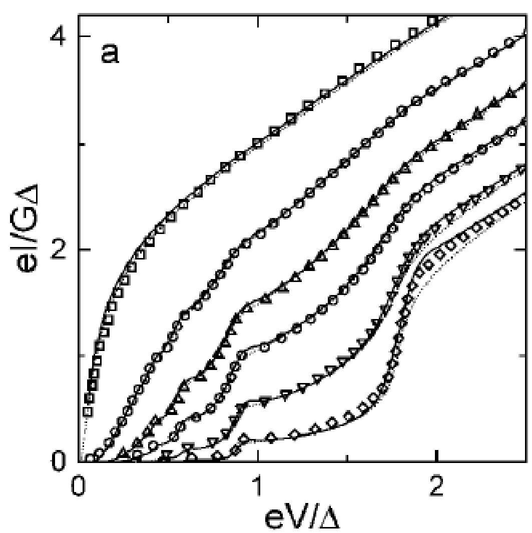


FIG. 2. Measured (symbols) and calculated (lines) IVs in the tunnel regime for Al-Au-Al samples No. 1 (triangles) and No. 2 (circles), and pure Al sample No. 3 (squares). The dotted line

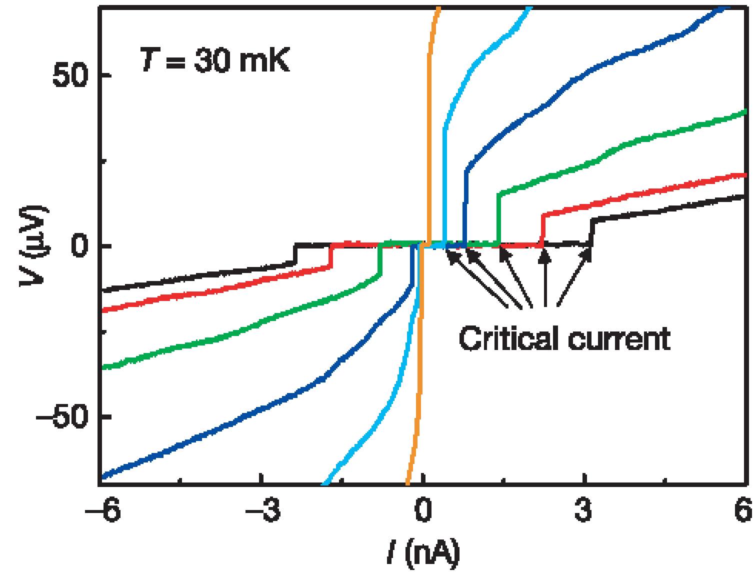


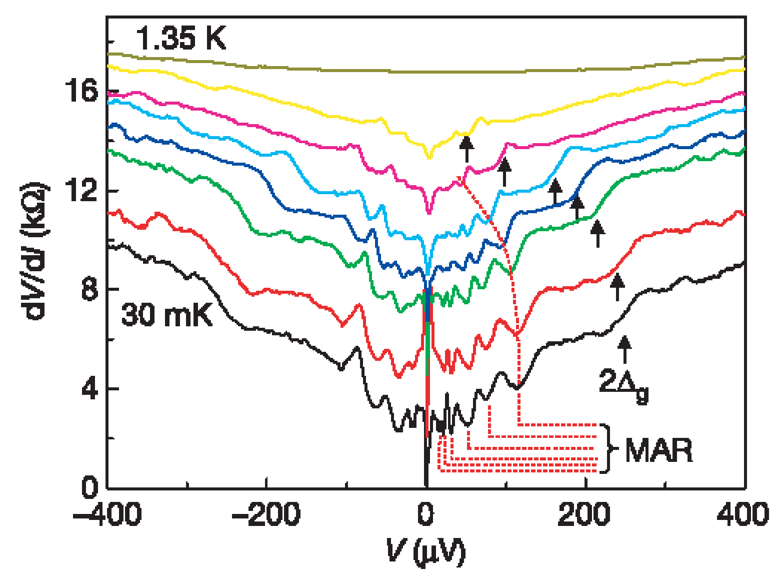
Au: only one transport channel

Quantum supercurrent transistors in carbon nanotubes

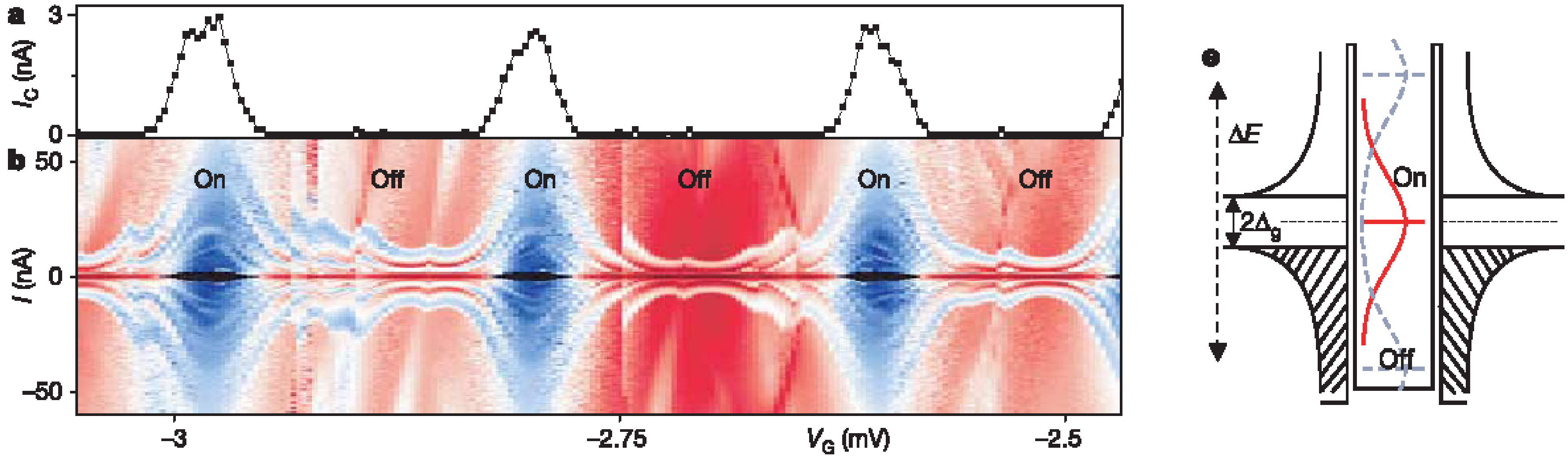
nature | Vol 439|23 February 2006|doi:10.1038/nature04550

Electronic transport through nanostructures is greatly affected by the presence of superconducting leads[1–3]. If the interface between the nanostructure and the superconductors is sufficiently transparent, a dissipationless current (supercurrent) can flow through the device owing to the Josephson effect[4,5]. A Josephson coupling, as measured by the zero-resistance supercurrent, has been obtained using tunnel barriers, superconducting constrictions, normal metals and semiconductors. The coupling mechanisms vary from tunnelling to Andreev reflection[5–8]. The latter process has hitherto been observed only in normal-type systems with a continuous density of electronic states. Here we investigate a supercurrent flowing through a discrete density of states—that is, the quantized single particle energy states of a quantum dot[9], or 'artificial atom', placed between superconducting electrodes. For this purpose, we exploit the quantum properties of finite-sized carbon nanotubes[10]. By means of a gate electrode, successive discrete energy states are tuned on- and off-resonance with the Fermi energy in the superconducting leads, resulting in a periodic modulation of the critical current and a non-trivial correlation between the conductance in the normal state and the supercurrent. We find, in good agreement with existing theory[11], that the product of the critical current and the normal state resistance becomes an oscillating function, in contrast to being constant as in previously explored regimes.





superconductivity when level aligns with middle of the gap (Fermi energy)



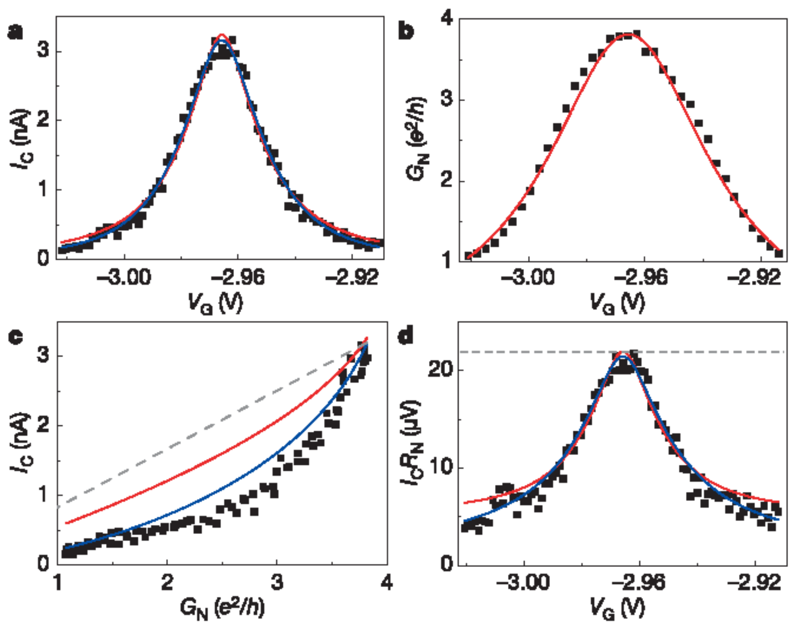
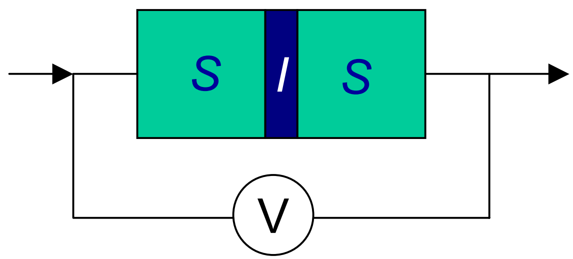


Figure 3 | Correlation between critical current and normal state conductance and modulation of the $I_C R_N$ product. In all panels, the black dots represent the experimental data points ($T = 30$ mK) and the red/blue curves are theoretical plots. **a**, Critical current, I_C, versus V_G for the resonance shown in Fig. 2c. The theoretical lines are fits to $I_C = I_0[1-(1-\Gamma_1\Gamma_2/((V_G - V_{GR})^2 + 0.25\Gamma^2))^{1/2}]$ (red curve) and $I_{CM} = I_{0M}[1-(1-\Gamma_1\Gamma_2/((V_G - V_{GR})^2 + 0.25\Gamma^2))^{1/2}]^{3/2}$ (blue), as explained in the main text. V_{GR} is the value of gate voltage on-resonance. All gate voltages and Γs are converted into energies by multiplying by the gate coupling factor, $\alpha = 0.02$ meV mV^{-1}, obtained from measurements in the nonlinear regime. **b**, Conductance, G_N, as a function of V_G in the normal state ($B = 40$ mT) and the corresponding fit to $G_N = 4e^2/h(\Gamma_1\Gamma_2/((V_G - V_{GR})^2 + 0.25\Gamma^2))$. **c**, I_C–G_N correlation plot. The data show a non-trivial correlation, with a stronger decrease of I_C than expected from the theoretical curve $I_C = I_0[1-(1-1/4G_N)^{1/2}]$ (red curve). The 1/4 factor simply denotes that G_N is measured in e^2/h units. The difference can be almost entirely accounted for by the influence of the electromagnetic environment, resulting in a measured $I_{CM} = I_0[1-(1-1/4G_N)^{1/2}]^{3/2}$ (blue curve). An ideal SNS junction, with N a normal metal with continuous density of states, would exhibit a linear I_C–G_N correlation curve (grey dashed curve). **d**, $I_C R_N$ product versus V_G, resulting from dividing the experimental data and theory curves from **a** and **b**. The grey dashed line indicates a constant $I_C R_N$ product such as in a SNS junction.

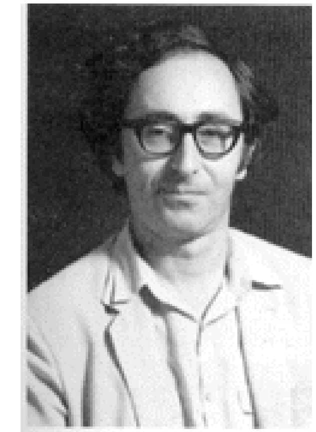
Josephson junction: S-I-S tunnel junction

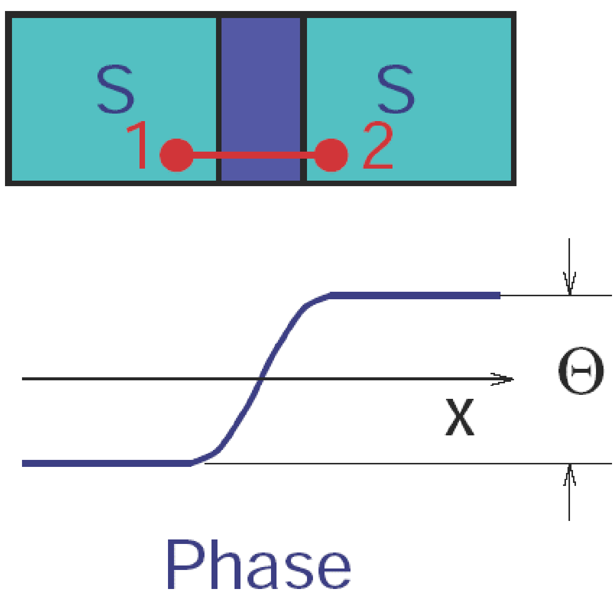


Josephson relations

$$U(t) = \frac{h}{2e}\frac{\partial \phi}{\partial t}$$

$$I(t) = I_c \sin(\phi(t))$$





Mesoscpoic Josephson circuits:

- **Competition between Josephson coupling energy (hIc/4p), which favors the flow of a supercurrent and the charging energy, which tries to localize the Cooper pairs.**

- **There exist a Heisenburg relation between the fluctuations in the phase on a superconducting island and fluctuations in the number of Cooper pairs on it:**

$$[\Delta\phi, \Delta N] \geq 1/2$$

- **The flux quantum bits (qubits) and charge qubits**

The three main effects predicted by Josephson follow from these relations:

1. The DC Josephson effect. This refers to the phenomenon of a direct current crossing the insulator in the absence of any external electromagnetic field, owing to tunneling. This DC Josephson current is proportional to the sine of the phase difference across the insulator.

2. The AC Josephson effect. With a fixed voltage across the junctions, the phase will vary linearly with time and the current will be an AC current with a frequency proportional to the applied voltage. The complete expression for the current drive I_{ext} becomes

$$I_{ext} = C_J \frac{dv}{dt} + I_J sin\phi + \frac{V}{R}$$

This means a Josephson junction can act as a perfect voltage-to-frequency converter.

3. The inverse AC Josephson effect. If the phase takes the form

$$\phi(t) = \phi_0 + n\omega t + a\sin(\omega t)$$

the voltage and current will be

$$U(t) = \frac{h}{2e}\omega(n + a\cos(\omega t)), \quad I(t) = I_c \sum_{m=-\infty}^{\infty} J_n(a)\sin(\phi_0 + (n+m)\omega t)$$

The DC components will then be $$U_{DC} = n\frac{h}{2e}\omega, \quad I(t) = I_c J_{-n}(a)\sin\phi_0$$

Hence, for distinct DC voltages, the junction may carry a DC current and the junction acts like a perfect frequency-to-voltage converter **(Shappiro steps; see next slide)**.

from Wikipedia

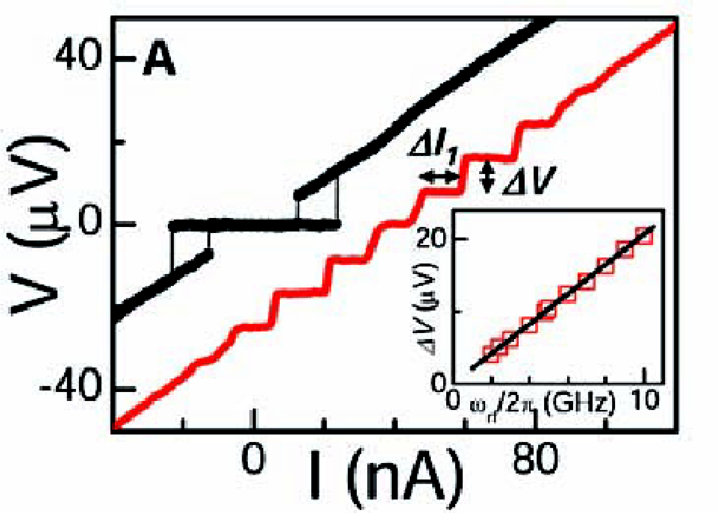
Applications of the Josephson effect

The Josephson effect has found wide usage, for example in the following areas:

- SQUIDs or superconducting quantum interference devices, are very sensitive magnetometers that operate via the Josephson effect. They are widely used in science (mesoscopic experiments) and engineering.

- In precision metrology, the Josephson effect provides an exactly reproducible conversion between frequency and voltage. Since the first is already defined precisely and practically by the caesium standard, the Josephson effect is used, for most practical purposes, to give the definition of a volt (although, as of July 2007, this is not the official BIPM definition).

- Single-electron transistors are often constructed of superconducting materials, allowing use to be made of the Josephson effect to achieve novel effects. The resulting device is called a "superconducting single-electron transistor".



from Wikipedia