Dynamics & Stability
AE3-914

Fictitious forces

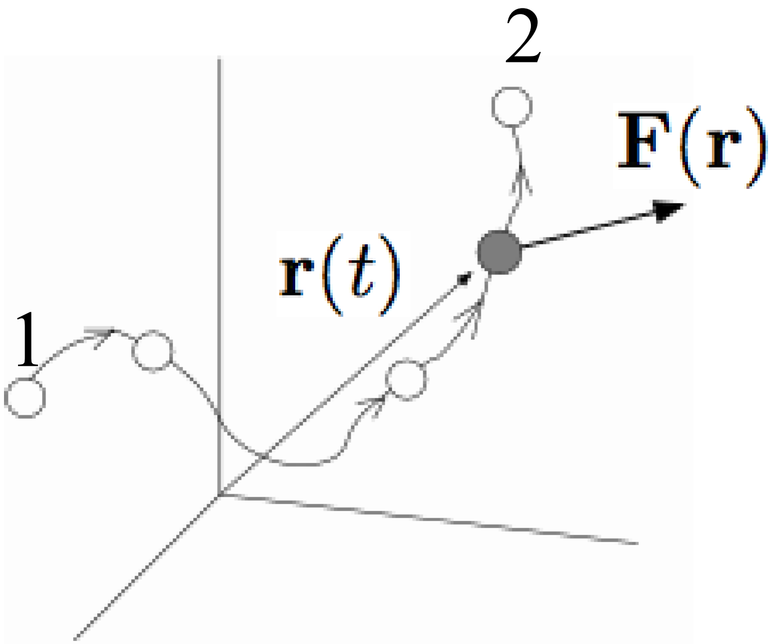
$$
\begin{aligned}
\mathbf{a}_P \quad = \mathbf{a}_{xyz} \quad & +\dot{\boldsymbol{\omega}}_{xyz} \times \mathbf{r}_{rel} \\
& +\boldsymbol{\omega}_{xyz} \times (\boldsymbol{\omega}_{xyz} \times \mathbf{r}_{rel}) \\
& +2(\boldsymbol{\omega}_{xyz} \times \mathbf{v}_{rel}) \qquad +\mathbf{a}_{rel}
\end{aligned}
$$

Double-deck train

Focault's Pendulum

1st demonstration of earth's rotation (1851)

Work & Energy



$$W_{1\to 2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r}$$

$$= \int_{t_1}^{t_2} \mathbf{F} \cdot \dot{\mathbf{r}}\, dt$$

Conservative force field

$$
\begin{aligned}
\mathbf{F} &= -\nabla V \\
&= -\frac{\partial V}{\partial x}\mathbf{i} - \frac{\partial V}{\partial y}\mathbf{j} - \frac{\partial V}{\partial z}\mathbf{k}
\end{aligned}
$$

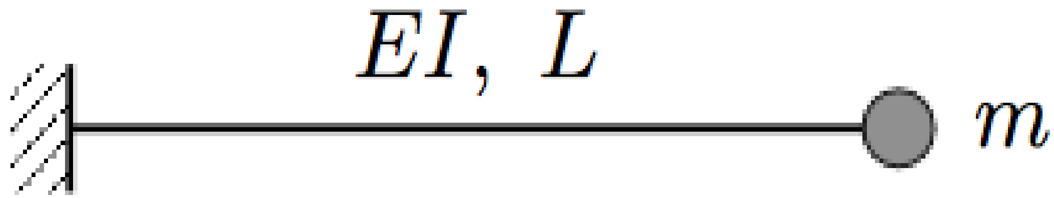
Conservative?

$$\mathbf{F} = -\frac{mgR^2}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$$

Conservative systems

$$T + V = \mathbb{E}$$ **= constant !**

Qualitative analysis



$$k = \frac{3EI}{L^3}$$

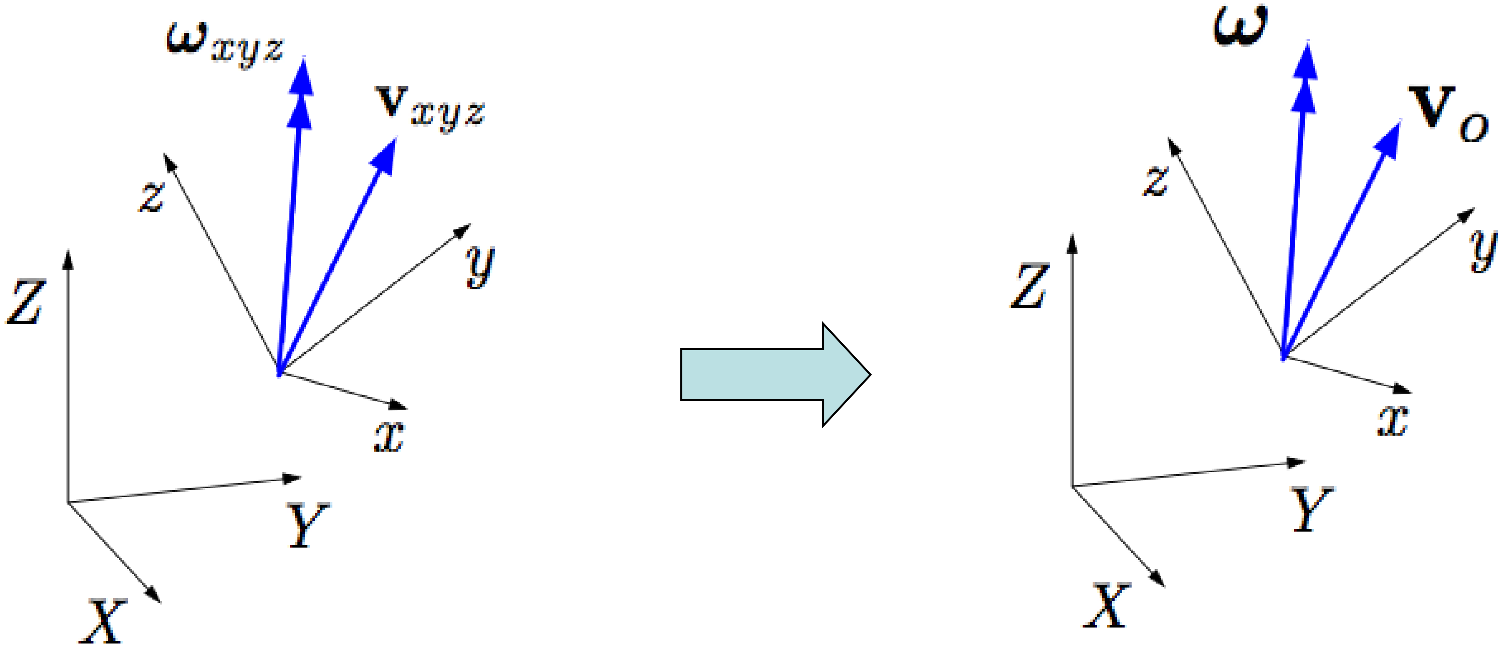
$$m\ddot{x} + kx = 0$$

$$V = \frac{1}{2}kx^2 \qquad T = \frac{1}{2}m\dot{x}^2$$

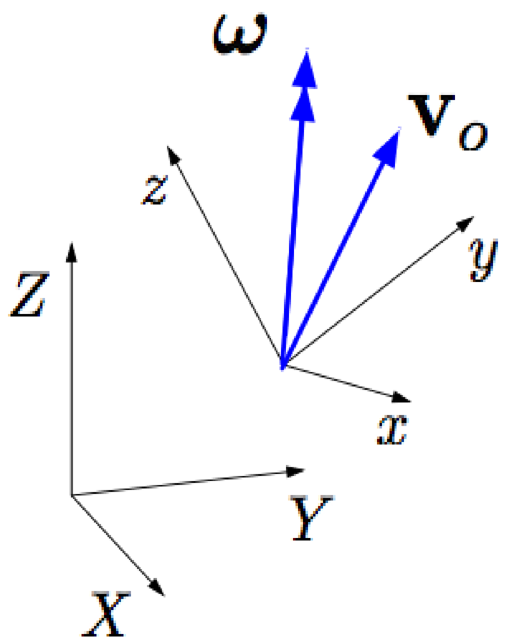
Kinetic energy of rigid bodies

$$dT = \frac{1}{2}\mathbf{v} \cdot \mathbf{v}\, dm$$

$$T = \frac{1}{2}\int \mathbf{v} \cdot \mathbf{v}\, dm$$



$\boldsymbol{\omega}_{xyz}$
$\mathbf{v}_{xyz}$
z
y
Z
x
Y
X
$\boldsymbol{\omega}$
$\mathbf{v}_O$
z
y
Z
x
Y
X



xyz is attached to the body, consequently

$$\mathbf{v}_{rel} = \mathbf{0}; \quad \mathbf{r}_{rel} = \mathbf{r}$$

$$\mathbf{v} = \mathbf{v}_o + \boldsymbol{\omega} \times \mathbf{r}$$

for any *dm*

$$\begin{aligned} T &= \frac{1}{2}\int (\mathbf{v}_o + \boldsymbol{\omega} \times \mathbf{r}) \cdot (\mathbf{v}_o + \boldsymbol{\omega} \times \mathbf{r})\, dm \\ &= \frac{1}{2} m v_o^2 + \mathbf{v}_o \cdot \boldsymbol{\omega} \times \int \mathbf{r}\, dm \\ &\qquad + \frac{1}{2}\int (\boldsymbol{\omega} \times \mathbf{r}) \cdot (\boldsymbol{\omega} \times \mathbf{r})\, dm \end{aligned}$$

$$\boldsymbol{\omega} \times \mathbf{r} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix}$$

$$= (\omega_y z - \omega_z y)\mathbf{i} + (\omega_z x - \omega_x z)\mathbf{j} + (\omega_x y - \omega_y x)\mathbf{k}$$

$$(\boldsymbol{\omega} \times \mathbf{r}) \cdot (\boldsymbol{\omega} \times \mathbf{r})$$

$$= \begin{pmatrix} \omega_x & \omega_y & \omega_z \end{pmatrix} \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$\mathbf{I} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

is the inertia tensor

Inertia tensor

$\mathbf{I}_O$ w.r.t. the fixed point O

$\mathbf{I}_G$ w.r.t. the mass centre G

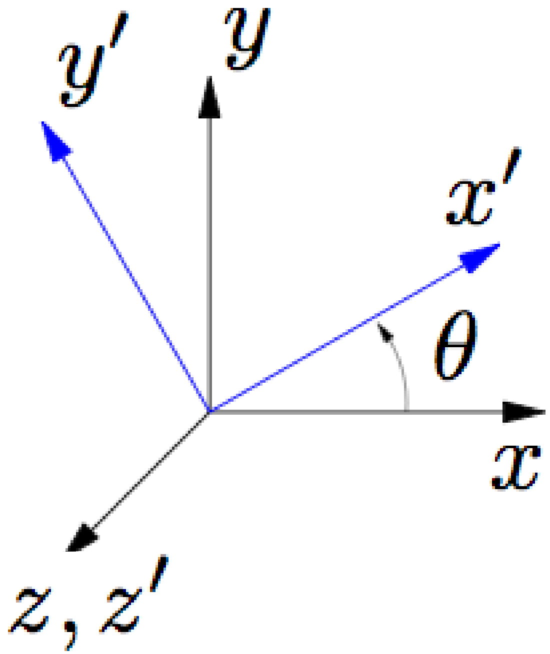
Fixed point rotation: $T = \frac{1}{2}\boldsymbol{\omega}^T \mathbf{I}_O \boldsymbol{\omega}$

General motion: $T = \frac{1}{2} m v_G^2 + \frac{1}{2}\boldsymbol{\omega}^T \mathbf{I}_G \boldsymbol{\omega}$

I depends on:

(a) Origin of coordinates
(b) Direction of axes

The coordinate system can be rotated



$$\mathbf{T} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = [\mathrm{T}] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$\mathrm{v}' = \mathrm{T}\mathrm{v}$$

$$\left[\mathrm{I}'\right] = [\mathrm{T}][\mathrm{I}][\mathrm{T}]^{T}$$

I is symmetric:

(a) **I** can be diagonalised (eigenvalues)
(b) Eigenvectors (principal directions) are orthogonal

which can be reached through a coordinate transformation

Principal moments of inertia:

$$\mathbf{I} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$$

Principal axes $\rightarrow$ Diagonal form

Symmetry axes are principal axes

Planar body in *xy*-plane:

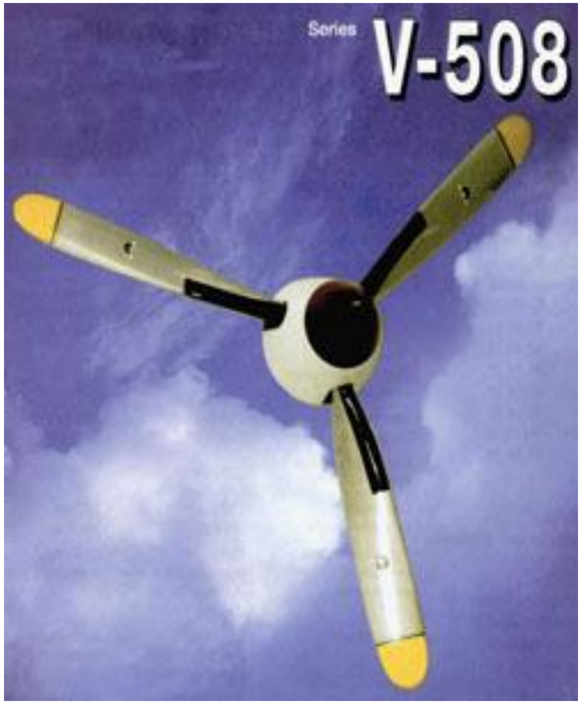
$$I_{zz} = I_{xx} + I_{yy}; \quad I_{xz} = I_{yz} = 0$$

Body with at least **three** symmetry axes in *xy*-plane:

$$I_{xx} = I_{yy}; \quad I_{xy} = 0$$

for any frame rotation about *z*-axis

Find the inertia tensor given the moment of inertia of one blade I_b about the shaft. Model the blade as a bar.



Virtual work

Please check the book!