Dynamics & Stability

AE3-914

Stability of Lagrangian systems

Equilibrium condition: $$\frac{\partial(V - T_0)}{\partial q} = 0$$

$$V_{\text{eff}} = V - T_0$$

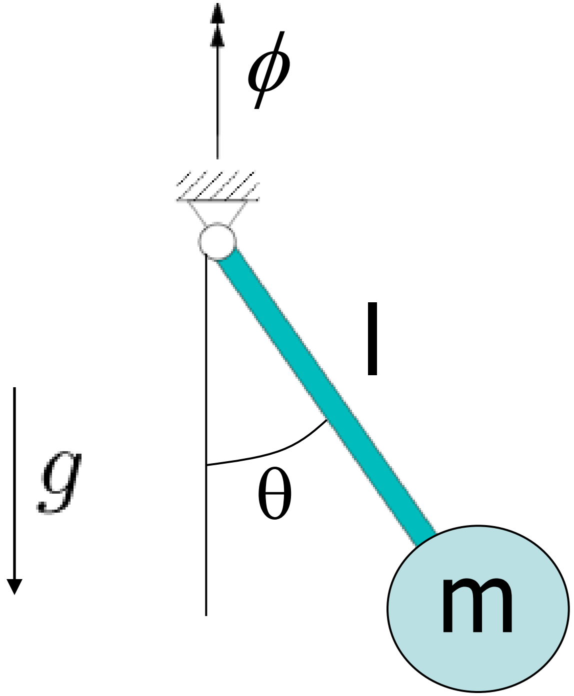
is defined as the **effective potential**

V_{eff} is a minimum: **STABLE**

V_{eff} is a maximum: **UNSTABLE**

The effective potential is also present in the Jacobi integral (Török, pg. 124)

$$h = T_2 - T_0 + V = T_2 + V_{\text{eff}}$$



Spherical pendulum

Is the steady motion stable?

Satellite system

$$V = -\frac{km}{r}$$

Dynamics of rotating bodies

Newton revisited

$$\sum \mathbf{F} = \dot{\mathbf{p}}$$

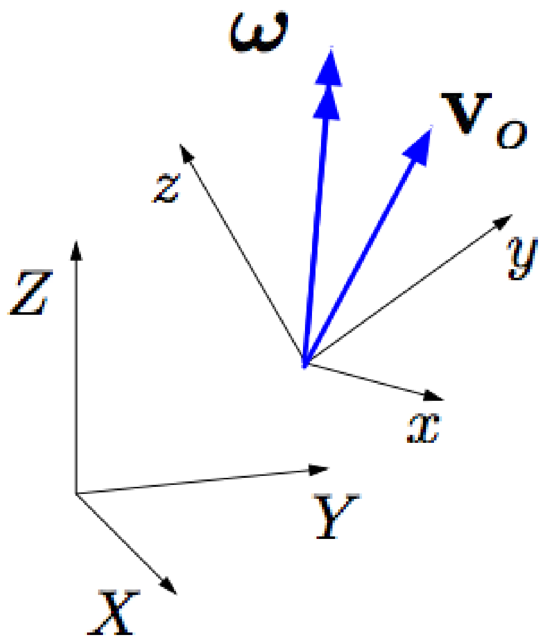
Newton’ s second law

$$\sum \mathbf{M}_O = \dot{\mathbf{L}}_O$$

[redacted]

Rigid body

$$\mathbf{p} = \int \mathbf{v} dm = m\mathbf{v}_G$$



xyz is attached to the body, consequently

$$\mathbf{v}_{rel} = \mathbf{0}; \quad \mathbf{r}_{rel} = \mathbf{r}$$

$$\mathbf{v} = \mathbf{v}_o + \boldsymbol{\omega} \times \mathbf{r}$$

for any dm

Angular momentum

$$\mathbf{L}_O = \int \mathbf{r} \times \mathbf{v} dm = \int \mathbf{r} \times (\mathbf{v}_O + \boldsymbol{\omega} \times \mathbf{r}) dm$$

Angular momentum

O is fixed or O=G

$$\mathbf{L}_0 = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \mathbf{I}_o \boldsymbol{\omega}$$

Newton’s second law

$$\sum \mathbf{M}_O = \dot{\mathbf{L}}_O$$

$$\mathbf{L}_O = L_x\mathbf{i} + L_y\mathbf{j} + L_z\mathbf{k}$$

Since **ijk** are rotating with angular velocity ω :

$$\dot{\mathbf{L}}_O = \dot{L}_x\mathbf{i} + \dot{L}_y\mathbf{j} + \dot{L}_z\mathbf{k} + \boldsymbol{\omega} \times \mathbf{L}_O$$

Take now principal axes of inertia:

$$\mathbf{I}_O = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$$

$$1 \equiv x$$
$$2 \equiv y \quad \text{(notation)}$$
$$3 \equiv z$$

$$\sum \mathbf{M_O} = \dot{\mathbf{L}}_O = I_1\dot{\omega}_1\mathbf{i} + I_2\dot{\omega}_2\mathbf{j} + I_3\dot{\omega}_3\mathbf{k} + \boldsymbol{\omega} \times (I_1\omega_1\mathbf{i} + I_2\omega_2\mathbf{j} + I_3\omega_3\mathbf{k})$$

Euler equations of motion

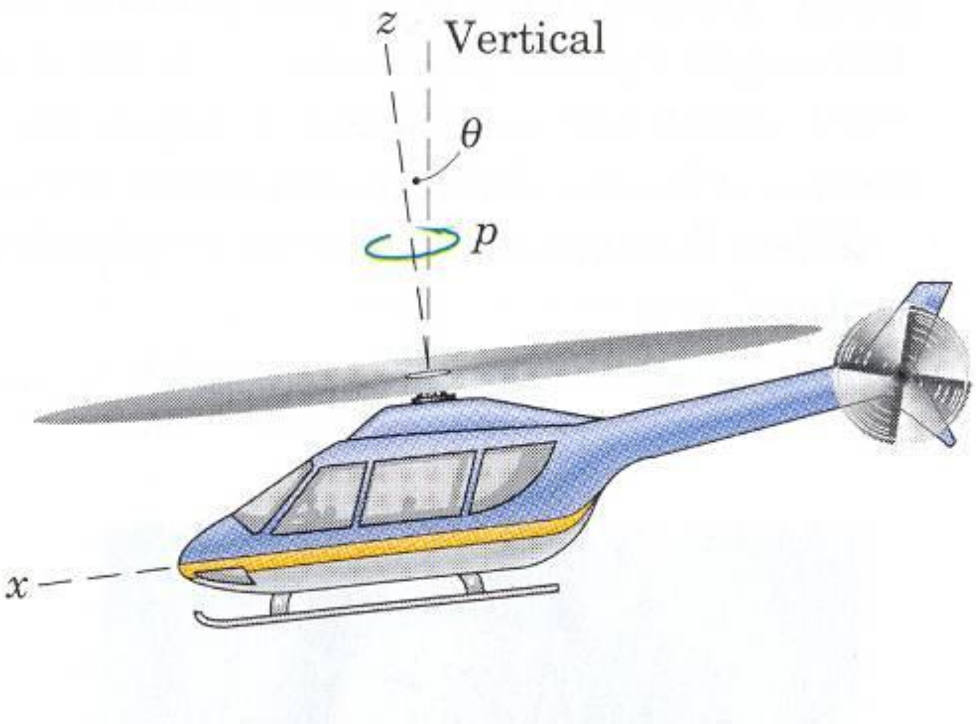
$$\sum M_1 = I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3$$

$$\sum M_2 = I_2\dot{\omega}_2 - (I_3 - I_1)\omega_1\omega_3$$

$$\sum M_3 = I_3\dot{\omega}_3 - (I_1 - I_2)\omega_1\omega_2$$

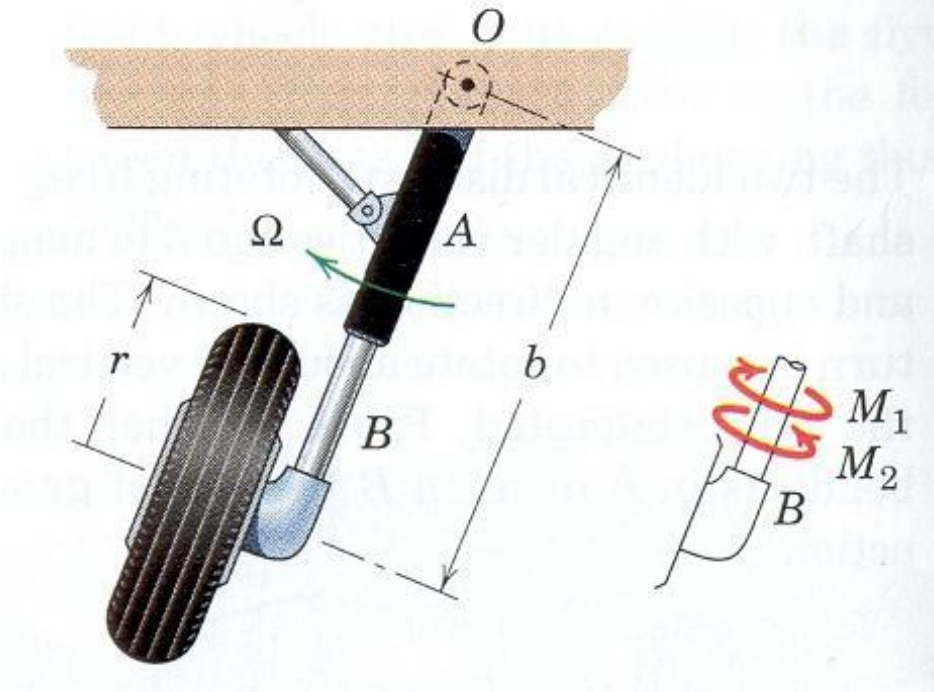
Unbalanced aft propeller

Consequences?



Pitching helicopter

What will the pilot experience?

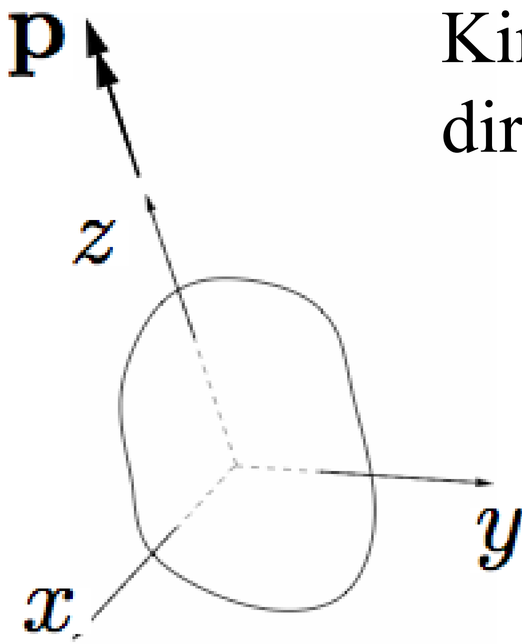


Radius of gyration k

Aeroplane took off with speed v

Torque M ?

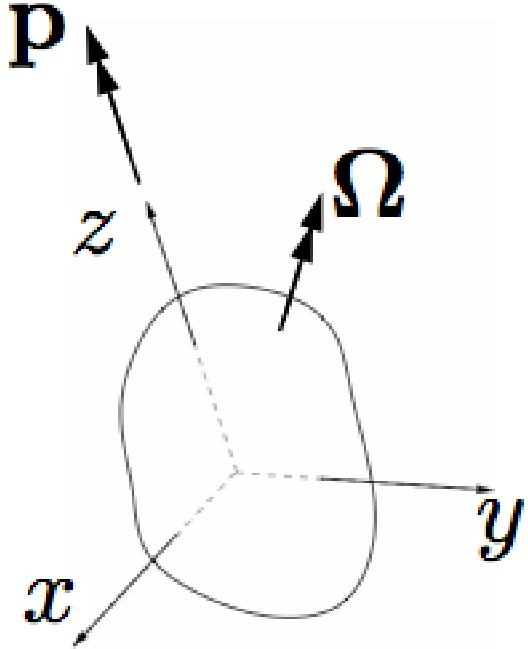
Kinematics of spinning body when direction of spin axis changes



$$\omega_1 = \omega_2 = 0$$

$$\omega_3 = p$$

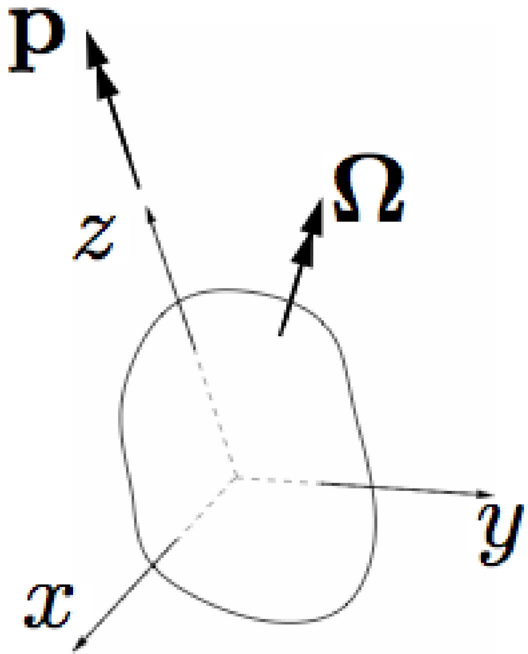
$$\dot{\omega}_1 = \dot{\omega}_2 = \dot{\omega}_3 = 0$$



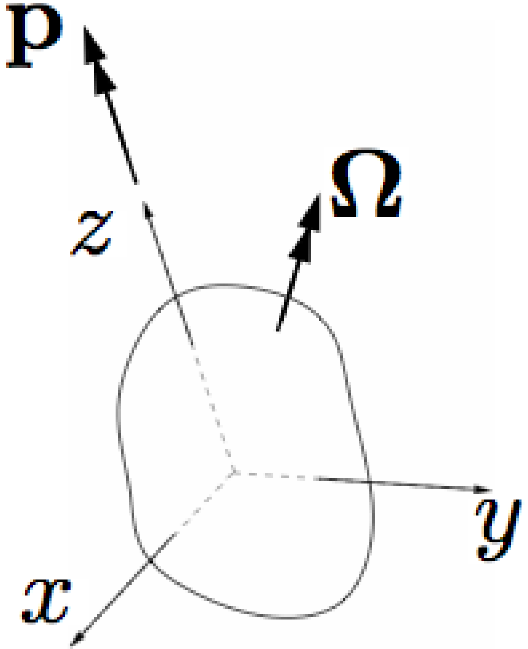
$$\omega_1 = \Omega_1$$

$$\omega_2 = \Omega_2$$

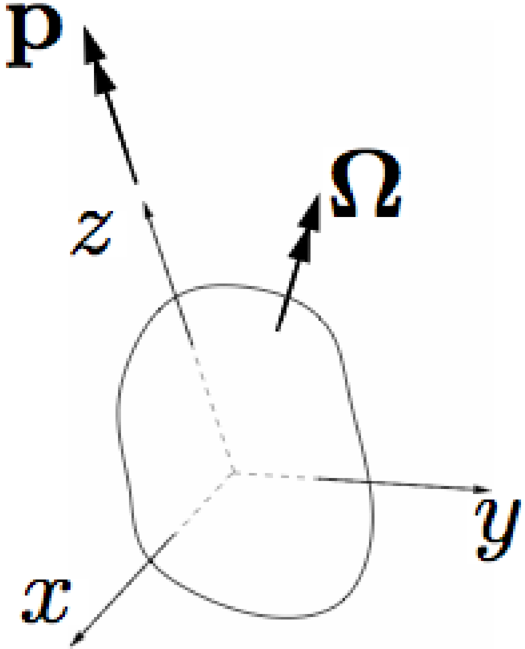
$$\omega_3 = \Omega_3 + p$$



$$\frac{d\boldsymbol{\Omega}}{dt} = \mathbf{0}$$



$$\dot{\Omega}_1\mathbf{i} + \dot{\Omega}_2\mathbf{j} + \dot{\Omega}_3\mathbf{k} + \boldsymbol{\omega} \times \mathbf{\Omega} = \mathbf{0}$$

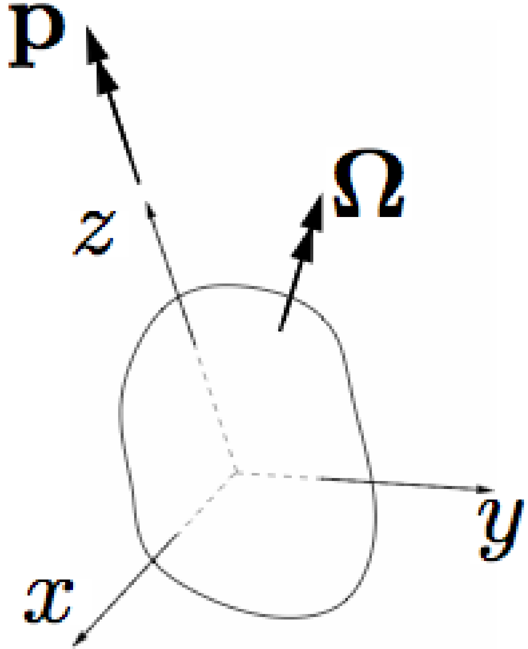


$$\dot{\Omega}_1 \mathbf{i} + \dot{\Omega}_2 \mathbf{j} + \dot{\Omega}_3 \mathbf{k} + \boldsymbol{\omega} \times \boldsymbol{\Omega} = \mathbf{0}$$

$$\dot{\Omega}_1 = p\Omega_2$$

$$\dot{\Omega}_2 = -p\Omega_1$$

$$\dot{\Omega}_3 = 0$$



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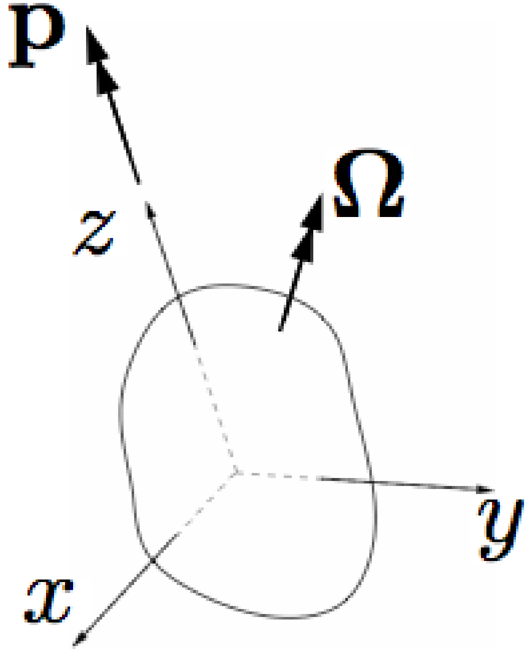
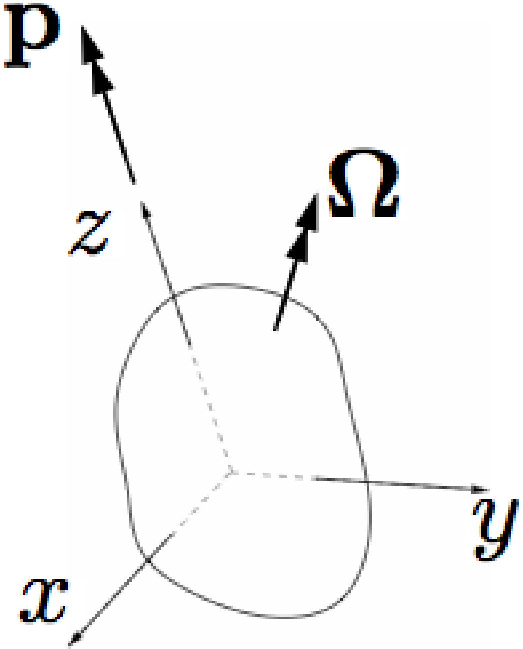


$$\dot{\omega}_1 = \dot{\Omega}_1 = p\Omega_2$$

$$\dot{\omega}_2 = \dot{\Omega}_2 = -p\Omega_1$$

$$\dot{\omega}_3 = 0$$



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