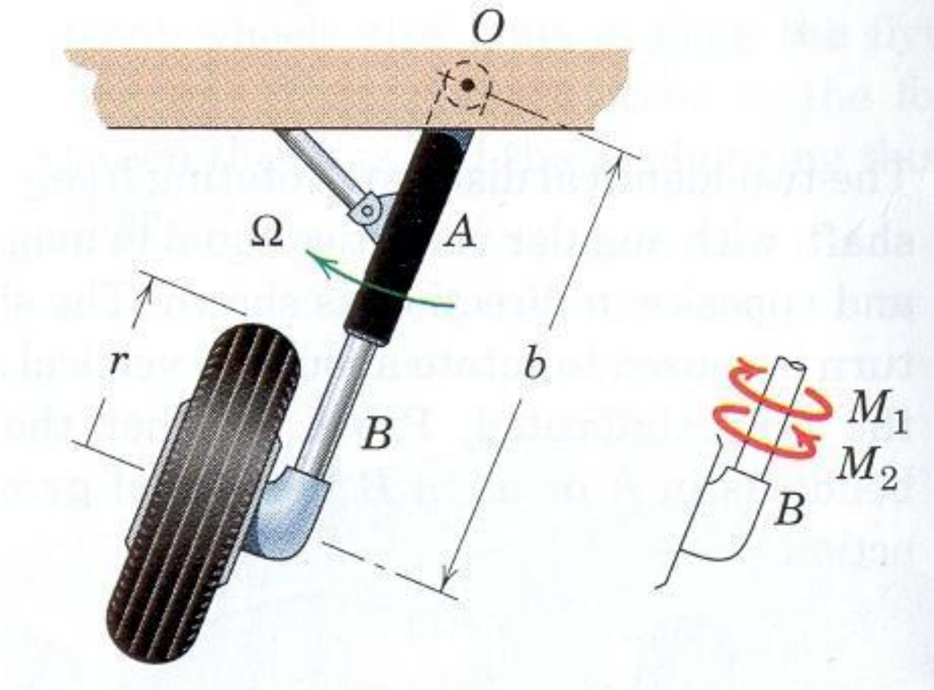
Dynamics & Stability

AE3-914

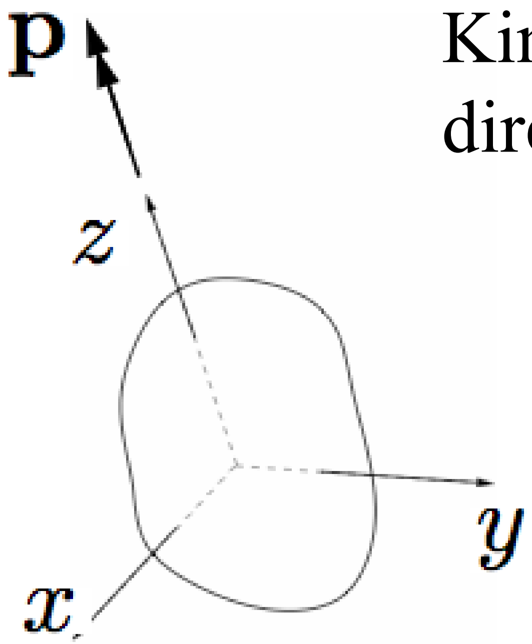


Radius of gyration k

Aeroplane took off with speed v

Torque M ?

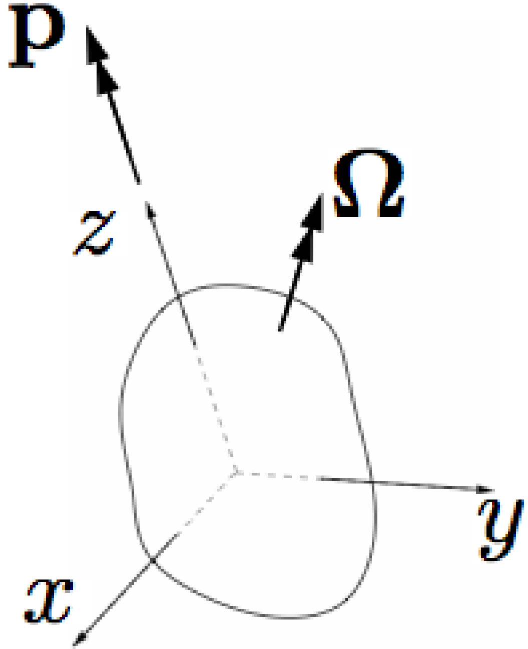
Kinematics of spinning body when direction of spin axis changes



$$\omega_1 = \omega_2 = 0$$

$$\omega_3 = p$$

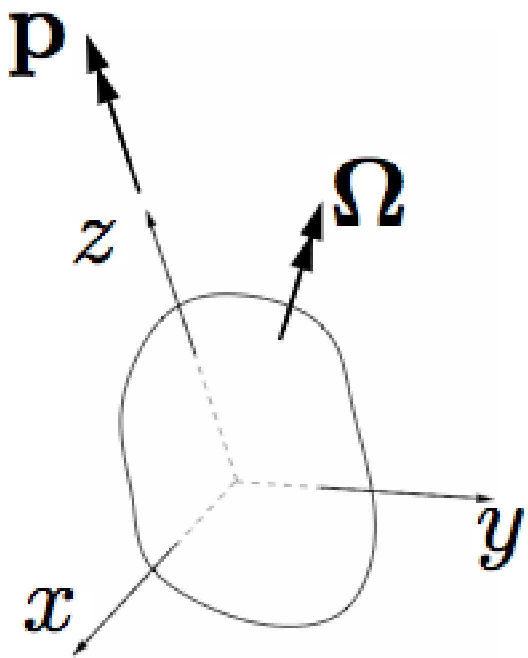
$$\dot{\omega}_1 = \dot{\omega}_2 = \dot{\omega}_3 = 0$$



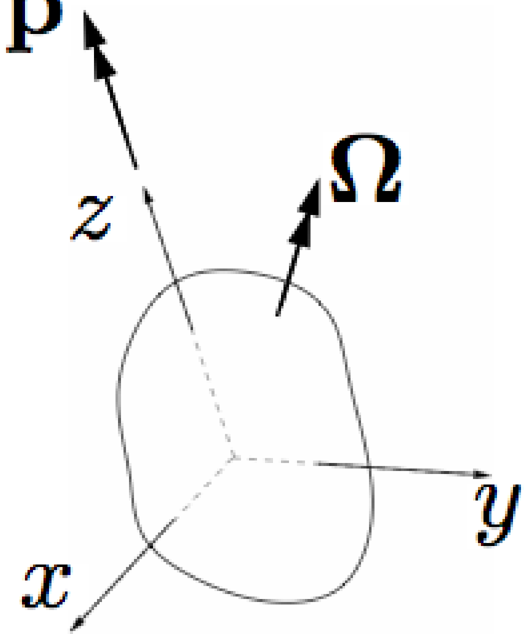
$$\omega_1 = \Omega_1$$

$$\omega_2 = \Omega_2$$

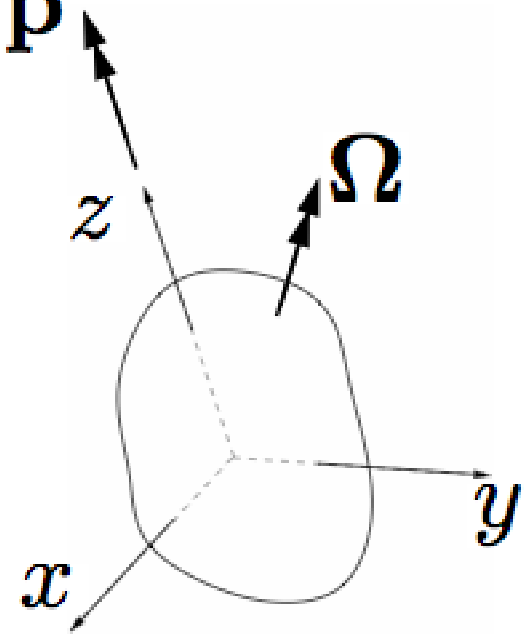
$$\omega_3 = \Omega_3 + p$$



$$\frac{d\mathbf{\Omega}}{dt} = \mathbf{0}$$



$$\dot{\Omega}_1\mathbf{i} + \dot{\Omega}_2\mathbf{j} + \dot{\Omega}_3\mathbf{k} + \boldsymbol{\omega} \times \boldsymbol{\Omega} = \mathbf{0}$$

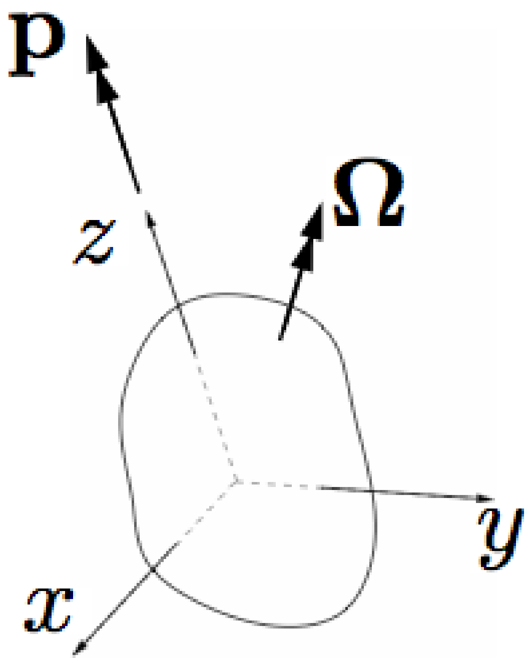


$$\dot{\Omega}_1 \mathbf{i} + \dot{\Omega}_2 \mathbf{j} + \dot{\Omega}_3 \mathbf{k} + \boldsymbol{\omega} \times \boldsymbol{\Omega} = \mathbf{0}$$

$$\dot{\Omega}_1 = p\Omega_2$$

$$\dot{\Omega}_2 = -p\Omega_1$$

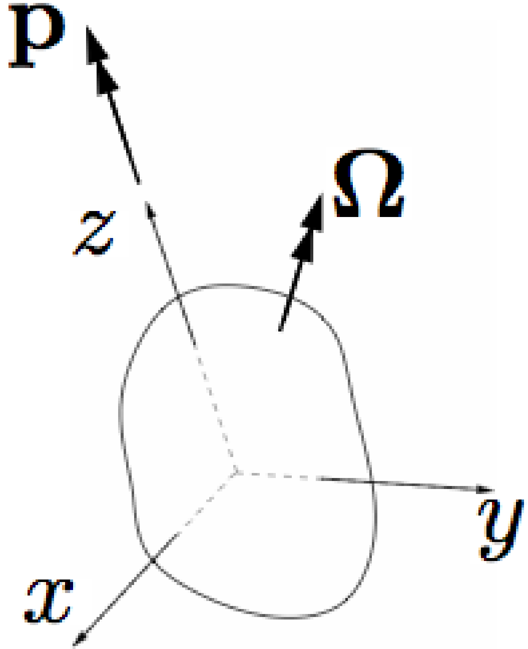
$$\dot{\Omega}_3 = 0$$



$$\omega_1 = \Omega_1$$

$$\omega_2 = \Omega_2$$

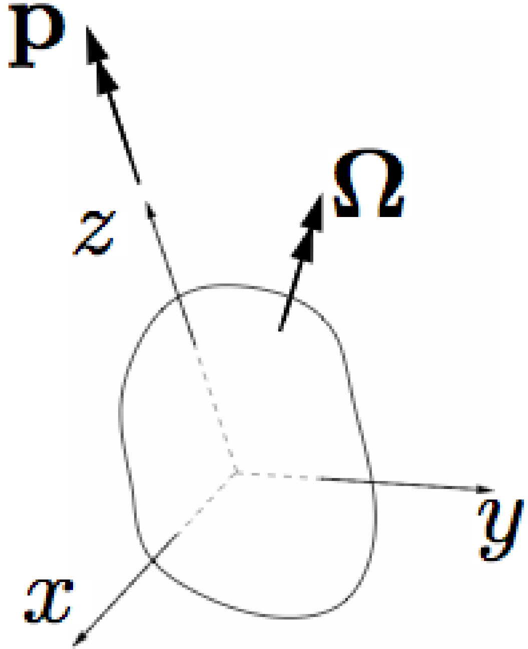
$$\omega_3 = \Omega_3 + p$$



$$\dot{\omega}_1 = \dot{\Omega}_1 \qquad = p\Omega_2$$

$$\dot{\omega}_2 = \dot{\Omega}_2 \qquad = -p\Omega_1$$

$$\dot{\omega}_3 = 0$$



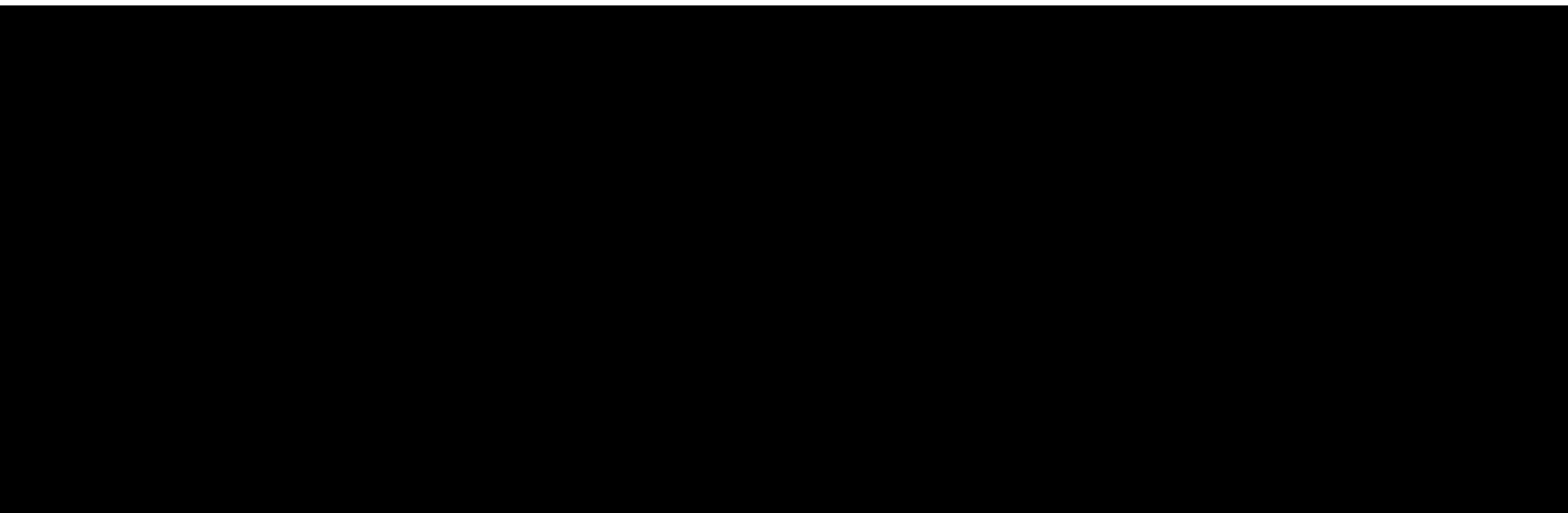
$$\dot{\omega}_1 = p\omega_2$$

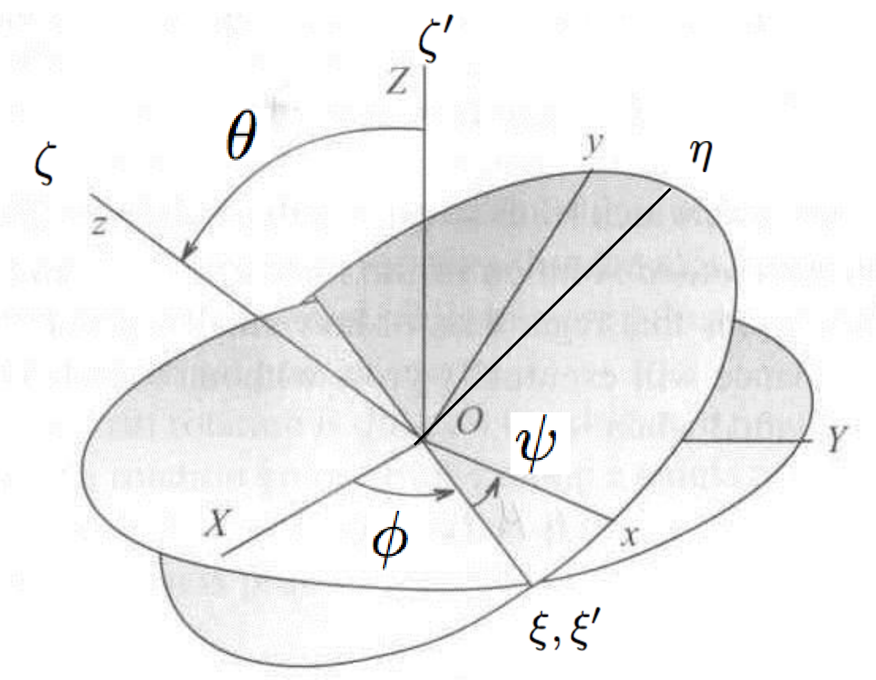
$$\dot{\omega}_2 = -p\omega_1$$

$$\dot{\omega}_3 = 0$$

Lagrangian dynamics

Euler angles




ζ'
Z
θ
ζ
z
y
η
O
ψ
Y
X
φ
x
ξ, ξ'

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\mathbf{R}(\psi) \qquad\qquad \mathbf{R}(\theta) \qquad\qquad \mathbf{R}(\phi)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} C_\phi C_\psi - S_\phi C_\theta S_\psi & S_\phi C_\psi + C_\phi C_\theta S_\psi & S_\theta S_\psi \\ -C_\phi S_\psi - S_\phi C_\theta C_\psi & -S_\phi S_\psi + C_\phi C_\theta C_\psi & S_\theta C_\psi \\ S_\phi S_\theta & -C_\phi S_\theta & C_\theta \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Kinetic energy (rotation)

$$\begin{aligned} T &= \tfrac{1}{2}\Big[I_1(\dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi)^2 \\ &\qquad + I_2(\dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi)^2 + I_3(\dot{\phi}\cos\theta + \dot{\psi})^2\Big] \end{aligned}$$

Gyrodynamics

$$I_3 = I_s \qquad\qquad I_1 = I_2 = I$$

$$T = \frac{1}{2}\left[I(\dot{\phi}^2 \sin^2\theta + \dot{\theta}^2) + I_s(\dot{\phi}\cos\theta + \dot{\psi})^2\right]$$

$$R\left(\theta,\dot{\theta},C_\phi,C_\psi\right)=-\frac{1}{2}I\dot{\theta}^2+\frac{\left(C_\phi-C_\psi\cos\theta\right)^2}{2I\sin^2\theta}+\frac{C_\psi^2}{2I_s}+mgl\cos\theta$$

$$V_{eff}(\theta)=\frac{\left(C_\phi-C_\psi\cos\theta\right)^2}{2I\sin^2\theta}+\frac{C_\psi^2}{2I_s}+mgl\cos\theta$$

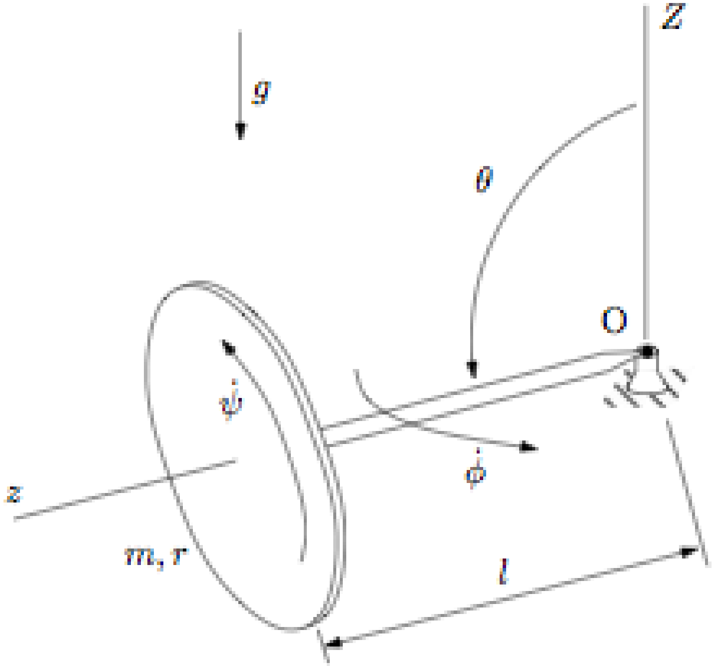
Eq. of motion: $$\frac{d}{dt}\left(\frac{\partial R}{\partial\dot{\theta}}\right)-\frac{\partial R}{\partial\theta}=0$$

Steady precession

$$\ddot{\theta} = \dot{\theta} = 0$$

$$\frac{\partial R}{\partial \theta} = \frac{dV_{eff}}{d\theta} = 0$$

There is an error in equation (4.38) and on top of page 223 of the textbook corresponding to this course.



Steady precession for $\theta = 90^\circ$?

Steady precession for $\theta = 60^\circ$?

This slide corresponds to a sample problem that can be found in the assignments section of this course